CDO models: Opening the black box – Part two
The Finite Homogenous Pool model

Domenico Picone
+44 (0)20 7475 3870
domenico.picone@dkib.com

Marco Stoeckle
+44 (0)20 7475 3599
marcosebastian.stoeckle@dkib.com

Priya Shah
+44 (0)20 7475 6839
priya.shah@dkib.com

Andrea Loddo
+44 (0)20 7475 8721
andrea.loddo@dkib.com

Structured credit research: Global

Please refer to the Disclosure Appendix for all relevant disclosures and our disclaimer. In respect of any compendium report covering six or more companies, all relevant disclosures are available on our website www.dresdnerkleinwort.com/research/disclosures or by contacting the Dresdner Kleinwort Research Department at the address below.

Dresdner Bank AG London Branch, authorised by the German Federal Financial Supervisory Authority and by the Financial Services Authority; regulated by the Financial Services Authority for the conduct of UK business, a Member Firm of the London Stock Exchange. Registered in England and Wales No FC007638. Located at: 30 Gresham Street, London EC2V 7BG. Telex: 885540 DRES BK G. Incorporated in Germany with limited liability. A Member of the Dresdner Bank Group.
After having released the Large Homogenous Pool Model in the first part of our series, we now move towards the finite homogenous pool model.

This publication builds up on the first publication of this series (CDO models: Opening the black box) and has been structured as a guide to be used in conjunction with the excel-based Finite Homogenous Pool model. Whilst we do briefly touch upon the main theoretical concepts, we do not go into detailed explanations and proofs, as this has been widely discussed and is readily available. Instead, we focus on how to implement the theory and apply the models.

We start by taking a closer look at how the loss distribution is constructed for a finite pool of homogenous assets.

Afterwards we address some important factors that have to be considered when using the Gauss-Hermite integration technique in combination with the binomial distribution.

Finite Homogeneous Pool Model:

The Binomial distribution is used to build up the default distribution

* The assumption of homogenous assets allows us to use the Binomial distribution to construct the default distribution
* The Binomial distribution is a discrete distribution used to model binary random variables (e.g., “tossing a coin”)
* For a given number of trials $n$, the probability of $k$ successful trials (observing exactly $k=20$ tails for $n=40$ coin tosses) is given by:

$$P(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

* Where $p$ denotes the likelihood to succeed in a given attempt (50% for a fair coin)
* Within the context of the finite homogenous pool model, we can therefore apply this distribution by parametrising:
  - $n =$ number of assets
  - $k =$ number of defaults,
  - $p =$ PD($t|m$), the probability of default by time $t$ conditional on the common factor $m$
The Binomial distribution is used to build up the default distribution

As we assume homogeneity, the PDs are identical for all assets and as before are given by

$$PD(t \mid m) = \Phi \left( \frac{\text{Default Barrier} (t) - \sqrt{\rho m}}{\sqrt{1 - \rho}} \right)$$

Applying the Binomial distribution, the probability of observing exactly $k$ defaults conditional on the realisation of a common factor $m$ is therefore given by

$$\text{Prob}(k \text{ defaults until } t \mid m) = \binom{n}{k} PD(t \mid m)^k (1 - PD(t \mid m))^{n-k}$$

The unconditional default distribution until time $t$ is then given by

$$\text{Prob} \left( \text{up to } k \text{ defaults by time } t' \right) = \sum_{n=0}^{k} \binom{n}{k} \int_{-\infty}^{\infty} \left( \Phi \left( \frac{\text{Default Barrier} (t) - \sqrt{\rho m}}{\sqrt{1 - \rho}} \right) \right)^k \left( 1 - \Phi \left( \frac{\text{Default Barrier} (t) - \sqrt{\rho m}}{\sqrt{1 - \rho}} \right) \right)^{n-k} \phi(m) \, dm$$

We integrate over the common factor to receive unconditional probabilities, for $k$ defaults

Then we sum over all defaults up to $k \ (0,1,\ldots,k-1,k)$ to calculate the cumulative default distribution until time $t$
CDO models: Opening the black box

Our spreadsheet – sheet “integration”

Screenshot (1/5) (spread 100 bps, 40% rec., 20% correlation)

As we assume flat CDS spreads, for a given spread $S$ and recovery $R$, the cumulative default probability is simply:

$$PD = 1 - \exp\left(-\frac{S}{1-R}\right) = 1 - \exp(-\lambda)$$

The default barrier $K$ for time period 3 is calculated using the PD of the asset:

$$Default\ \text{Barrier} \ (t = 3) = \Phi^{-1}(0.908\%)$$

Realisation of common factor and weights for each realisation. Integral calculated using $P(M=m)=f(m)*\text{step size}$

For each point of the default distribution, the matrix of conditional PDs is calculated. For 125 assets, this results in 126 iteration steps ($0, 1, 2, ..., 124, 125$ defaults).

The screenshot shows the sixth iteration step, i.e., five defaults.

The unconditional probability for exactly five defaults up until 20/09/13 ($t=21$) is then simply the sum over the corresponding conditional probabilities:

$$0.0000\% + 0.0000\% + ... + 2.5327\% + 2.0914\% = 6.2715\%$$
CDO models: Opening the black box

Building the loss distribution – sheet “integration”

Screenshot (2/5) (spread 100 bps, 40% rec., 20% correlation)

Expected portfolio default rate until 20 Mar 09:

\[ E[PD(t)] = 0.0\% \cdot 0.5451 + 0.8\% \cdot 0.2132 + \ldots + 99.2\% \cdot 0.0000 + 100.0\% \cdot 0.0000 = 0.908\% \]

For each iteration step, the macro copies the unconditional probabilities of default to the matrix below.

Once completed, the portfolio’s expected default rate as well as the default distribution can be obtained easily.

The kth point of the portfolio’s unconditional default rate distribution until 20 Mar 2013 is calculated by summing over all defaults up to k (0,1,..,k-1,k)

The portfolio’s default rate distribution at maturity is displayed in the sheet “pricing”
CDO models: Opening the black box

Loss and recovery waterfalls – sheet “pricing”

Screenshot (3/5) (spread 100 bps, 40% rec., 20% correlation)

Portfolio loss for one default (40% Recovery):
\[
\frac{\text{Fraction of Portfolio in Default} \cdot (1 - \text{Recovery Rate})}{\text{Fraction of Portfolio in Default} \cdot (1 - 40\%)} = 0.8\% \cdot (1 - 40\%) = 0.48\%
\]

Portfolio recovery for one default (40% Recovery):
\[
\frac{\text{Fraction of Portfolio in Default} \cdot \text{Recovery Rate}}{\text{Fraction of Portfolio in Default} \cdot \text{Recovery Rate}} = 0.8\% \cdot 40\% = 0.32\%
\]

For 40% RR, 124 defaults (equal to a portfolio default rate of 99.2%) result in a Portfolio loss rate of 59.52%. The 22%-100% tranche therefore experiences a loss of 48.1% of its notional:

For 40% RR, 124 defaults (equal to a portfolio default rate of 99.2%) result in a Portfolio loss rate of 59.52%. The 22%-100% tranche therefore recovers 50.87% of its notional:

\[
\frac{\text{Min} \left( \text{Max} \left( 0; \text{Portfolio Loss Rate} - \text{Attachment Point} \right), \text{Tranche Width} \right)}{\text{Tranche Width}}
\]

\[
= \frac{\text{Min} \left( \text{Max} \left( 0; 59.52\%-22\% \right), 78\% \right)}{78\%}
\]

\[
= \frac{37.52\%}{78\%} = 48.10\%
\]

\[
\frac{\text{Min} \left( \text{Max} \left( 0; \text{Portfolio Recovery Rate} - \text{Detachment Point} \right), \text{Tranche Width} \right)}{\text{Tranche Width}}
\]

\[
= \frac{\text{Min} \left( \text{Max} \left( 0; 39.68\%-1-100\% \right), 78\% \right)}{78\%}
\]

\[
= \frac{39.68\%}{78\%} = 50.87\%
\]
CDO models: Opening the black box

Tranche pricing

From conditional to unconditional tranche loss and recovery

The way the calculation of the tranche losses and recoveries is implemented differs slightly from the previously published LHP models.

We now directly multiply the tranche’s principal loss for a given number of defaults (i.e., a specific point of the loss distribution) with the corresponding unconditional probability of observing precisely this event.

The cumulative loss is therefore calculated by multiplying the default probability matrix (shown on slide 5 in pink) with the tranche’s principal loss (shown on slide 6).

The cumulative recoveries are obtained analogously. The default probability matrix is now multiplied with the tranche’s recovered amount (shown on slide 6).

Screenshot (4/5) (spread 100 bps, 40% rec., 20% correlation)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>98.52%</td>
<td>1.48%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.48%</td>
</tr>
<tr>
<td>2</td>
<td>90.56%</td>
<td>7.96%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>9.44%</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>20</td>
<td>25.19%</td>
<td>1.66%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>74.81%</td>
</tr>
<tr>
<td>21</td>
<td>23.65%</td>
<td>1.54%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>76.35%</td>
</tr>
</tbody>
</table>

Marginal loss for time period $t$

$$\text{MarLoss}_{E_t}(t) = TL_{E_t}(t) = TL_{E_t}(t-1) = 76.35\% - 74.81\%$$

Similar calculation for marginal recovery.

Model outputs for specific tranches

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99.96%</td>
<td>0.00%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.00%</td>
</tr>
<tr>
<td>2</td>
<td>99.75%</td>
<td>0.00%</td>
<td>0.22%</td>
<td>0.25%</td>
<td>0.00%</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>20</td>
<td>95.97%</td>
<td>0.01%</td>
<td>0.20%</td>
<td>3.99%</td>
<td>0.04%</td>
</tr>
<tr>
<td>21</td>
<td>95.76%</td>
<td>0.01%</td>
<td>0.20%</td>
<td>4.19%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

End period tranche notional is simply the difference between the notional at the start of the period and the marginal loss and marginal recovery for that period.
Tranche pricing: equivalent to pricing single name CDS

Once the loss distribution has been built and the losses and recoveries for each tranche have been determined, tranche pricing is straightforward and implemented identical to the previous models.

To ensure no arbitrage, the final index spread calculated using the tranche spread should be the same as the initial spread input.

\[ S = \frac{\text{Contingent Leg}}{\text{Fee Leg}} \]
A variant of the Gaussian quadrature, the Gauss Hermite integration can be used to approximate integrals ranging from \((-\infty, +\infty)\)

As demonstrated in the our first two spreadsheets for the Large homogenous pool model (LHPM), the basic idea is to reduce the number of points that are used to approximate the distribution of the common factor

While in the basic version of the LHPM we integrated over the interval (-5, 5) using decimal steps, resulting in 101 points, in the LHPM with the Gaussian-Hermite we reduced this to 30 points

While significantly reducing the number of calculations that have to be carried out and therefore increasing speed, we still achieve a high level of accuracy because of the way the points are chosen within the Gaussian Hermite integration approach

The 30 points we use in model two as well as in this model achieve almost the same level of accuracy as the 101 equally spaced points in model one, and a considerably higher level of precision than would be achieved with 30 equally spaced points.
The Gauss-Hermite integration technique – some caveats (2/2)

A numerical procedure to increase speed

► However, as with any approximation, the gain in speed comes at a cost

► For a low number of points for the Gauss-Hermite, especially in combination with high spreads and correlations, the loss distribution will exhibit waves. The higher the number of assets, the more pronounced this behaviour

► These waves do however not result from the use of the Gaussian-Hermite technique per se but are rather induced by the fact that we now look at a finite rather than infinite pool of homogenous assets

► For given model inputs with respect to number of assets, spreads, recoveries and correlations, increasing the number of points results in a smoother default distribution.

► As this comes at the cost of slower computation, when choosing the adequate number of points for the Gauss-Hermite, the trade-off between calculation speed and the required degree of accuracy has to be kept in mind
Disclosure appendix

Disclosures

The relevant research analyst(s), as named on the front cover of this report, certify that (a) the views expressed in this research report accurately reflect their personal views about the securities and companies mentioned in this report; and (b) no part of their compensation was, is, or will be directly or indirectly related to the specific recommendation(s) or views expressed by them contained in this report. The relevant research analyst(s) named on this report are not registered/qualified as research analysts with FINRA. The research analyst(s) may not be associated persons of the Dresdner Kleinwort Securities LLC and therefore may not be subject to the NASD Rule 2711 and Incorporated NYSE Rule 472 restrictions on communications with a subject company, public appearances and trading securities held by a research analyst account.

Any forecasts or price targets shown for companies and/or securities discussed in this presentation may not be achieved due to multiple risk factors including without limitation market volatility, sector volatility, corporate actions, the unavailability of complete and accurate information and/or the subsequent transpiration that underlying assumptions made by Dresdner Kleinwort or by other sources relied upon in the presentation were inapposite.

Recommendation history tables

Past performance is not an indicator of future performance.

Please refer to our website www.dresdnerkleinwort.com/research/disclosures for our tables of previous fundamental credit opinions

Dresdner Kleinwort Research - Explanation of fundamental credit opinions

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overweight</td>
<td>We expect the issuer to outperform sector peers over a 6-months horizon and would suggest holding more of the issuer’s instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer’s instruments and fundamentals compared to sector peers.</td>
</tr>
<tr>
<td>Marketweight</td>
<td>We expect the issuer to perform in line with sector peers over a 6-months horizon and would suggest holding an amount of the issuer’s instruments in line with what the market would hold on average. The recommendation reflects our weighted view on all of an issuer’s instruments and fundamentals compared to sector peers.</td>
</tr>
<tr>
<td>Underweight</td>
<td>We expect the issuer to underperform sector peers over a 6-months horizon and would suggest holding less of the issuer’s instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer’s instruments and fundamentals compared to sector peers.</td>
</tr>
</tbody>
</table>

We started tracking our trading recommendation history in compliance with the requirements of the Market Abuse Directive on 8 April 2005.

Distribution of Dresdner Kleinwort credit research recommendations as at 30 Jun 2008

<table>
<thead>
<tr>
<th></th>
<th>All covered companies</th>
<th>Companies where a Dresdner Kleinwort company has provided investment banking services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overweight</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>Marketweight</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>Underweight</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>51</td>
<td>24</td>
</tr>
</tbody>
</table>

Source: Dresdner Kleinwort Research
Disclaimer

This presentation has been prepared by Dresdner Kleinwort, by the specific legal entity named on the cover or inside cover page. iBoxx and iTraxx are brand names of the International Index Company (IIC). Dresdner Kleinwort is one of 9 investment banks which are shareholders in IIC. An employee of Dresdner Kleinwort or an affiliate is a member of the board of IIC.

United Kingdom: This report is a communication made, or approved for communication in the UK, by Dresdner Bank AG London Branch (authorised by the German Federal Financial Supervisory Authority and by the Financial Services Authority; regulated by the Financial Services Authority for the conduct of UK business, a Member Firm of the London Stock Exchange and incorporated in Germany with limited liability). It is directed exclusively to eligible counterparties and professional clients. It is not directed at retail clients and any investments or services to which the report may relate are not available to retail clients. No persons other than an eligible counterparty or a professional client should read or rely on any information in this report. Dresdner Bank AG London Branch does not deal for, or advise or otherwise offer any investment services to retail clients.

European Economic Area: Where this presentation has been produced by a legal entity outside of the EEA, the presentation has been re-issued by Dresdner Bank AG London Branch for distribution into the EEA. Dresdner Kleinwort Research GmbH is regulated by the Federal Financial Supervisory Authority ("BaFin") by the laws of Germany.

United States: Where this presentation has been approved for distribution in the US, such distribution is by either: (i) Dresdner Kleinwort Securities LLC; or (ii) other Dresdner Kleinwort companies to US Institutional Investors and Major US Institutional Investors only; or (iii) if the presentation relates to non-US exchange traded futures, Dresdner Kleinwort Limited. Dresdner Kleinwort Securities LLC, or in case (iii) Dresdner Kleinwort Limited, accepts responsibility for this presentation in the US. Any US persons wishing to effect a transaction through Dresdner Kleinwort (a) in any security mentioned in this presentation may only do so through Dresdner Kleinwort Securities LLC, telephone: (+1 212) 429 2000; or (b) in a non-US exchange traded future may only do so through Dresdner Kleinwort Limited, telephone: (+1 212) 969 2700. Dresdner Kleinwort may provide hyperlinks to web-sites of entities mentioned in this presentation, including the Disclosure Appendix but notes that, excluding (i) Dresdner Kleinwort Securities LLC and (ii) the research analyst(s) responsible for this presentation unless specifically addressed in the disclosures: (a) Dresdner Kleinwort and its directors, officers, representatives and employees may have positions in or options on the securities mentioned in this presentation or any related investments or may buy, sell or offer to buy or sell such securities or any related investments as principal or agent on the open market or otherwise; and (b) Dresdner Kleinwort may conduct, solicit and/or engage in other investment and/or commercial banking business (including without limitation loans, debt securities and/or derivative, currency and commodity transactions) with the issuers or relating to the securities mentioned in this presentation. Accordingly, information may be available to Dresdner Kleinwort, which is not reflected in this presentation or the disclosures. In this notice “Dresdner Kleinwort” means Dresdner Bank AG and any of its affiliated or associated companies and their directors, officers, representatives or employees and/or any persons connected with them. Additional information on the contents of this presentation is available at www.dresdnerkleinwort.com/research and on request.

© Dresdner Bank AG London Branch 2008