CDO models: Opening the black box – Part four

Coping with Copulas .. Managing tail risk

October 2008

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Structured credit research: Global

Dresdner Kleinwort

A risk management tool for CDOs

Analysing default risk of credit portfolios and CDOs

- In our CDO model series we have so far released various models with increasing flexibility and sophistication
- While they all have the advantage of being analytical / closed form solution models, they require simplifying assumptions with respect to the way credits may default together
- Most importantly, they all use a factor model to generate joint default events
- If the way credits default together is introduced without a factor model:
    - We can't apply the recursive algorithm anymore (as it depends on conditioning on a common factor)
    - However, Monte Carlo (MC) techniques can be used to simulate joint default events
- Within a MC approach, copulas allow for a very general and flexible way to directly model the dependency within the portfolio
- The current credit crisis has highlighted the importance of tail events in risk management. This model gives credit investors the tool to analyse the behaviour of their credit portfolio under stressed market conditions

Copula model:

https://research.dresdnerkleinwort.com/document/FILE.pdf?SYSTEM=1020&REF=249388

Keep in mind the fine difference between correlation and dependency!!!

- In everyday usage, the terms correlation and dependency are often used interchangeably
- Linear correlation is the correlation measure we are used to (from the Markowitz portfolio theory)
- However, linear correlation will not be able to capture other forms of dependency correctly
- When we leave the Gaussian world, the linear correlation measure looses its validity and has to be handled with care

Section 1

Copula 101

Dresdner Kleinwort

The market's choice…

Copulas have become the workhorse of dependency modelling

- Since its introduction to credit derivatives pricing by Li (1999), copulas have become the market's choice to model the dependency structure inherent in credit portfolios
- The reasons for that are manifold and include ease of implementation and calibration
- The main benefit however is the simple and intuitive way in which even complex dependency structures can be incorporated into a generic model
- The results are dependent default times, which also can be easily calibrated to individual, market based, credit spreads
- This is achieved by splitting the description of the specific default behaviour of each credit in the portfolio (the marginal distributions) from their joint behaviour. This means that we can model separately the individual marginals and the correlation structure (or in more general terms their dependency structure)
- For simplicity, we restrict our model to cases where the marginal and the joint distribution are of the same kind, while with very little extra work, the user can modify this aspect
- Copula models are powerful tools for pricing and risk management of different correlation products, ranging from FTD's to synthetic CDO tranches. In all cases the loss distributions are derived from the simulated default times, based on the assumptions used for spreads, recoveries and dependency

What is a Copula? The maths..

Copulas allow expressing joint probability distributions independent from the shape of their marginals..

- The joint distribution function C of m uniform random variables $U_1, U_2, \ldots, U_m$ can be referred to as copula function:

$$C(u_1, u_2, \ldots, u_m, \rho) = \Pr\left[U_1 \leq u_1, U_2 \leq u_2, \ldots, U_m \leq u_m\right]$$

- A set of univariate marginal distribution functions can be linked via a copula function, resulting in a multivariate distribution function:

$$C\left(F_1(x_1), F_2(x_2), \ldots, F_m(x_m)\right) = F(x_1, x_2, \ldots, x_m)$$

- Sklar's theorem (1959) established the converse, showing that any multivariate distribution function F can be expressed via a copula function
  - For any multivariate distribution, the univariate marginal distributions and their dependency structure can be separated
- Let $F(x_1, x_2, \ldots, x_m)$ be a joint multivariate distribution function with univariate marginal distribution functions $F_1(x_1), F_2(x_2), \ldots, F_m(x_m)$
- Then there exists a copula function $C(u_1, u_2, \ldots, u_m)$ such that:

$$F(x_1, x_2, \ldots, x_m) = C\left(F_1(x_1), F_2(x_2), \ldots, F_m(x_m)\right)$$

- C is unique if each F_i is continuous
- Depending on the type of copula, the dependency structure will be expressed in terms of a correlation matrix, as in case for the normal copula, or via a smaller number of parameters as in case of Archimedean copulas (such as the Clayton or Gumbel copula)

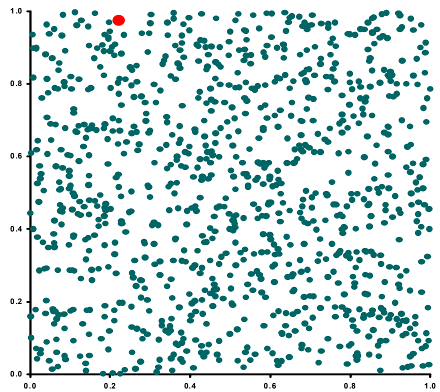
Generating correlated default times - the intuition behind the Gaussian copula

Copulas to generate correlated default times

First step:

Generate independent uniform random numbers
Excel "=rand()"

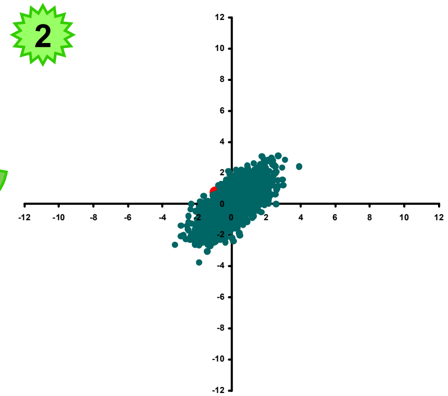
Starting with uncorrelated uniforms..



Second step:

Transform uniforms in normal numbers and then impose correlation structure as given by the correlation matrix via Cholesky transformation (see next slide) to get correlated normal numbers

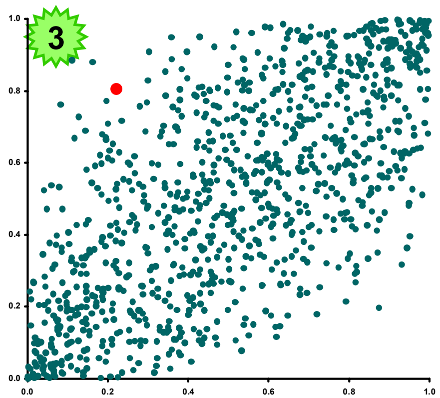
..to correlated normals..



Third step:

Map the correlated normals back to uniforms (Excel "normsdist(z)"). These correlated uniforms now effectively represent correlated survival rates

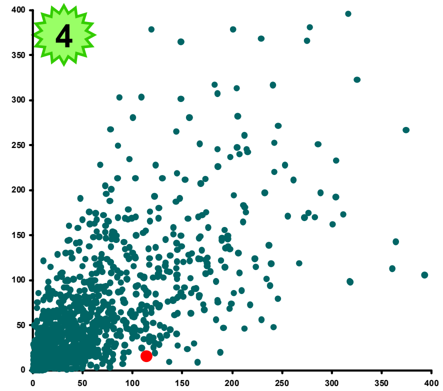
.. via correlated uniforms..



Final step:

Translate survival rates into default times using the individual survival term structures

.. we finally arrive at correlated default times..



Normal Copula

Draws	U(0,1) ①	U(0,1) ①	N(0,1)	N(0,1)	Corr N(0,1) ②	Corr N(0,1) ②	Corr U(0,1) ③	Corr U(0,1) ③	Corr DT ④	Corr DT ④
1	0.07	0.86	-1.46	1.07	-1.46	-0.26	0.07	0.40	197	69
2	0.72	0.31	0.58	-0.49	0.58	0.05	0.72	0.52	25	49
3	0.44	0.14	-0.15	-1.08	-0.15	-0.88	0.44	0.19	62	125

Doing it in Excel..

Spreadsheet screenshots - The Gaussian copula (sheet "Normal and Student-t Copula")

1.) Input:

Choose spreads and recoveries, the level of correlation

	Credit 1	Credit 2
Spread (bps)	80	80
Recovery	40%	40%
Clean Spread (bps)	133.3	133.3

Correlation input
70%

2.) Cholesky transformation:

The correlation matrix is transformed into a lower triangular matrix via the Cholesky transformation

Correlation Matrix	
100%	70%
70%	100%

Cholesky Matrix	
100%	0%
70%	71.41%

$$\sqrt{\left(\sigma_B^2 - \frac{\sigma_{AB}^2}{\sigma_A^2}\right)} = \sqrt{\left(100\% - \frac{70\%^2}{100\%}\right)}$$

3.) From uncorrelated uniform random numbers to correlated default times: three simulations for two credits

Normal Copula

Draws	U(0,1)	U(0,1)	N(0,1)	N(0,1)	Corr N(0,1)	Corr N(0,1)	Corr U(0,1)	Corr U(0,1)	Corr DT	Corr DT
1	0.07	0.86	-1.46	1.07	-1.46	-0.26	0.07	0.40	197	69
2	0.72	0.31	0.58	-0.49	0.58	0.05	0.72	0.52	25	49
3	0.44	0.14	-0.15	-1.08	-0.15	-0.88	0.44	0.19	62	125

=RAND()

=NORMSINV(0.72)

=0.58*100%+(-0.49)*0%

=0.58*70%+(-0.49)*71.41%

=NORMSDIST(0.05)

Using the intensity (ie the clean spread), the default times can be backed out from the survivor rates via:

$$= -\frac{\ln(0.52)}{133.33\,bps}$$

Student-t copula..

Symmetric like the Normal, but fatter tails..

- The Student-t copula is a well understood alternative to the Gaussian copula
- While also a symmetric distribution, the Student-t distribution exhibits fatter tails (a higher kurtosis) than the normal distribution
- While converging towards the normal distribution for an infinite number of degrees of freedom (df), this behaviour becomes more and more pronounced for lower dfs
- The Student-t copula therefore is easy to implement and at the same time, it is a powerful tool to analyse the risk inherent in extreme scenarios (“tail events”), especially when used against the Gaussian copula as a benchmark
- To introduce dependency via the Student-t copula we use the following transformation

$$t(n) = \frac{z}{\sqrt{x/n}}$$

- Where
  - $z \sim N(0,1)$
  - $x \sim \chi^2(n)$ with *n* denoting the degrees of freedom

Doing it in Excel..

Spreadsheet screenshots - Student-t copula (sheet "Normal and Student-t Copula")

1.) Input:

Choose spreads and recoveries, the level of correlation, and the degrees of freedom for the Student-t copula

Spread Curve Parameter Inputs

	Credit 1	Credit 2
Spread (bps)	80	80
Recovery	40%	40%
Clean Spread (bps)	133.3	133.3

Correlation input
70%

Student-t Degrees of Freedom (df)
3

Specify the number of degrees of freedom (df)

2.) Cholesky transformation

Identical to slide 6

3.) From uncorrelated random numbers to correlated default times:

As described on the previous slide, correlated student-t random variables are generated by drawing from 2 normal distributions, correlating them and then dividing by *sqrt(chi2/df)*.

As demonstrated before for the Gaussian copula these can then be easily converted into correlated default times

Student-t Copula

Draws	U(0,1)	U(0,1)	N(0,1)	N(0,1)	Corr N(0,1)	Corr N(0,1)	U(0,1)	Chi2	sqrt(DoF/Chi2)	Corr T(0,1)	Corr T(0,1)	Corr U(0,1)	Corr U(0,1)	Corr DT	Corr DT
1	0.07	0.86	-1.46	1.07	-1.46	-0.26	0.95	0.38	2.83	-4.12	-0.73	0.01	0.26	250	101
2	0.72	0.31	0.58	-0.49	0.58	0.05	0.54	2.16	1.18	0.68	0.06	0.73	0.52	24	49
3	0.44	0.14	-0.15	-1.08	-0.15	-0.88	0.50	2.39	1.12	-0.17	-0.98	0.44	0.20	62	121

Analogously to Gaussian copula (slide 6)

=*RAND()*

=*CHIINV(0.54 ; 3)*

=*sqrt (3 / 2.16)*

=*0.58*1.18*

The cumulative probability for the Student-t

$$\text{Prob}\ \left(T \le (t = 0.68);\ 3\,df\right)$$

Using the intensity (ie the clean spread), the default times can be backed out from the survivor rates via:

$$= -\frac{\ln(0.73)}{133.33\,bps}$$

Section 2

CDO pricing with copulas

Monte Carlo cookbook recipe..

..a very simple algorithm

- For pricing purposes, the above described approach is applied on portfolio level
  - For each Monte-Carlo run
    - Draw a random number for *each* credit
    - Generate dependent default times using the copula of choice
    - A given tranche experiences a default event, resulting in a principal loss, if the total loss corresponding to the number of defaults occurring before the tranche's maturity is higher than the attachment point
    - For each tranche, store the PV of losses
  - Repeat *n* times, where *n* is the number of simulations
  - Once finished:
    - Calculate the expected loss for each tranche, by summing all discounted losses and dividing this by the number of simulations
    - Calculate the DV01
    - Divide the tranche EL by the DV01 to arrive at the fair spread

Pricing CSO tranches (1/4)

CDO model inputs (1/2)

- Our spreadsheet model allows to model and price CSO tranches using the Gaussian as well as the Student-t copula
- In addition to specifying the number of assets (up to 125), the maturity (up to 5 years), correlation and the discount rate, this Monte Carlo based model also requires an input for the number of simulations
- Being a teaching tool aimed to increase transparency and therefore entirely Excel/VBA based, we have limited the number of simulations to 50,000
- While 10,000 simulations are already accurate enough to analyse CSO tranches and to gain a correct understanding of their behaviour when input parameters are changed, outputs such as the expected loss or the 99% VaR will exhibit some variance, especially for tranches with very high attachment points such as the super senior
- For more stable and accurate results, a higher number of simulations (eg 1,000,000) has to be used. A purely Excel/VBA based model reaches its limitations at this point – we therefore recommend using a C++ version of the model for these purposes

Spreadsheet screenshots – deal and copula inputs in sheet "CDO model"

Specify the number of reference credits and the discount rate

Averages for the recovery rate and spreads (see next slide)

Deal Parameters

No of Credits	125
Discount Rate (%)	5%
Coupon payment frequency (p.a.)	4
Average Recovery (%)	40.00%
Average Spread (bps)	100.00
Hazard Rate ~ Clean spread	166.67
Maximum individual hazard rate	166.67
Cumulative Default Probability	8.17%
Total Portfolio Notional	1,000,000
Value Date	04-Sep-08
Maturity Date	20-Sep-13
Next Coupon Date	20-Sep-08
Horizon (as a year fraction)	5.1167
Maturity in months	60
Maturity in quarters	21

Copula Model Inputs

Simulation	10,000
Correlation	20.00%
Copula model	Normal
Student-t df	3
Dump loss distribution	TRUE
Number of Bins	20

Available Copula *(do not delete this area)*	
1	
1	Normal
2	Student-t

Monte Carlo
(before running ensure spreadsheet set to manual ...ation in Tools-Options)

Select number of runs for the Monte Carlo simulation, the correlation as well as the type of copula. If the Student-t is chosen, also select the number of degrees of freedom.
To display the portfolio and tranche loss distributions, set *Dump loss distribution* to *"TRUE"*

Pricing CSO tranches (2/4)

CDO model input (2/2)

- The model allows asset specific inputs for notional, recovery rates and CDS spreads and then backs out the corresponding default probabilities
- The discount factors are assuming a flat interest rate curve based on the discount factor specified in the control box “Deal Parameters” (see previous slide)
- However, any term structure of default probabilities as well as interest rates can be easily implemented by directly typing the values in the corresponding cells

Spreadsheet screenshots – credit inputs in sheet “CDO model”

Input asset specific values for notional, recovery rate and the CDS spread

Pool Details

Obligor ID	Notional	Recovery Rate	CDS spread	DV01
1	8,000	40.0%	100	4.3739
2	8,000	40.0%	100	4.3739
3	8,000	40.0%	100	4.3739
…	…	…	…	…
123	8,000	40.0%	100	4.3739
124	8,000	40.0%	100	4.3739
125	8,000	40.0%	100	4.3739
	1,000,000			

Individual CDS default probabilities

Pay Dates	20-Sep-08	20-Dec-08	20-Mar-09		20-Jun-13	20-Sep-13
Day Count	0.044	0.253	0.250		0.256	0.256
Discount Factor	99.78%	98.52%	97.30%		78.42%	77.43%
Periods	1	2	3		20	21
	0.0740%	0.4941%	0.9079%	…	7.7823%	8.1743%
	0.0740%	0.4941%	0.9079%	…	7.7823%	8.1743%
	0.0740%	0.4941%	0.9079%	…	7.7823%	8.1743%
	…	…	..	…	…	…
	0.0740%	0.4941%	0.9079%	…	7.7823%	8.1743%
	0.0740%	0.4941%	0.9079%	…	7.7823%	8.1743%
	0.0740%	0.4941%	0.9079%	…	7.7823%	8.1743%

In the standard setting, discount factors are based on a flat interest rate curve as specified in the box “Deal Parameters”, but by manually overwriting discount factors other term structures can be incorporated

In the standard setting, default probabilities are based on a flat CDS term structure. They can be manually overwritten to incorporate asset specific term structures

Pricing CSO tranches (3/4)

Output summary – Capital structure, VaR and spreads

- The model calculates the fair spreads, DV01 and Value at Risk based on the portfolio's parameters as well as the selected capital structure
- For the contingent leg, the standard error is shown, giving an indication of the accuracy of the results. To increase accuracy and decrease standard errors, the number of simulation has to be increased
- Compared with the contingent leg, the results for the DV01 will be more accurate

Spreadsheet screenshots – capital structure inputs and results in sheet "CDO model"

Capital Structure and Pricing

						Contingent Leg				Fee leg ~ DV01				VAR £.		
Tranche Types	Attach-ment	Detach-ment	UpFront	Running Spread	Tranche Size	£	Standard Error (£)	%	Cont. Leg + SE (%)	Coupon Leg	Accrual on Default	Fee leg ~DV01	Fair Spread	95%	97%	99%
Equity	0%	3.00%	58.02%	5%	30,000	20,905	94	69.68%	70.0%	2.25	0.087	2.333	29.87%	29,279	29,414	29,618
Mezz Jun	3%	6.00%			30,000	11,019	119	36.73%	37.1%	3.62	0.047	3.668	10.01%	28,057	28,470	29,013
Mezz Sen	6%	9.00%			30,000	5,762	100	19.21%	19.5%	4.10	0.024	4.129	4.65%	26,718	27,317	28,255
Senior Junior	9%	12.00%			30,000	3,038	78	10.13%	10.4%	4.31	0.013	4.320	2.34%	25,200	26,086	27,356
Senior	12%	22.00%			100,000	2,851	122	2.85%	3.0%	4.44	0.004	4.446	0.64%	18,967	38,870	80,904
Super Senior	22%	100.00%			780,000	292	41	0.04%	0.0%	4.48	0.005	4.484	0.01%	0	0	0
Index	0.00%	100.00%			1,000,000	43,867				4.37	0.009	4.375	1.0026%	127,4	149,530	194,097

Specify the capital structure

Choose the running spread for the equity

The contingent leg and the corresponding standard error

The Value at Risk at for various confidence levels

Note: Monte Carlo with 10,000 runs (see slide 11 for inputs)

Pricing CSO tranches (4/4)

Output summary – Spreads, DV01 and testing

- Fair Spreads and DV01s, the main outputs, are also summarised in the box "Key Model Outputs"
- The box "Testing" gives an indication of the overall accuracy of the model for a given number of simulation. In this instance, for 10,000 simulations, the error is only a quarter of a bp

Spreadsheet screenshots – result summary and testing in sheet "CDO model"

Key Model Outputs

	Upfront	Running Spread (bps)	DV01
Equity	58.02%	500.0	2.33
Mezz Jun		1,001.3	3.67
Mezz Sen		465.2	4.13
Senior Junior		234.4	4.32
Senior		64.1	4.45
Super Senior		0.8	4.48
Index		**100.3**	**4.38**

Testing

Test on the Index spread		
Tranche model	100.26bps	
Index initial input	100.00bps	**0.260bps**

The final test on the index spread. While up to 50,000 simulations are possible, the overall error is already only a quarter bp for 10,000 simulations

Section 3

Risk Management – Understanding tail risk

Dresdner Kleinwort

The Student-t copula

Loading the tails

- In the box "Copula Model Inputs" in addition to the Gaussian copula, the Student-t copula can be selected
- As pointed out on slide 2, we assume that the marginal distribution is of the same kind as the joint distribution
- Changing the copula alters the individual and joint behaviour of the pools constituents and therefore has a significant impact on the risk/return characteristics of the portfolio as a whole but particularly for the individual tranches
- Switching to the Student-t copula puts more weight on the tails, therefore spreads of junior tranches will be lowered, whereas fair spreads for senior tranches will be significantly higher
- On the following slides, we will analyse these risk/return characteristics and how they are altered when the copula is modified
- We compare the standard Gaussian copula with the Student-t copula with 3 and 10 degrees of freedom (df)
- For all three copula assumptions, we used the same parameters for the portfolio (5yrs, 100bps, 40% RR, 5% Interest rate) and correlation (20%), together with 50,000 simulations

Copula Model Inputs

Simulation	10,000
Correlation	20.00%
Copula model	Student-t
Student-t df	3
Dump loss distribution	TRUE
Number of Bins	20

Select the Student-t copula and specify the number of degrees of freedom.

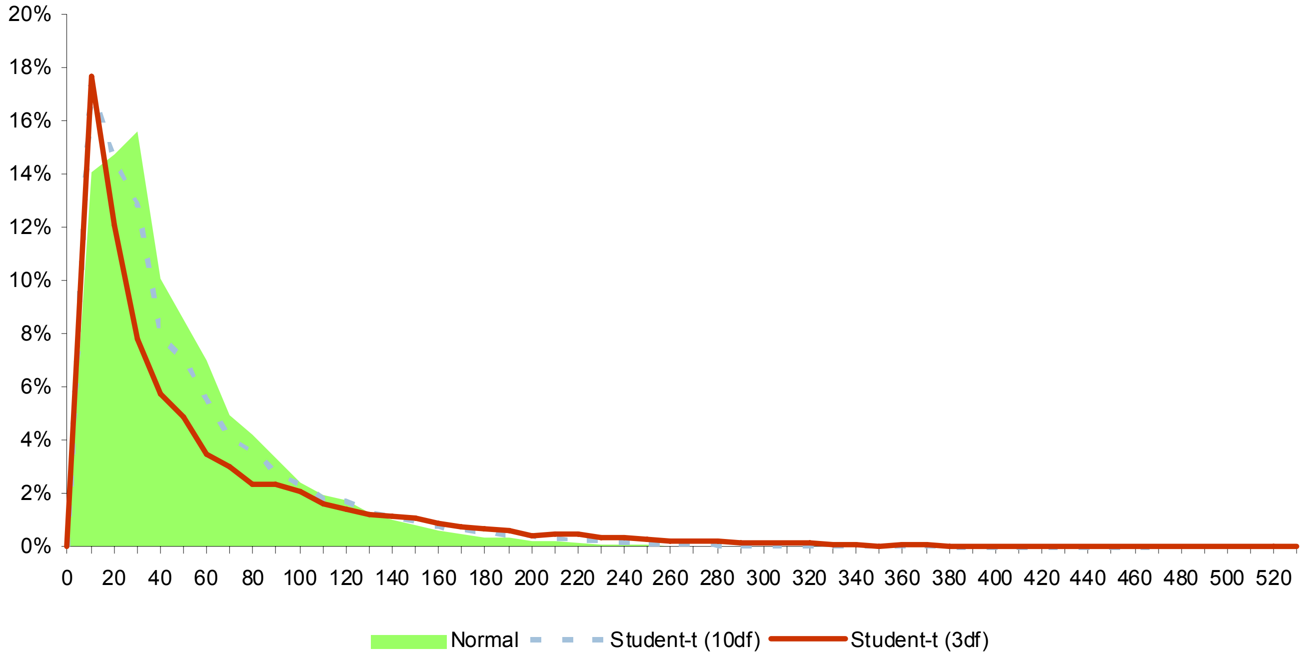
Available Copula (do not delete this area)	
1	
1	Normal
2	Student-t

Student-t vs. Normal

Switching to the Student-t copula puts more weight on the tails..

- Below we show the relative frequency of given discounted cumulative portfolio losses
- As intuition suggests, compared with the Gaussian copula, the likelihood of observing very low but more importantly extremely high cumulative losses is considerably higher for the Student-t copulas
- In addition, when using the Student-t copula the tails are not only fatter, but also considerably longer, as indicated by the chart

Relative frequency of discounted cumulative portfolio losses ('000)



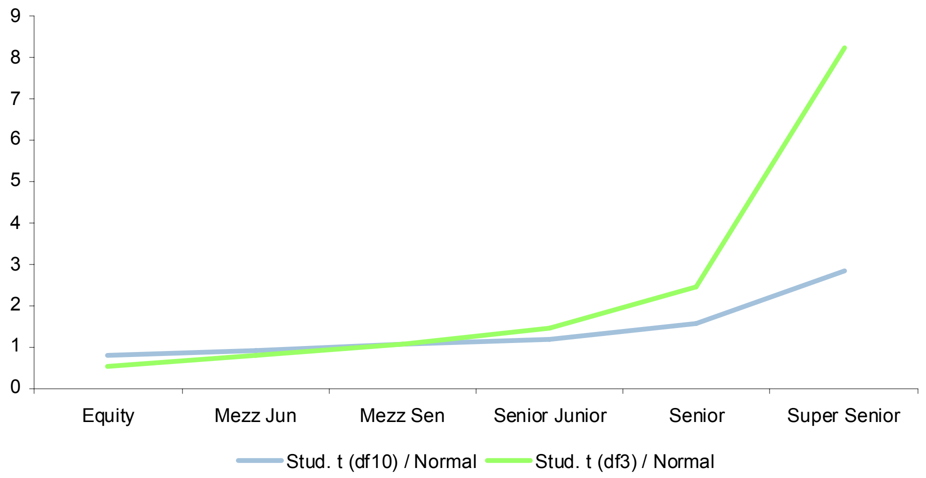
Source: Dresdner Kleinwort Research

Different copulas, different spreads

Fatter tails can be interpreted as higher systemic risk, leading to higher spreads for senior tranches

- The different shapes of the loss distributions are reflected in the tranche spreads
- The equity upfront is considerably lower in Student-t copulas
- For the 6-9% and the more senior tranches Tranches, spreads are increasingly higher than under the Normal copula, by several multiples in case of the super senior tranche
- The chart to the left shows the ratio of tranche spreads (upfronts in case of the equity)
- The attributes of the Student-t copula (fat tails, tail dependency) lead to an extreme redistribution of risk throughout the capital structure, especially for the super senior tranche

Student-t copula tranche spreads (relative to Normal copula)



Source: Dresdner Kleinwort Research

Tranche spreads with different copulas

Tranche	Normal	Stud. t (df10)	Stud. t (df3)	Stud. t (df10) - Normal	Stud. t (df3) - Normal
0-3%	29.58%	23.59%	16.00%	-6.00%	-13.58%
3-6%	973.5	901.2	773.8	-72.4	-199.7
6-9%	451.1	473.5	485.2	22.4	34.2
9-12%	226.7	271.8	326.6	45.1	99.9
12-22%	62.6	97.3	152.7	34.7	90.1
22-100%	0.8	2.3	6.7	1.5	5.9

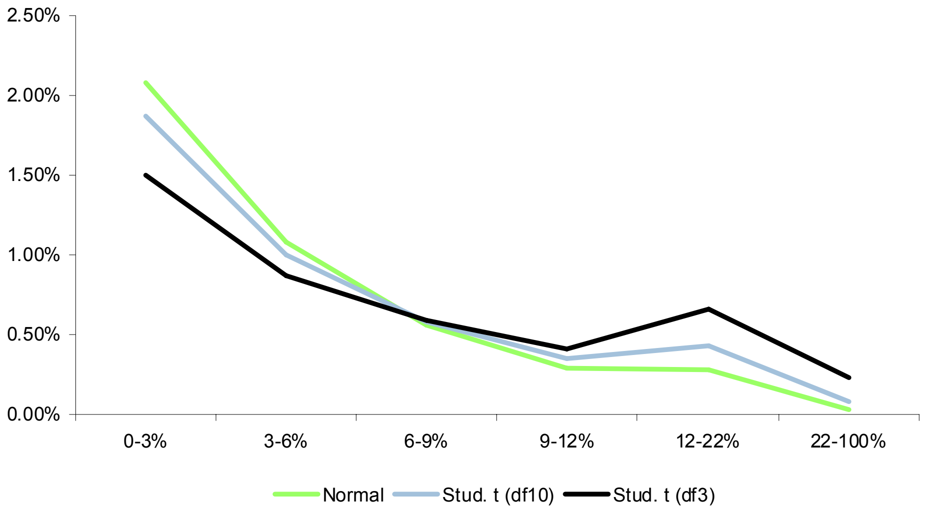
Source: Dresdner Kleinwort Research

Redistribution of risk across the tranches

..and more losses for senior tranches

- For the expected losses (EL) of the tranches we observe the same pattern as for the spreads
- In general it makes sense to look at ELs as a fraction of the corresponding tranche's notional
- We also show below tranche ELs as a fraction of the portfolio notional
- As a further remark, we point out that the differences for the portfolio EL are very small and only due to the Monte Carlo approach. The portfolio EL should stay the same between these two copulas

Tranche ELs as a fraction of portfolio notional



Source: Dresdner Kleinwort Research

Expected losses

	EL (as fraction of tranche notional)			EL (as fraction of index notional)		
	Normal	Stud. t (df10)	Stud. t (df3)	Normal	Stud. t (df10)	Stud. t (df3)
Index	4.32%	4.31%	4.27%	4.32%	4.31%	4.27%
0-3%	69.51%	62.52%	50.05%	2.09%	1.88%	1.50%
3-6%	35.93%	33.35%	29.15%	1.08%	1.00%	0.87%
6-9%	18.67%	19.29%	19.51%	0.56%	0.58%	0.59%
9-12%	9.80%	11.57%	13.62%	0.29%	0.35%	0.41%
12-22%	2.78%	4.29%	6.62%	0.28%	0.43%	0.66%
22-100%	0.04%	0.10%	0.30%	0.03%	0.08%	0.23%

Source: Dresdner Kleinwort Research

For the risk managers ... VaR and Expected Shortfall

Differences become more obvious as we look at the key credit portfolio management tools

- The Value at Risk (VaR) and Expected Shortfall of the portfolio for various confidence levels are shown below
- Again the elevated likelihood of extreme events under the Student-t copula is clearly visible
- Also, the tail length of the different copulas can be compared very good using the Expected Shortfall

Portfolio VaR and Expected Shortfall

	VaR			Expected Shortfall		
Confid. Level	Normal	Stud. t (df10)	Stud. t (df3)	Normal	Stud. t (df10)	Stud. t (df3)
0%	0	0	0	43,246	43,117	42,653
10%	4,273	0	0	47,842	47,908	47,392
25%	12,800	8,233	3,717	55,570	56,415	56,854
50%	30,224	25,089	15,857	72,497	76,396	81,382
75%	59,432	58,753	55,673	101,476	112,686	130,759
90%	98,471	108,816	125,876	139,538	162,191	199,516
95%	127,288	146,705	179,419	167,368	198,890	249,309
97.5%	155,560	183,540	232,512	195,029	234,362	295,555
99%	192,761	231,090	295,854	230,448	280,151	347,569
99.5%	220,142	267,068	333,678	255,656	313,067	383,012
99.99%	357,085	420,698	503,976	372,144	447,155	514,088
99.999%	383,414	462,130	521,318	384,486	463,222	525,330
100%	384,486	463,222	525,330			

Source: Dresdner Kleinwort Research

Section 4

Archimedean copulas

Dresdner Kleinwort

Variations on a theme

Various copula families with differing properties exist

- The Gaussian and Student-t copulas both belong to the family of elliptical copulas
- While both are symmetrical and introduce dependency via a matrix containing pairwise correlations, they however differ as the Student-t copula exhibits lower and upper tail dependence
- The Gumbel and the Clayton copulas, which we will introduce on the following slides, are both examples for Archimedean copulas
- Archimedean copulas are different from the copulas we looked at so far, as they allow dependency without the classic linear correlation concept and introduce lower and upper dependency usually via using one or two parameters only
- Depending on the characteristics of the portfolio that needs to modelled, especially with respect to the number of constituents and the degree of heterogeneity, this can either be an advantage or a limiting factor
- For large, granular and relatively homogenous portfolios, Archimedean copulas can be a very powerful and parsimonious alternative to the Gaussian or Student-t copulas

Comparing dependency..

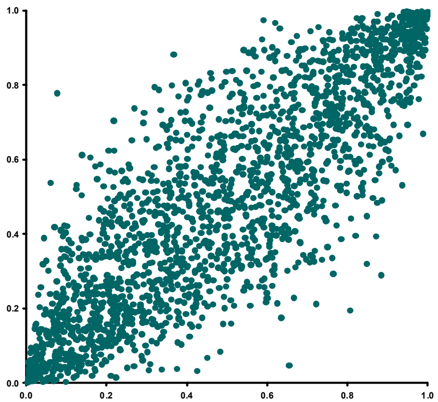
Some important copulas.. (sheet "Test Copula")

- Below we show the results for the Gaussian, Student-t with 3df, Gumbel and Clayton copulas
- To compare the level of dependency across copulas, rank correlation coefficients such as Kendall's Tau should be used, as the basic linear correlation coefficient (Pearson's rho) fails to capture non-linear dependency correctly
- In addition, tail dependency ratio's can be used to express the extent of lower or upper tail dependency numerically

Parameters	
Number of Simulation	2,000
Correlation	85.00%
Student-t Degree of freedom	3

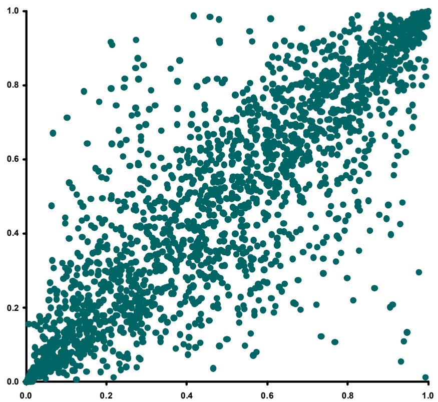
Number of Credits	2
Gumbel Alpha	2.8312
Clayton Alpha	3.6625
Tau	64.68%

Gaussian Copula:
No tail dependence



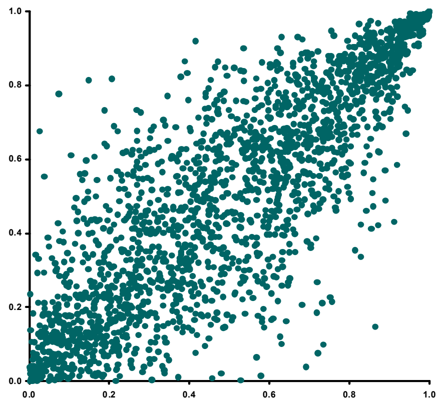
Statistical outputs	Credit 1	Credit 2
Average	0.497	0.496
St Dev	0.292	0.288
Correlation	84.94%	
Kendall's Tau	65.86%	

Student-t Copula (3df):
both lower and upper tail dependence



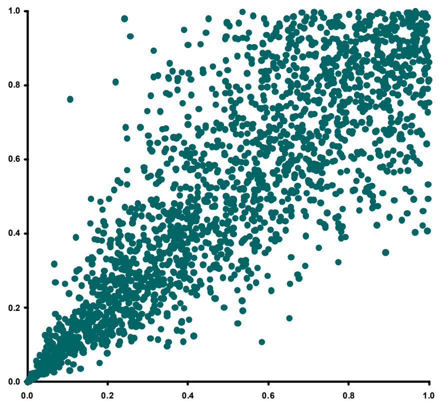
Statistical outputs	Credit 1	Credit 2
Average	0.504	0.503
St Dev	0.292	0.293
Correlation	83.46%	
Kendall's Tau	66.31%	

Gumbel Copula:
upper tail, no lower tail dependence



Statistical outputs	Credit 1	Credit 2
Average	0.491	0.491
St Dev	0.284	0.288
Correlation	83.66%	
Kendall's Tau	65.26%	

Clayton Copula:
lower tail, no upper tail dependence



Statistical outputs	Credit 1	Credit 2
Average	0.496	0.503
St Dev	0.287	0.290
Correlation	83.76%	
Kendall's Tau	65.60%	

Comparing the dependency in default times..

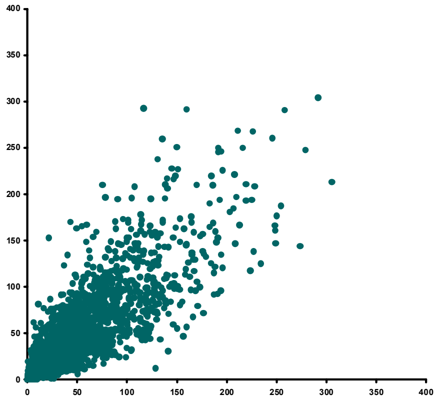
A wide range of different dependency patterns can be generated using copulas

- Below we show the results of the previous slide converted into default times, assuming a flat CDS spread of 120 bps and 40% recovery rate
- The below charts illustrate the dependency pattern in the default times each of the four copulas generates
- While the upper tail dependence of the Gumbel copula results in a clustering of early defaults, it is the other way round for the Clayton copula
- As expected, the transformation from survivor rates to default times affects the linear correlation, especially in case of the Gumbel and Clayton copula
- Kendall's Tau however remains unchanged as the ranking structure is not altered via this transformation

Parameters	
Number of Simulation	2,000
Correlation	85.00%
Student-t Degree of freedom	3

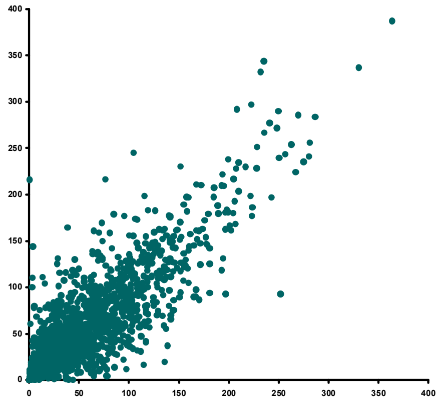
Number of Credits	2
Gumbel Alpha	2.8312
Clayton Alpha	3.6625
Tau	64.68%

Gaussian Copula:



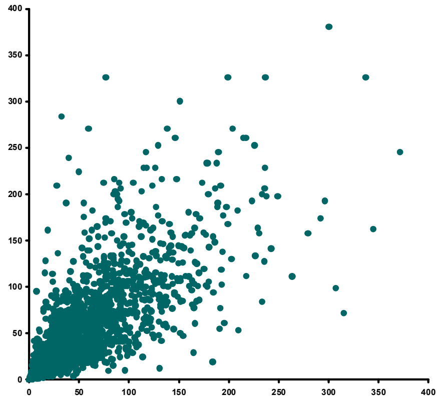
Statistical outputs	Credit 1	Credit 2
Average	15.34	20.34
St Dev	47.52	47.57
Correlation	81.93%	
Kendall's Tau	65.86%	

Student-t Copula (3df):



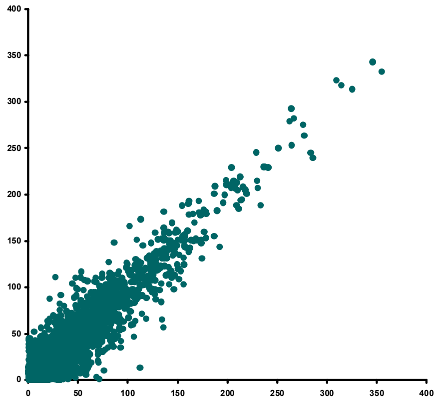
Statistical outputs	Credit 1	Credit 2
Average	14.92	20.34
St Dev	50.27	51.54
Correlation	86.26%	
Kendall's Tau	66.31%	

Gumbel Copula:



Statistical outputs	Credit 1	Credit 2
Average	85.17	81.24
St Dev	49.13	50.96
Correlation	75.13%	
Kendall's Tau	65.26%	

Clayton Copula:



Statistical outputs	Credit 1	Credit 2
Average	15.75	18.82
St Dev	49.75	49.55
Correlation	93.21%	
Kendall's Tau	65.60%	

Disclosure appendix

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Underweight	We expect the issuer to underperform sector peers over a 6-months horizon and would suggest holding less of the issuer's instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer's instruments and fundamentals compared to sector peers

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	All covered companies		Companies where a Dresdner Kleinwort company ha provided investment banking service	
Overweight	16	33%	9	56%
Marketweight	20	42%	7	35%
Underweight	12	25%	6	50%
Total	48		22	

Source: Dresdner Kleinwort Research

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