A model for longevity swaps
Pricing life expectancy

Marco Stoeckle
+44 (0)20 7475 3599
marcosebastian.stoeckle@dkib.com

Andrea Loddo
+44 (0)20 7475 8721
andrea.loddo@dkib.com

Domenico Picone
+44 (0)20 7475 3870
domenico.picone@dkib.com

Structured credit research: Global

Please refer to the Disclosure Appendix for all relevant disclosures and our disclaimer. In respect of any compendium report covering six or more companies, all relevant disclosures are available on our website www.dresdnerkleinwort.com/research/disclosures or by contacting the Dresdner Kleinwort Research Department at the address below.

Dresdner Bank AG London Branch, authorised by the German Federal Financial Supervisory Authority and by the Financial Services Authority; regulated by the Financial Services Authority for the conduct of UK business, a Member Firm of the London Stock Exchange. Registered in England and Wales No FC007638. Located at: 30 Gresham Street, London EC2V 7BG. Telex: 885540 DRES BK G. Incorporated in Germany with limited liability. A Member of the Dresdner Bank Group.

A member of Allianz AG
Rising life expectancy creates new opportunities

Designing a hedging tool for pension funds

- In addition to the obvious exposure each and everyone of us has to mortality, economic agents such as pension funds, insurers and governments are exposed to mortality and to its flip-side longevity risk.

- OECD pension funds had about USD 18tr of assets under management in 2007, and through their defined benefit pension schemes are massively exposed to longevity risk, as increases in life expectancy create additional costs. They are “short longevity”.

- In Europe, the UK and the Netherlands, because of the size of their pension fund markets, would be the greatest beneficiaries of a liquid longevity market.

- Life settlements and mortality cat bonds have been used in the past to transfer very specific forms of longevity risk. Annuity buyouts have also been offered some form of risk relief for UK DB pension funds.

- With regulators also forcing pension funds and life insurers to take a more active stance in managing longevity risk, longevity derivatives are likely to become the instrument of choice to manage this risk.

- The longevity swap offers the simplest and easiest way to standardise the transfer of longevity risk between pension funds, insurers and new longevity investors looking at this market for diversification benefits.

- With the attached spreadsheet to price the longevity swap, we distribute our implementation of the Lee Carter ‘92 model, which we used to forecast future German mortality rates, the main input for pricing.

Longevity swap model

The cost of aging

In the western world, life expectancy has increased significantly in the last century

- In addition to the obvious exposure each and everyone of us has to mortality, economic agents such as pension funds, insurers and governments, are exposed to mortality and to its flip-side longevity risk.

- Within the private sector, pension funds have the highest economic exposure to longevity risk. Through their defined-benefit (DB) pension liabilities, increases in life expectancy create additional costs. They are “short longevity”.

- Insurers with annuity liabilities (short longevity) and term life insurance (long) can be anything from flat to short or long, depending on their business mix. On aggregate, however, insurers are relatively balanced.

- Overall, particularly in countries with an established pension fund sector, such as the UK and the US, the market is net short longevity, resulting in an rising shortfall between pension funds assets and liabilities, as life expectancy increases.

- The pressure from regulators, rating agencies and changes in accounting rules are forcing pension funds and life insurers to take a more active stance in managing this risk.

- Pension funds so far could not efficiently hedge their longevity risk.

- Bulk annuity buyouts have in the past provided some form of risk transfer for UK DB pension funds (fund buys annuities from insurer and can then close down the DB pension scheme).

- Life settlements and mortality cat bonds are also used to transfer longevity risks. However, they only provide ways to transfer very specific forms of longevity risk.

### Longevity market participants

<table>
<thead>
<tr>
<th></th>
<th>Short Longevity</th>
<th>Long Longevity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pension Funds</strong></td>
<td>Defined benefit pension liabilities</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td>Annuity liabilities</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Term life insurance policies</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Governments</strong></td>
<td>Social Security</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>PAUG pension systems</td>
<td>✓</td>
</tr>
</tbody>
</table>

Source: Dresdner Kleinwort Research
Many market participants are just starting to realise that they are massively exposed to longevity risk

- The creation of a liquid and transparent market for longevity risk would help market participants, such as pension funds and insurers, to manage their risk more efficiently.

- New longevity/mortality investors would have to emerge to fill the gap that is resulting from the massive demand overhang by the pension funds’ need to hedge longevity risk.

- While hard to predict how rapid a market for longevity risk in developed economies will emerge, we think that in Europe, the UK and the Netherlands offer the most promising conditions for this market to take off.

- This is based on various factors ranging from regulatory pressure to availability of data, but primarily driven by economic necessity.

- Within Europe, the UK’s pension funds is the biggest in absolute terms, followed by the Netherlands. For pension fund’s assets as a ratio of GDP, the Netherlands have the top position, with the UK fourth after Switzerland and Austria.

- Because of their massive size, UK pension funds, with just over US$2tr. of assets under management in 2007 (more than the rest of Europe combined), would be the greatest beneficiaries of a liquid longevity market in Europe.

- The longevity market in countries such as Germany, Italy and France, because of the stronger public component in their pension sector, will be more driven by life insurers and other “long” longevity players, and therefore grow at a lower pace.

Source: Dresdner Kleinwort Research, Bloomberg, OECD.
Rule of thumb to size the longevity risk of the pension fund industry

One additional year of life expectancy creates costs of USD 50bn

- Across developed countries pension fund assets have increased on average well over 150%, between 2001 and 2007.
- The UK pension regulator estimated in 2007, that the PV of the UK’s Pension fund liabilities increases by 3% per additional year of life expectancy.
- We use this “rule of thumb” to measure the effect a one year increase in life expectancy would have on industry-wide pension fund liabilities.
- The resulting shortfall is equivalent to US$50bn for the UK market alone.
- To put this into perspective, in the US and in the UK over the last 40 years, life expectancy for 65 year old men has on average increased by more than two months a year.

Source: Dresdner Kleinwort Research, Bloomberg, OECD
** Assets for 2001 (FRA) and 2007 (UK) estimated
The longevity index

The German example

- Within the LifeMetrics framework, the historical mortality rates are themselves considered as the index. The advantage of this approach is transparency and simplicity in the definition of the index. For consistency, we keep the same approach.

- However, the index is at the moment restricted to only few countries and age groups.

- For our model, using a similar approach, we used extensive data, which we received from the Federal Statistical Office of Germany and constructed a German index for several age groups.

- Our German index, disclosed in the sheet “Historical Mortality” can be read in two different ways:
  - The “Age group” approach tracks historic mortality rates for a constant age group and how they change over time, for example 50 year old males.
  - The “Cohort” approach tracks the development of historic mortality rates for a given year of birth over time, for example males born in 1980.

- In both instances, estimation regarding the future path of mortality rates can be based on their historical behaviour and volatility.
To simulate future mortality rates we use the Lee Carter model

Easy to calibrate, intuitive but still with some limits

First introduced by Lee and Carter in 1992, the model emerged as the benchmark for forecasting mortality rates. It is a one factor stochastic model where the mortality rate is a function of three parameters

\[ m(t, \text{age}) = e^{a_{\text{age}} + b_{\text{age}} k t} \]

- \( a \), the drift term, is a function of a particular age group and describes its overall evolution over time. \( b \), which again is a function of the age group, indicates the sensitivity of an age group to time. \( k \) describes the change in mortality rates over time without any differentiation between age groups.

The intuition behind this formulation is clear: historical observations of mortality data reveal that mortality, by and large, increases exponentially in time.

The model is calibrated on historical data, namely population and number of deaths. It is also possible to specify a 0 weight for specific years which exhibit a low number of observations or inaccurate data.

The model is extremely easy to calibrate, given the limited number of parameters and their intuitive meaning. More sophisticated models can better fit historical data but the larger number of parameters makes calibration an extremely delicate process. The Lee Carter model has become the standard as institutions, such as the US Social Security Technical Advisory Panels, recommend its use.

However, given its extreme simplicity, it cannot capture specific cohort behaviours which, instead, can be modelled within more sophisticated frameworks such as the Currie model. Also, depending on the historical data used for the calibration, the model can exhibit inversion of mortality rates across age groups for long term simulations*. This means, for example, that in 40 years time the mortality rate of the age group 40 can be lower than the mortality rate of the age group 38. This is clearly hard to justify.

*Understanding the Lee-Carter Mortality Forecasting Method, Federico Girosi, Gary King, 2007

Source: Dresdner Kleinwort Research, Federal Statistical Office of Germany
The easiest way to hedge longevity risk: the swap

What is and how to price a longevity swap

- As for interest rate swaps, with a longevity swap, a stream of cash flows is exchanged for an alternative one.
- This technology has been successfully replicated in other asset classes such as credit and volatility, and we will show that it can be easily implemented also in the longevity market.
- By entering a swap contract, a counterparty can hedge its exposure against fluctuations in future mortality rates. For example, a pension fund, typically exposed to increasing longevity rates, would like to enter into a swap, receiving actual floating "index" payments and paying fixed according to the expected mortality rate, for a given cohort-age at inception.
- Clearly, the fixed rate depends on the evolution of future mortality rates, as measured by the future level of the index. By analogy, a traditional interest rate swap depends on the forward curve. Following exactly the same logic, we price a longevity swap by simulating future mortality rates, i.e. the "index".
- Therefore, the main difference between an interest rate and longevity swap is in the way the forward curves are determined: supply and demand mechanics in the first case, simulating the "index" in the second. Until liquidity will not affirm in the longevity market, a simulation based approach will remain the predominant approach.
- There are two main different categories of longevity swap. The age group swap hedges the longevity exposure of a defined age group, for example males aged 30. On the contrary, the cohort swap hedges the longevity risk of a particular cohort which is therefore subject to aging as time goes by.
- The nature of the portfolio to hedge plays an important role in defining the best fitting hedging strategy. A closed pension fund will not know the exact mortality profile of its members, and therefore would benefit the most by hedging with a cohort longevity swap.
- In our model we only focus on the cohort longevity swap, however it could be adapted with little effort to price any alternative structure.
The pricing components: mortality and survival rates, fixed and floating leg

Our model simulates the future paths of mortality rates for a given cohort (for example males aged 40 in 2007). From the expected mortality rates, we calculate the survival curve for that given cohort.

The swap rate is the rate which makes the PV of the floating rate paying strategy equivalent to the PV of the fixed rate one. The longevity swap rate is calculated by simply dividing the floating leg by the duration of the fixed leg.

Initially the value of the swap contract is zero as the PVs of floating and fixed legs are equal. As the actual mortality rates evolve and are reported in the German index, the contract will have a MTM impact depending on the realized index mortality rates.

Spreadsheet screenshot – “Pricing” sheet

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Pay Date</th>
<th>Day Count</th>
<th>Discount Factor</th>
<th>Expected central mortality rates</th>
<th>Survival rate until the end of period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31-Dec-08</td>
<td>1.017</td>
<td>95.04%</td>
<td>0.1484%</td>
<td>99.852%</td>
</tr>
<tr>
<td>2</td>
<td>31-Dec-09</td>
<td>1.014</td>
<td>90.35%</td>
<td>0.1636%</td>
<td>99.688%</td>
</tr>
<tr>
<td>3</td>
<td>31-Dec-10</td>
<td>1.014</td>
<td>85.88%</td>
<td>0.1943%</td>
<td>99.495%</td>
</tr>
<tr>
<td>4</td>
<td>31-Dec-11</td>
<td>1.014</td>
<td>81.63%</td>
<td>0.2078%</td>
<td>99.288%</td>
</tr>
<tr>
<td>5</td>
<td>31-Dec-12</td>
<td>1.017</td>
<td>77.59%</td>
<td>0.2295%</td>
<td>99.060%</td>
</tr>
<tr>
<td>6</td>
<td>31-Dec-13</td>
<td>1.014</td>
<td>73.75%</td>
<td>0.2606%</td>
<td>98.802%</td>
</tr>
<tr>
<td>7</td>
<td>31-Dec-14</td>
<td>1.014</td>
<td>70.11%</td>
<td>0.2871%</td>
<td>98.518%</td>
</tr>
<tr>
<td>8</td>
<td>31-Dec-15</td>
<td>1.014</td>
<td>66.64%</td>
<td>0.3312%</td>
<td>98.192%</td>
</tr>
<tr>
<td>9</td>
<td>31-Dec-16</td>
<td>1.017</td>
<td>63.34%</td>
<td>0.3494%</td>
<td>97.849%</td>
</tr>
<tr>
<td>10</td>
<td>31-Dec-17</td>
<td>1.014</td>
<td>60.21%</td>
<td>0.3719%</td>
<td>97.485%</td>
</tr>
<tr>
<td>11</td>
<td>31-Dec-18</td>
<td>1.014</td>
<td>57.23%</td>
<td>0.4039%</td>
<td>97.091%</td>
</tr>
<tr>
<td>12</td>
<td>31-Dec-19</td>
<td>1.014</td>
<td>54.40%</td>
<td>0.4423%</td>
<td>96.662%</td>
</tr>
</tbody>
</table>
The specific longevity risk profiles of pension funds and life insurers can substantially differ from the index profile, thus making the hedge largely ineffective. This risk component, called basis risk, can therefore compromise the effectiveness of the swap strategy.

A first step to mitigate basis risk is by comparing how well the historic index mortality rates have tracked the mortality rates of a specific pension fund portfolio.

Also, the evolution on the mortality rate within the framework of the Lee Carter model is based on a normal distribution. Under this assumption, catastrophic events are not captured by the model and, as a result, this tail risk component is not accurately reflected by the swap price.

Our model can be used to price more complex longevity derivatives. In addition to the already described longevity swap, a second generation of longevity derivatives such as options, swaptions, caps and floors on either age group or cohort can be structured within our framework.
Disclosure appendix

Disclosures

The relevant research analyst(s), as named on the front cover of this report, certify that (a) the views expressed in this research report accurately reflect their personal views about the securities and companies mentioned in this report; and (b) no part of their compensation was, is, or will be directly or indirectly related to the specific recommendation(s) or views expressed by them contained in this report. The relevant research analyst(s) named on this report are not registered/qualified as research analysts with FINRA. The research analyst(s) may not be associated persons of the Dresdner Kleinwort Securities LLC and therefore may not be subject to the NASD Rule 2711 and Incorporated NYSE Rule 472 restrictions on communications with a subject company, public appearances and trading securities held by a research analyst account.

Any forecasts or price targets shown for companies and/or securities discussed in this presentation may not be achieved due to multiple risk factors including without limitation market volatility, sector volatility, corporate actions, the unavailability of complete and accurate information and/or the subsequent transpiration that underlying assumptions made by Dresdner Kleinwort or by other sources relied upon in the presentation were inapposite.

Credit Recommendation history tables

Past performance is not an indicator of future performance.

Please refer to our website www.dresdnerkleinwort.com/research/disclosures for our tables of previous fundamental credit opinions

Dresdner Kleinwort Research - Explanation of fundamental credit opinions

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overweight</td>
<td>We expect the issuer to outperform sector peers over a 6-months horizon and would suggest holding more of the issuer’s instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer’s instruments and fundamentals compared to sector peers</td>
</tr>
<tr>
<td>Marketweight</td>
<td>We expect the issuer to perform in line with sector peers over a 6-months horizon and would suggest holding an amount of the issuer’s instruments in line with what the market would hold on average. The recommendation reflects our weighted view on all of an issuer’s instruments and fundamentals compared to sector peers</td>
</tr>
<tr>
<td>Underweight</td>
<td>We expect the issuer to underperform sector peers over a 6-months horizon and would suggest holding less of the issuer’s instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer’s instruments and fundamentals compared to sector peers</td>
</tr>
</tbody>
</table>

We started tracking our trading recommendation history in compliance with the requirements of the Market Abuse Directive on 8 April 2005.

Distribution of Dresdner Kleinwort credit research recommendations as at 30 Sep 2008

<table>
<thead>
<tr>
<th></th>
<th>All covered companies</th>
<th>Companies where a Dresdner Kleinwort company has provided investment banking service</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overweight</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>33%</td>
<td>56%</td>
</tr>
<tr>
<td>Marketweight</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>42%</td>
<td>35%</td>
</tr>
<tr>
<td>Underweight</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td>50%</td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>22</td>
</tr>
</tbody>
</table>

Source: Dresdner Kleinwort Research