

# Closing out DVA?

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*Counterparty risk valuation often includes the debt value adjustment (DVA) component linked to an institution's own default which is viewed by some as contentious. However, there are several possible practical ways that an institution can monetise their DVA. One is, perversely, when their counterparty defaults where standard documentation seems to dictate that DVA may be considered in any amounts owed to or claims against the defaulted counterparty ("risky closeout"). However, doing this adds complexity to the already difficult problem of counterparty risk valuation. In this article, we analyse the complex interaction between CVA, DVA and closeout assumptions and the general accurate calculation of CVA and DVA with risky closeout assumptions. We also consider if there is a simpler formula that can be used as a good approximation for risky closeout, without the need for adding complexity to CVA quantification.*

## 1. Introduction

Institutions are allowed by international accountancy standards to consider their own default in the valuation of liabilities. This can be included by pricing of counterparty risk bilaterally, including what is often known as the DVA (debt value adjustment) component. DVA is a double edged sword. On the one hand, it creates a symmetric world where counterparties can readily trade with one another even when their underlying default probabilities are high. On the other hand, the nature of DVA and its implications and potential unintended consequences creates some additional complexity and potential discomfort. The controversy over DVA can be seen when comparing accountancy standards and capital rules. Whilst accounting rules (IFRS 13, FASB 157) require DVA, the Basel III committee have decided to ignore any DVA relief in capital calculations<sup>2</sup>.

An additional theoretical complexity brought about by the use of bilateral CVA (BCVA) is that it implies that the CVA alone depends on the credit quality of the institution in question. This is because the probability of default of the counterparty must be weighted by the probability that the institution has not previously defaulted. This captures the "first to default" nature of a contract with respect to the default of the institution and counterparty and avoids a double counting. However, it also means that even a pure asset appears to bear the credit risk of both parties which is counterintuitive. For this reason, some institutions calculate both CVA and DVA unconditionally. Whilst this appears somewhat naïve at first glance, we will show that this method is actually commonly rather close to the (more complex) actual case.

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<sup>2</sup> "Application of own credit risk adjustments to derivatives - consultative document", Basel Committee on Banking Supervision, <http://www.bis.org/press/p111221.htm>

The debate over DVA usage centres on whether or not institutions can monetise their own default. Ways to do this include selling CDS protection on similar counterparties<sup>3</sup>, buying back own debt and unwinding or novating trades (e.g. see Gregory [2009], Burgard and Kjaer [2011]). Another way to realise DVA is when closing out trades in the event of the default of the counterparty. In such a case, there is usually a realistic need to enter into replacement transactions with other market participants. Such market participants will charge CVA to the institution, which corresponds to the institution's own DVA on the replacement trade. Following this logic, it is possible to incorporate various material economic factors, such as DVA, into the closeout amount. Note that an institution will suffer the reverse experience when they default, since counterparties can deduct DVA from amounts owed (or add it to claims). There is therefore a need to fully understand the relationship between BCVA and closeout assumptions.

## 2. Closeout and DVA

In deriving the formulas for CVA and DVA, a standard assumption is that, in the event of default, the closeout value of transactions (whether positive or negative) will be based on risk-free valuation. This is an approximation that makes quantification more straightforward but the actual payoff is more complex and subtle. The more natural proxy for a closeout amount is the cost of replacing the transaction with another party. Documentation tends to follow this approach, for example, under ISDA (2009) protocol, the determination of a closeout amount "*may take into account the creditworthiness of the Determining Party*", which suggests that an institution may consider their own DVA in determining the amount to be settled. Documentation allows for actual transaction replacement costs to define the closeout amount, provided enough market makers were asked for bids on the transaction and majority of them look reasonably fair.<sup>4</sup> The DVA of the surviving party would seem reasonably included in this method since this would correspond to the CVA charged on a replacement trade in the market. Following Brigo and Morini [2007] and Carver [2007], we will refer to this assumption as "risky closeout".

Let us consider the situation when a counterparty defaults. The CVA has disappeared but the DVA component still remains. Suppose the valuation is negative, say  $-\$900$ , with a DVA component making it  $-\$800$ . A risk-free closeout would require the institution to pay  $\$900$  and also make an immediate loss of  $\$100$ . If the DVA can be included in the closeout calculation then the institution pays only  $\$800$  and has no jump in their PnL that would otherwise occur (Brigo and Morini [2010]). However, if instead the institution has a positive valuation of  $\$1,000$ , of which  $\$900$  is risk-free value and  $\$100$  is DVA<sup>5</sup> then a risk-free closeout amount is based on  $\$900$ , leading to a certain loss of  $\$100$ . On the other hand, a risky closeout allows a claim of  $\$1,000$  which matches perfectly<sup>6</sup>. There are two possible implications of these examples: either DVA should be ignored in valuation or it should be incorporated into the closeout procedure.

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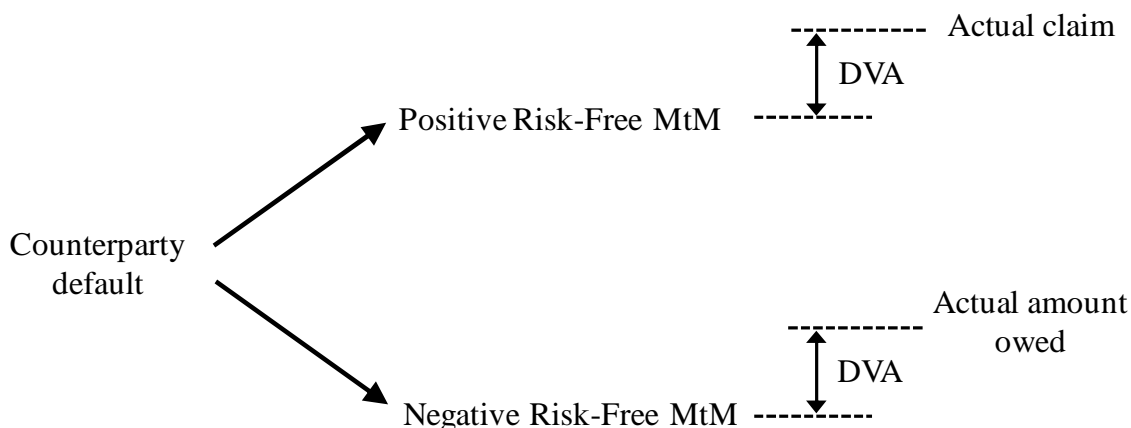
<sup>3</sup> Meaning those with credit spreads which are highly correlated to those of the institution.

<sup>4</sup> Market quotation in 1992 ISDA. Also included among broader options in the closeout definition of 2002 ISDA.

<sup>5</sup> This could arise from two outstanding payments where the institution receives  $\$1,900$ , and pays  $\$1,000$ . The  $\$100$  DVA is coming from this  $\$1,000$  payment.

<sup>6</sup> However, there is a problem with hedging as there will still be a loss due to the non-recovered amount of the DVA. The amount of CDS protection required is  $\$900 + \$100/(1-\text{Recovery})$ , where the second component hedges the DVA loss. Note that the CDS hedge depends on the highly uncertain recovery value.

In a world where DVA is considered real then its inclusion in the closeout amount, as is seemingly supported by standard documentation, makes sense. The new situation is represented in Figure 1, which summarises both cases of positive or negative risk-free mark to market (MtM) of the portfolio at the time of counterparty default. Here, a positive value leads to a claim on the amount owed, which includes the cost of DVA that would be incurred on a replacement transaction. Note that only a recovery fraction of this DVA will be received. A negative value requires a settlement of the amount to the counterparty which is offset by the DVA<sup>7</sup>.



**Figure 1.** Illustration of the impact of DVA on the closeout amount when a counterparty defaults.

An institution also needs to consider the symmetric case which occurs when they themselves default. In this case, the counterparty can increase their valuation in exactly the same way. To the institution, this *increase* in valuation from DVA appears as a *reduction* in valuation by CVA. Having CVA and DVA appear in their own payoff is complex but seemingly unavoidable. Indeed, similar effects occur in cases such as the exercise of physically settled options where the CVA and DVA of the underlying impact the exercise boundary<sup>8</sup>.

However, risky closeout has been violated historically. For, example in the *Peregrine Fixed Income Limited v Robinson Department Store plc* case (e.g. see Parker and McGarry 2009), the judge ruled that the Market Quotation method (used essentially to derive a risky closeout) did not produce a commercially reasonable result. In this case, the surviving party (Robinson) essentially received market quotes with significant DVA components due to their poor credit quality. The court ruling did not agree that this large DVA should be subtracted from the amount owed to the Peregrine. More recent evidence such as the bankruptcy of Lehman Brothers suggests that risky closeout is common, at least as long as the surviving party is not itself close to default. This has been likely aided by changes in the 2002 ISDA Master Agreement.

An intuitive criticism of risky closeout could be the lack of recognition of the CVA of the replacement transaction, i.e. the implicit assumption that the replacement counterparty is risk-free. Consider a dealer market with homogeneous credit quality and symmetric exposures.

<sup>7</sup> Note that this could turn the amount owed into a claim. This is accounted for in the formulas given below.

<sup>8</sup> See, for example, Arvanitis and Gregory [2001].

Here, CVA and DVA are reduced by the use of collateral and should in any case cancel so that the correct replacement cost (ignoring transaction costs) would simply be the risk-free amount. Other suggestions (see Dehapiot and Patry [2012]) could be that the fair replacement cost be defined with reference to the credit quality of the institution at the *start* of the transaction (only the change in DVA is considered)<sup>9</sup> or that the replacement counterparty is assumed to have the same credit quality as the surviving institution.

Despite the potential objections and the lack of any consistent practice in actual default situation, we will analyse risky closeout in more detail and show that it has some pleasant properties and may therefore be the most appropriate method to use from a theoretical point of view. In particular, we will show that the strong “first to default” effect of bilateral CVA valuation is largely removed when assuming risky closeout. However, in contrast to previous research, we will also show that risky closeout is not a perfectly clean theoretical solution in that aspects such as default correlation are still important.

### 3. Risky closeout

Risky closeout has been recently discussed by Brigo and Morini [2011] who show that the inclusion of DVA in the closeout amount generally leads to a more intuitive result than a risk-free closeout. These authors illustrate the impact on a zero coupon bond and discuss the special cases of independence and perfect correlation of default times. The zero coupon bond alone (one sided payoff profile) might be quite a limiting simplification, since it naturally neglects one side of the CVA/DVA pair. There are three potential ways in which to extend such an analysis. The first of these is to consider the impact of default correlation (and/or spread volatility and spread correlation) on the results. The second is to look at the recursive nature of this effect (the closeout amount has an impact on the current CVA and DVA *and* vice versa). The third and very important point of interest is to calculate the impact on bilateral derivatives exposures.

In order to account for risky closeout in counterparty risk valuation, an institution should quantify the additional gain arising when their counterparty defaults. This comes from two components; the first is an increased claim in the event of a positive future value (of which a recovery will be achieved). The second is a gain resulting from offsetting any amount owed by the DVA. The equivalent additional losses stemming from the counterparty using risky closeout in the event of the institution’s own default should also be considered. The four resulting cases are shown in Table 1 (these four cases are incorporated directly into the bilateral CVA formula in equation 8 in Brigo, Buescu and Morini [2011]).

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<sup>9</sup> This essentially happens in the bond market via the claim being on the par value and not the risk-free value of the bond. For example, suppose a risky bond is issued at par with a large coupon (compared to treasuries). The claim on default is on the par amount and therefore relates only to the *deterioration* in credit quality of the issuer. However, it would be difficult to define for derivatives since there is no associated measure comparable to a bond’s par value

**Table 1.** Comparison of payoffs using risk-free and risky closeout. Risk-free closeout is defined in the usual way via the risk-free mark-to-market (denoted MtM). In risky closeout, when the counterparty defaults, the institution increases the valuation by their own DVA (which is negative by convention). When the institution themselves defaults, the counterparty can reduce the valuation by their DVA (which from the institution's point of view is their CVA).  $R_C$  and  $R_I$  represent recovery values of the counterparty and institution respectively.

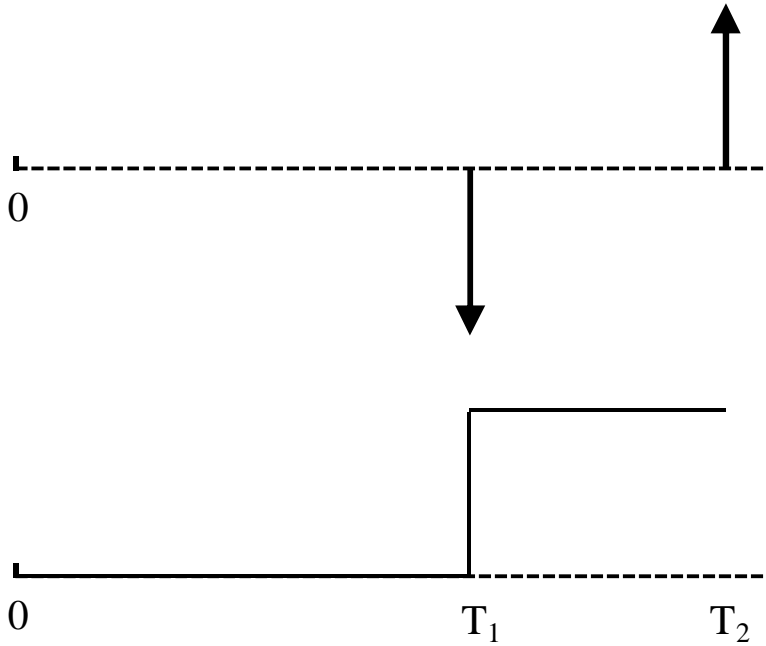
	Risk-free closeout		Risky closeout	
	Positive Exposure	Negative Exposure	Positive Exposure	Negative Exposure
Counterparty defaults	$R_C \times \text{Max}(\text{MtM}, 0)$	$\text{Min}(\text{MtM}, 0)$	$R_C \times \text{Max}(\text{MtM} - \text{DVA}, 0)$	$\text{Min}(\text{MtM} - \text{DVA}, 0)$
Institution defaults	$\text{Max}(\text{MtM}, 0)$	$R_I \times \text{Min}(\text{MtM}, 0)$	$\text{Max}(\text{MtM} - \text{CVA}, 0)$	$R_I \times \text{Min}(\text{MtM} - \text{CVA}, 0)$

#### 4. Simple example

Under the usual assumptions for pricing derivatives, valuation is the expectation of all future cashflows using some pricing measure. CVA and DVA are the corresponding changes in this expectation, coming from default of the counterparty or institution themselves. CVA and DVA are driven respectively by positive and negative valuations (from the institution's point of view). A good understanding of CVA, DVA and closeout interdependence should therefore come from analysing a simple case of cashflows in opposite directions. We therefore begin by looking at the relatively simple, but in no way restrictive example of two cashflows in opposite directions. The logic would be the same regardless of the sizes of those cashflows<sup>10</sup>, so to simplify the exposition we assume them to be equal.

Assume an institution pays a unit cashflow at time  $T_1$  and receives a unit cashflow at a later time  $T_2$  (Figure 2) We assume that both the institution ( $I$ ) and their counterparty ( $C$ ) can default and have associated fixed hazard rates of  $h_I$  and  $h_C$  respectively. Percentage recovery rates are given by  $R_I$  and  $R_C$  and interest rates are assumed to be zero. The exposure based on risk-free closeout is zero until  $T_1$  and +1 from  $T_1$  to  $T_2$ . The fact that the above case represents only positive exposure is not a concern due to the inherent symmetry of the problem (although we deal with the more general case below). The aim now is to compute the formula for the CVA.

<sup>10</sup> Unless the case degenerates due to very significant difference in cashflow sizes.



**Figure 2.** Illustration of the simple example showing the cashflows (top) and exposure (bottom).

Note that the representation below, for ease of exposition, assumes independence of defaults but the more general case is an easy extension, for example we can represent the hazard rates under conditional independence as in some factor model. We define  $F(T_1, T_2)$  as the default probability between dates  $T_1$  and  $T_2$  and  $S(T_1, T_2)$  as the associated survival probability. We denote the first to default probability and associated survival functions as  $F^1(\cdot)$  and  $S^1(\cdot)$  respectively. With a standard closeout based on the risk-free value of the claim, the CVA at time zero, which intuitively should reflect the fact that if the counterparty defaults first in the interval  $[T_1, T_2]$  then the institution makes a loss due to not receiving the final cashflow, can be written as:

$$(1 - R_C)h_c \int_{T_1}^{T_2} \exp(-(h_c + h_I)s) ds = (1 - R_C) \frac{h_c}{h_c + h_I} F^1(T_1, T_2), \quad (1)$$

where  $F^1(T_1, T_2) = \exp(-(h_c + h_I)T_1) - \exp(-(h_c + h_I)T_2)$  is the first to default probability within the interval  $[T_1, T_2]$ . The ratio  $h_c/(h_c + h_I)$  gives the probability that the counterparty is the first to default. This formula has an unpleasant dependence on the institution's own default probability via the first to default probability. As the institution's default probability increases, the CVA tends to zero.<sup>11</sup>

Let us now look at the impact of "risky closeout" (including DVA) on the above calculation. If the institution defaults, the counterparty will include DVA (from the institution's point of view). We have to therefore consider two additional components corresponding to the two different time periods<sup>12</sup>.

<sup>11</sup> We note that if we consider the risk-free closeout, this case is not different from the zero bond case mentioned above, apart from the reduction in the relevant time frame.

<sup>12</sup> Note that we do not need to consider the default of counterparty because they cannot be a creditor in this example.

*i) Institution defaults first in the period  $[0, T_1]$ .*

Here the counterparty will claim their DVA benefit (which is the institution's CVA) but will receive only a recovery fraction of it. This requires an addition term of:

$$R_I h_I \int_0^{T_1} CVA_{\tau_I=s}(s) \exp(-(h_c + h_I)s) ds,$$

which evaluates the CVA component at the default time of the institution. Since the institution has defaulted, its hazard rate will drop to zero<sup>13</sup> and the CVA will become  $CVA_{\tau_I=s}(s) = (1 - R_C)[\exp(-h_c(T_1 - s)) - \exp(-h_c(T_2 - s))]$ . Substituting this into the above and integrating again, we obtain:

$$R_I(1 - R_C)F_C(T_1, T_2)F_I(0, T_1). \quad (2)$$

The intuition behind this is that if the institution defaults before  $T_1$  and then the counterparty defaults in the interval  $[T_1, T_2]$  then the counterparty will claim their DVA on the remaining cashflows and the institution (because they are in default) will pay only a recovery fraction of this. Another way to look at this is to consider how much it will cost the counterparty to replace the transaction in case of the institution defaulting prior to  $T_1$ . A party providing the replacement transaction will have to assess the probability of the counterparty default in the interval  $[T_1, T_2]$  and will incorporate this in the price.

*ii) Institution defaults first in the period  $[T_1, T_2]$ .*

Here the counterparty will subtract their own DVA from the unit payment they are obliged to make. Since they owe the institution then there is no recovery value as in the previous case. This gives an additional term of:

$$h_I \int_{T_1}^{T_2} CVA_{\tau_I=s}(s) \exp(-(h_c + h_I)s) ds,$$

The CVA at this point will be  $CVA_{\tau_I=s}(s) = (1 - R_C)[1 - \exp(-h_c(T_2 - s))]$ . Again evaluating the integral gives:

$$(1 - R_C) \left[ \frac{h_I}{h_c + h_I} F^1(T_1, T_2) - S_C(0, T_2)F_I(T_1, T_2) \right] \quad (3)$$

The probability in the brackets gives the probability that the institution defaults in the interval  $[T_1, T_2]$  and the counterparty defaults second but before  $T_2$ . The CVA with risky closeout is found by adding the terms in equations (1-3) above, giving:

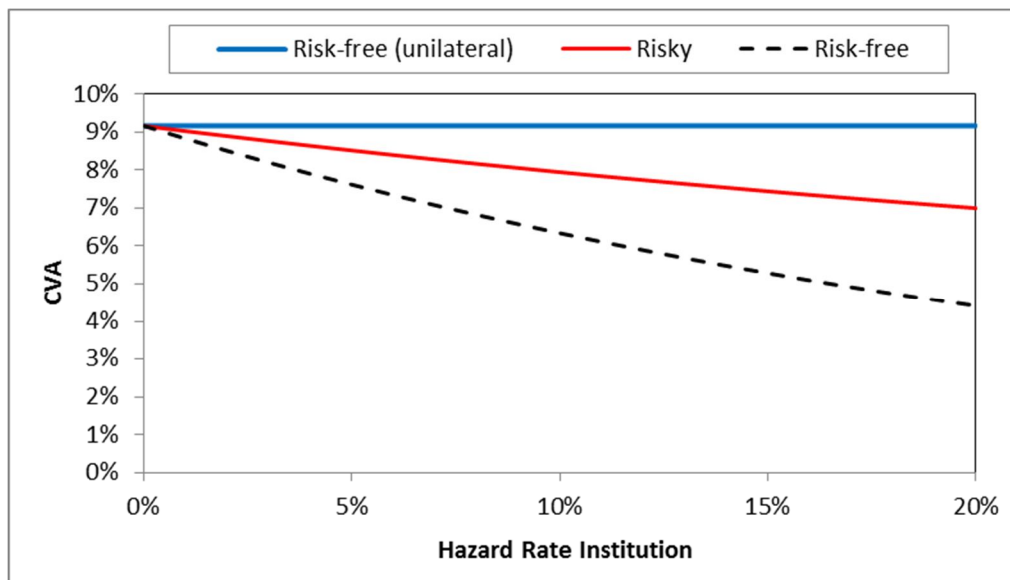
$$= (1 - R_C)F_C(T_1, T_2) - (1 - R_I)(1 - R_C)F_C(T_1, T_2)F_I(0, T_1), \quad (4)$$

where the first term is the unilateral CVA (without any reference to the institution credit quality) whilst the second is a correction due to the fact that, in the event of the institution's own default, the counterparty may claim a recovery fraction of their DVA benefit. As in previous research, the natural approximation for the CVA with risky closeout is the unilateral one given by the first term in equation (4). However, in this bilateral example, there is an

<sup>13</sup> This arises since we assume the replacement trade will be with a risk-free counterparty.

adjustment term which will be significant unless  $F_I(0, T_1)$  is small. However, in a more symmetric case (negative exposure also) then there will be an opposite term that will counteract this effect. We will look at the more general case below. If  $T_1 = 0$ , or equivalently, when the institution has no liability then, as also shown by Brigo and Morini [2011], the risky closeout in the case of a fixed exposure leads to the more natural CVA formula (the term on the left hand side only) with no sensitivity to the institution's own hazard rate.

The above shows that an unnatural consequence of DVA, which is that an institution's own default probability defines the value of a claim they have, can be partially corrected for by the correct choice of documentation (risky rather than risk-free closeout). However, whilst this seems to work perfectly in the asset only case (only positive exposure as discussed by Brigo and Morini [2011]), it does not in a more general case. In Figure 3, we compare the different closeout assumptions for this simple example. The three cases shown are risk-free closeout (equation 1), risky closeout (equation 4) and risk-free unilateral (the first term in equation 4). The risky closeout is between the extremes of risk-free closeout (including the survival probability of the institution) and unilateral risk-free closeout (excluding this survival probability).



**Figure 3.** CVA for the simple two cashflow example with  $T_1=2.5$  years and  $T_2 = 5$  years, computed with both risk-free and risky closeout as a function of the hazard rate of the institution. The counterparty hazard rate is 8.33% and recovery rates are 40%.

## 5. Impact of correlation

In the case with correlation, the product of probabilities (coming from zero correlation assumption) should be replaced with proper joint default probabilities. But equation (4) and logic behind it remains the same. One way to compute the correct conditional probabilities is via Monte Carlo simulation. However, a faster approach is via the computational of conditional survival probabilities. The most common way to correlate defaults is via the Gaussian copula approach attributed to Li (2000). We need to compute the conditional survival probability of the institution at the default time of the counterparty (and vice versa).



Following Laurent and Gregory [2005], this can be done via the conditional survival probability function:

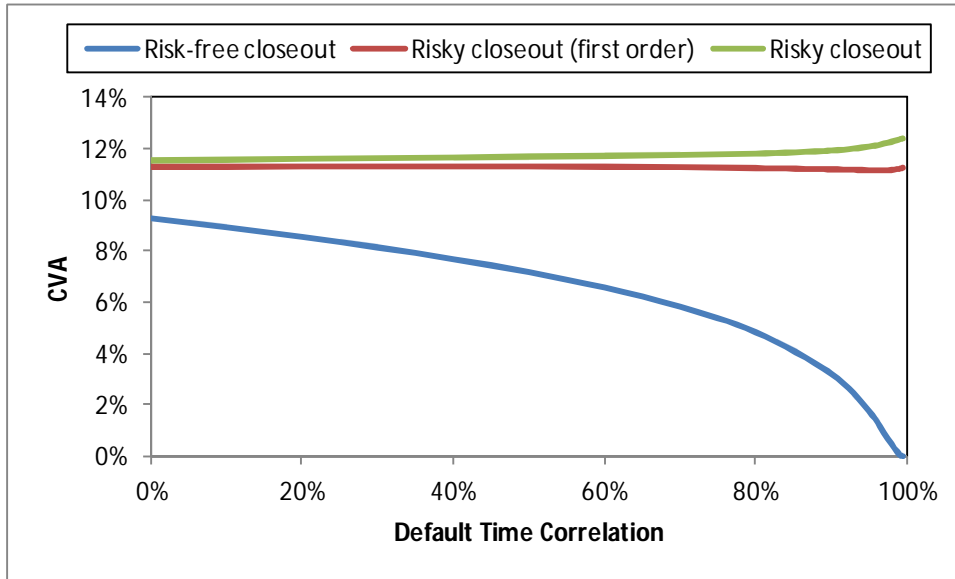
$$Q(\tau_I \geq t_2 | \tau_C = t_1) = \frac{\int_{-\infty}^{\infty} \Phi\left(\frac{\sqrt{\rho(1-\rho)}u - \rho\Phi^{-1}(S_C(0, t_1)) + \Phi^{-1}(S_I(0, t_1))}{\sqrt{1-\rho}}\right) \varphi(u) du}{\int_{-\infty}^{\infty} \Phi\left(\frac{\sqrt{\rho(1-\rho)}u - \rho\Phi^{-1}(S_C(0, t_1)) + \Phi^{-1}(S_I(0, t_2))}{\sqrt{1-\rho}}\right) \varphi(u) du},$$

where  $\rho$  is the correlation parameter,  $\varphi(\cdot)$  represents a normal distribution density function and  $\Phi(\cdot)$  and  $\Phi^{-1}(\cdot)$  represent the corresponding cumulative distribution function and its inverse. This model incorporates a jump in the default of the surviving party at the default time of the correlated party. We note that there is no other randomness of credit spreads and other approaches could be adopted. However, this will capture the main point of interest, which is the potential spread widening of the institution at the default of their counterparty (and vice versa).

## 6. Non linearity

Another aspect to consider is that the calculation above is only a first order approximation to the actual situation. In reality, the CVA (or equivalently DVA) defined at the time of closeout should naturally include the value of any future closeout adjustments (on the replacement transaction). This leads to a recursive problem. An obvious way to try and solve this is simply to calculate the above integrals numerically and iteratively solve until a convergence is reached.

We illustrate this with another example. Assume an exposure linked to a single unit cashflow in 5-years time. In the base case scenario, the hazard rates are assumed to be  $h_I=8.33\%$  and  $h_C = 4.17\%$  with  $R_I=R_C = 40\%$  with zero interest rates. This corresponds approximately to CDS spreads on the institution and counterparty of 500 bps and 250 bps respectively. The CVA with risk-free closeout is 9.29% and with risky closeout increases to 11.28%. The results as a function of correlation are shown in Figure 4. The risk-free closeout CVA reduces to zero as the correlation increases which is due to a well-known aspect of such a model where defaults become monotonic at 100% correlation, meaning that the more risky institution is certain to default first and therefore there is no CVA. With risky closeout, this (perhaps strange) effect is counteracted and the CVA is correlation independent in the first order case. However, true CVA based on risky closeout has some sensitivity to correlation and increases to 12.41% as the default times become comonotonic. This is because we assume the impact of risky closeout would itself be included in the price of a replacement transaction (and so on). Hence, even in the unilateral case, the evaluation of risky closeout is not trivial.



**Figure 4.** Illustration of the impact of risky closeout on the CVA for a single cashflow as a function of the correlation between the default of the institution and their counterparty.

## 7. Bilateral example

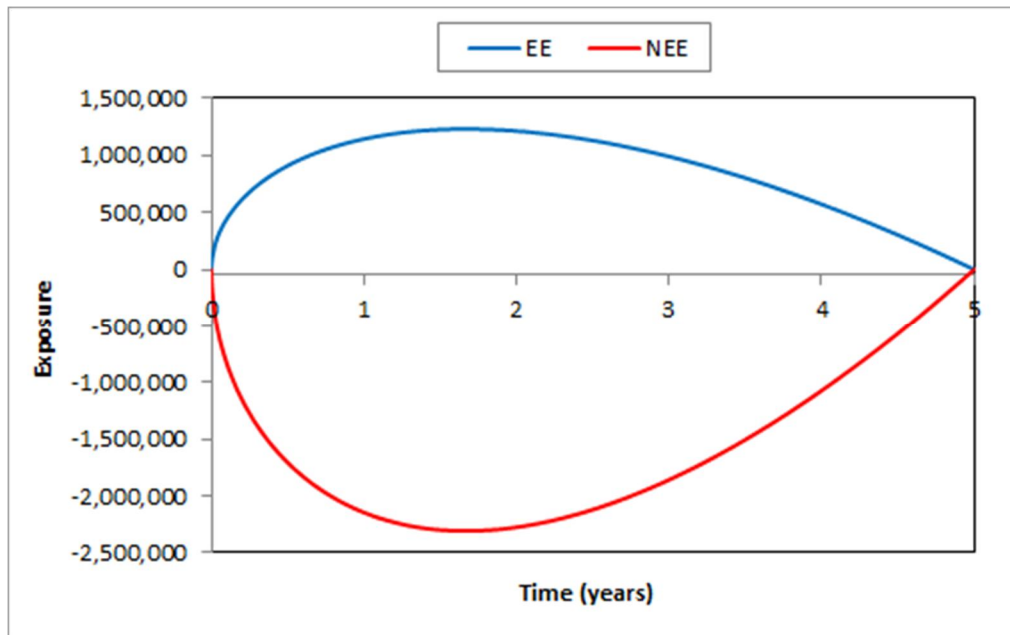
We now take an example with bilateral exposures based on the exposure profiles shown in Figure 5 which are representative of a typical swap<sup>14</sup>. In this portfolio the negative expected exposure (which drives the DVA) is greater in absolute terms than the expected exposure (which drives the CVA). If we assume that  $h_c=8.33\%$  and  $h_I = 4.17\%$  so that the counterparty is more risky than the institution then this gives an case where the CVA and DVA are approximately equal and opposite and the BCVA is close to zero (Table 2). We also show the unilateral values (UCVA and UDVA) which arise from not including survival probabilities of the counterparty and institution in the CVA and DVA formulas respectively<sup>15</sup>. There is a reasonably significant difference between the BCVA and UBCVA and it is important to consider which is the most appropriate calculation to use.

**Table 2.** Unilateral and bilateral CVA values and the corresponding unconditional values for the swap portfolio assuming independence between default events.

Conditional		Unconditional	
CVA	149,800	UCVA	162,407
DVA	-140,213	UDVA	-165,179
BCVA	9,587	UBCVA	-2,772

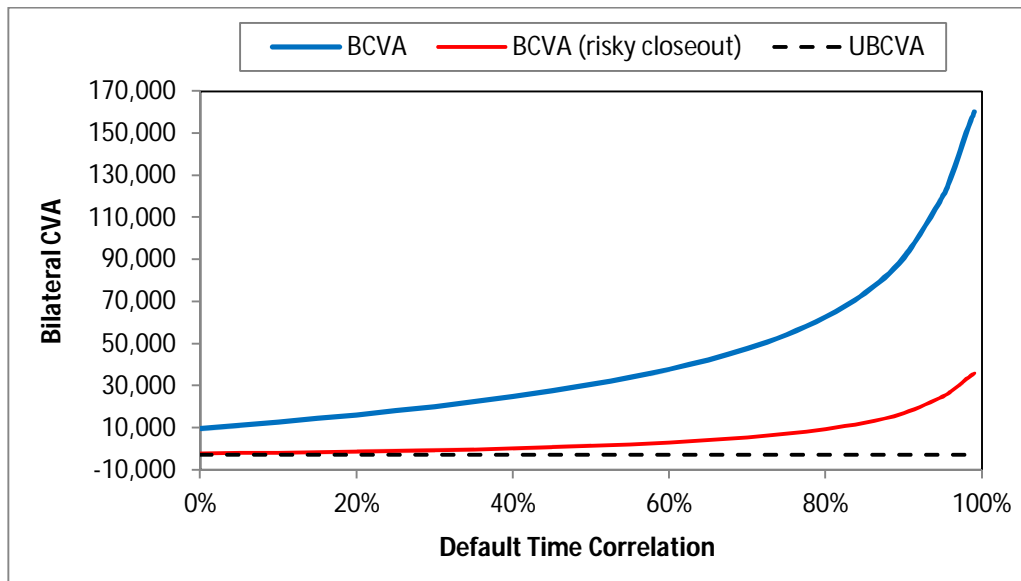
<sup>14</sup> These profiles are generated via  $EE(t) = -0.25(T-t)\sqrt{t}\Phi(-0.25) + (T-t)\sqrt{t}\phi(-0.25)$  and  $NEE(t) = -0.25(T-t)\sqrt{t}\Phi(0.25) - (T-t)\sqrt{t}\phi(0.25)$  which arises via the assumption that the future value at each date  $t$  is normally distributed with mean  $-0.25 \times (T-t)\sqrt{t}$  and standard deviation  $(T-t)\sqrt{t}$ .

<sup>15</sup> Note that closeout is always risk-free in such a case since it is assumed (inconsistently) that the second party to default is risk-free.



**Figure 5.** *Expected exposure (EE) and negative expected exposure (NEE) profiles used for the bilateral calculations.*

The impact of correlation on the BCVA (Figure 6) shows a strong effect with BCVA increasing towards the unilateral value as the correlation increases to 100%. This is due to the aforementioned comonotonic feature where the most risky name is certain to default first and therefore the DVA benefit is lost. The “first to default” impact on BCVA is clearly very significant. On the other hand, the results of the BCVA with a risky closeout (including the impact of DVA and CVA and the recursive effect) show that default correlation now has a much smaller impact on the BCVA. This is due to the fact that the institution can benefit from their DVA even in the event that the counterparty defaults first. For example, consider the large difference at a high correlation value. Here, the more risky counterparty is significantly more likely to default first and the institution’s DVA benefit is likely to be large at this time since their CDS spread is expected to have widened. Under risk-free closeout, a large amount of the DVA benefit is lost causing the overall BCVA to be large and positive but under risky closeout the DVA benefit can be realised even if the counterparty does default first.

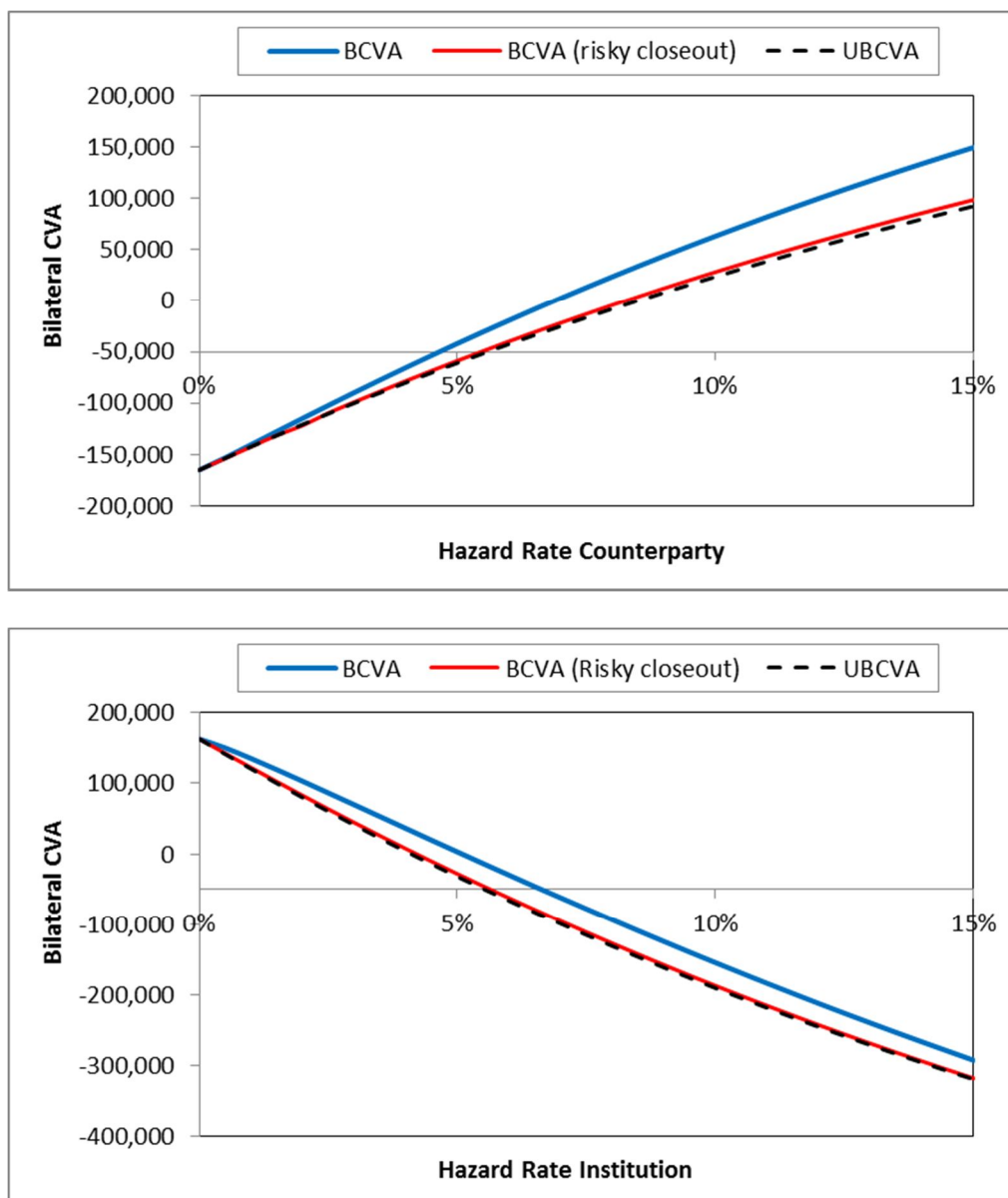


**Figure 6.** Illustration of the impact of risky closeout on the BCVA for the bilateral example as a function of the correlation between the default of the institution and their counterparty. Also shown is the approximation arising from using the unconditional BCVA (UBCVA).

Finally, we consider the unconditional result via UBCVA (calculated unilaterally and therefore without reference to the survival probabilities of the non-defaulting party). Figure 7 shows the same quantities as a function of the hazard rate of the counterparty and institution for a fixed default time correlation of 50%. Whilst this method is theoretically inconsistent, since it does not reflect the first to default nature of the problem, it gives very close (although not perfect) agreement with the case of risky closeout<sup>16</sup>. Whilst it should not be expected that UBCVA would agree perfectly with the more complex risky closeout calculation (as illustrated for example in the simple case represented by equation 4), it does give a much better agreement than the BCVA calculation.<sup>17</sup>

<sup>16</sup> We have verified that this is also true for other cases, including actual portfolios.

<sup>17</sup> However, there are cases where the UBCVA formula is not as close and the zero correlation BCVA may be a better approximation. An example is an exposure profile that flips dramatically in sign such as in the simple example in section 4. Such situations may occur, for example, with cross-currency swap portfolios.



**Figure 7.** Illustration of the impact of risky closeout on the BCVA for the bilateral example as a function of the hazard rate of the counterparty (top) and institution (bottom). Also shown is the approximation arising from using the unconditional BCVA (UBCVA). A fixed default time correlation of 50% is used.

## 8. Conclusion

In this article, we have discussed the impact of closeout conventions on CVA and DVA computation. One of the unpleasant features of quantifying bilateral counterparty risk is that it introduces a dependence on an institution’s own default probability even in the case of the institution having no liability. For similar reasons, the bilateral CVA can be very sensitive to the default correlation between the institution and their counterparty due to the “first to default” nature of the problem.

Risky closeout tends to cancel out some of the complicated features created by the use of DVA. In particular the impact of correlation between defaults is far less important. However, the impact of risky closeout does not completely cancel the dependence and it is still difficult to quantify it precisely, since it depends on the precise distribution of default times. This should not be surprising as, whatever the closeout, when the defaulted party has some obligations at default there is always a jump between some risky valuation and a recovery value. Furthermore, there is a non-linear effect for CVA/DVA similar to the impact of aspects such as option exercise where the CVA and DVA depend on their value at a future default time and vice versa. The correct solution therefore involves a recursive calculation. However, we have shown that a more simple calculation using unconditional CVA and DVA (UBCVA), as used by some market practitioners, gives a result which is typically close to this much more complex although this is not always the case in some extreme cases (as seen in Figure 3).

Risky closeout can be interpreted as market standards evolving to partially counteract unnatural features of accounting regulations around DVA. Excluding DVA on its own completely, as seemingly favoured by the Basel committee would seem to be an even simpler solution to the problem.

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