Toward a Better Estimation of Wrong-Way Credit Exposure

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Abstract

In counterparty credit risk management for swaps, forwards, and other derivative contracts, it is recognized that most common applications of credit exposure measures suffer from the deficiency of assuming that counterparty default is independent of the amount exposed. Stress tests are often proposed to handle this deficiency, but stress test measures cannot be applied to pricing and limit setting in the same vein as the standard measures. We introduce a framework to condition standard measures of counterparty exposure on default. The conditional measures thus account for "wrong way" exposures, but fit naturally into current applications.
1 Introduction

The market upheavals of 1998 brought greater attention to market and credit risk management alike. On the credit side, last year’s events pointed out that as crucial as monitoring the credit quality of counterparties is the seemingly simple task of monitoring the amounts actually exposed to these counterparties. Exposure estimation, while straightforward for traditional credit products, becomes more complex when the exposure is contingent on a market factor (e.g. an exchange rate) and, as we will see in this article, more complex still when there is a dependency between counterparty credit quality and the relevant market factor.

Regulators have explicitly recognized the uncertain future credit exposure on swaps, forwards, and other derivative contracts. The Basle Capital Accord requires regulatory capital for current exposure – roughly, the amount which would be lost should the counterparty default today – plus additional "add-on" capital to account for the potential future exposure – the cost of replacing a contract some time in the future – due to moves in the underlying market factor. As to estimating and monitoring exposure, sophistication among practitioners has varied greatly. To address these discrepancies, twelve large commercial and investment banks formed the Counterparty Risk Management Policy Group (CRMPG) in January 1999, and produced a report in June 1999. Fifth of the group’s twelve recommendations was that financial intermediaries "should upgrade their ability to monitor and, as appropriate, set limits for various exposure measures”.

The CRMPG report also highlights four issues that complicate the analysis of credit exposure. The issues read like a list of risk management themes in general. Liquidity, event, and operational concerns are the first three issues. The fourth is the typical assumption that the credit quality of the counterparty is independent of the market factors that underlie the exposure to the counterparty. In fact, the report is not the first criticism of this assumption. Duffee (1996) investigates the assumption empirically, and concludes that over the period 1971-1992, corporate defaults in the U.S. tended to cluster in periods of falling interest rates. For the receive fixed side of U.S. interest rate swaps, this produced a significant positive correlation between exposure size and counterparty default. In Duffee’s example, an exposure measure that accounts for the correlation was on average 65% greater than a comparable measure that maintains the independence assumption.

To address the independence assumption, the CRMPG report proposes stress tests that simultaneously shock
the market factors underlying exposure amounts and the credit factors influencing default. Unfortunately, it is difficult to reconcile stress measures of exposures with the common applications of exposure measures today. In this article, we will examine a number of current applications of exposure measures, and show that it is possible to relax the independence assumption and extend the standard measures in a natural way. The extension allows us to begin with any assumption about the distribution of the underlying risk factor, and to account for dependency between credit quality and market moves without resorting to stress tests.

The remainder of this article is structured as follows: in the Section 2, we define a number of standard measures of credit exposure and discuss their applications; in Section 3, we develop a framework to extend these measures by considering the dependency between counterparty credit quality and the underlying market factor; in Section 4, we present an example exposure calculation using this framework; in Section 5, we discuss a technique to calibrate the parameters of the model; lastly, we summarize and conclude.

## 2 Definitions and uses of exposure measures

In this section, we define a number of exposure measures and discuss their applications. For further details on the definitions and calculations, see Zangari (1997a) and (1997b), or Duffee (1996).

The first distinction between measures is whether they are estimates of current or potential exposures. The definition of *current exposure*, if not its calculation, is straightforward: the current exposure of a contract is the cost of replacing the contract, should the counterparty default today. Intuitively, this is just the current mark to market value of the contract, if the value is positive. If the mark to market value is negative, the current exposure is zero, since the counterparty has no obligations, and there is no cost to replace the contract.

In practice, as the CRMPG report points out, in illiquid markets or for large positions, the replacement cost of a contract will likely be greater than its mark to market value. We will not address this issue here, and will assume that at any time, the mark to market value of a contract gives an accurate assessment of its replacement cost.

In this article, we will treat measures of *potential exposure*. For these measures, we are concerned with the consequences of a counterparty default some time in the future. Thus, we would like to estimate the
replacement cost of the contract, given that the counterparty defaults at some future date. If we fix the date, then we may treat the value (and replacement cost) of the contract on that date as a random variable, and define two basic exposure measures. The expected exposure is the expected replacement cost of the contract in the case of a counterparty default; the maximum exposure is the worst replacement cost, given some level of confidence, that we might incur should the counterparty default.

To make these definitions more concrete, and to aid our discussion later, we introduce some notation. For simplicity, we will assume that only one risk factor underlies our contract. Suppose we wish to estimate exposure at some future date \( t \). Let \( R_t \) denote the (random) value of the risk factor at time \( t \), and \( f_t \) denote the probability density function\(^1\) for \( R_t \). Let \( v_t(r) \) be the mark to market value of the contract, and \( E_t(r) = \max\{0, v_t(r)\} \) be the exposure at time \( t \) given that the risk factor at that time is equal to \( r \). The expected exposure at \( t \) is

\[
E_t[Et(R_t)] = \int_0^\infty dr f_t(r) \cdot E_t(r). \tag{1}
\]

The maximum exposure at \( t \), at confidence level \( q \), is the level \( x \) satisfying

\[
q = P\{E_t(R_t) < x\} = \int_{\{r: E_t(r) < x\}} dr f_t(r). \tag{2}
\]

Since expected and maximum exposure refer to a specific date, there are actually entire profiles of these measures over the life of the contract. The profile of expected exposure, for example, consists of the measure defined in (1) for every \( t \) between today and the maturity of the contract. For practical reasons, it is common to aggregate these profiles into one number. For the expected exposure profile, the aggregate measure is referred to as average exposure, and is defined as the weighted average of the expected exposure measures, with weights proportional to the discount factors for each \( t \). The aggregate measure for the maximum exposure profile is referred to as peak exposure and is defined as the maximum value over the entire maximum exposure profile. We will not concern ourselves further with these aggregate measures, but point out that the methods introduced below can easily be applied in the aggregate, as well as in the single horizon case.

\(^1\)In this article, we will assume \( f_t \) is known. For details on modeling risk factors for exposure estimation, see Zangari (1997a) or Jamshidian and Zhu (1997).
Generically, all exposure measures are utilized to facilitate comparisons between traditional credit products (where the exposure is a fixed quantity) and contracts where the exposure is contingent on one or more underlying market factors. For this reason, it is common to see any of the exposure measures defined above referred to as *loan equivalent exposures*. The basic idea is that all of the effects of market volatility are embedded in the exposure measure, so for credit risk management purposes, a contract with a loan equivalent exposure of 100 can be treated like a loan with a face value of 100.

That the basic premise of loan equivalence is not exactly true is evident in that banks typically use different exposure measures for different purposes. The CRMPG report\(^2\) mentions the emerging practice of credit charges, whereby the credit risk on a swap or derivative contract is transferred within the institution, and the valuation of the contract is adjusted via an internal charge for the "cost of credit". In a sense, this practice can be thought of as the business that originates the contract buying credit protection from another part of the organization. The cost of this protection, as well as the pricing impacts it may have on the contract itself, are most often based on notions of expected default loss, and therefore use expected or average exposure measures. On the other hand, firm-wide counterparty exposure limits are designed to bound the worst case loss in the case of default. Thus, it is typical to apply maximum or peak exposure measures against these limits.

Though the assumption is seldom stated explicitly, these applications, using the exposure measure definitions we have defined, depend on the independence of the credit quality of the counterparty and the market factor underlying the contract. For example, for a contract with an expected exposure of USD 100,000 and a default probability of 1%, we must assume that the default event and exposure amount are independent in order to conclude that the expected default loss on the contract is USD 1,000. Similarly, investing in a contract with a maximum exposure of USD 500,000 and concluding that this contract adds precisely USD 500,000 to our worst case loss in a default assumes that default has nothing to do with exposure. We will see below that the independence assumption is not realistic, prompting us to consider other exposure measures that can serve in the pricing and limit setting applications mentioned here.

\(^2\)Section II D, "Valuation and Exposure Management", pages 29-31.
3 A framework for conditional exposure measurement

As we have already discussed, the typical application of standard exposure measures assumes independence between counterparty default and the value of the contract. To see why this assumption might fail, consider the case where an emerging markets counterparty seeks inexpensive US Dollar funding through a cross currency swap referencing the counterparty’s home currency, or where a highly leveraged counterparty seeks to receive fixed payments in an interest rate swap. Both of these transactions are "wrong-way", in that the scenarios in which the transactions are in-the-money to us (rising interest rates or a strengthening dollar) are likely to coincide with the scenarios in which the two counterparties have difficulty fulfilling their obligations. In these cases, we would still like some measure of exposure that, when multiplied by the counterparty default probability, gives us the expected default loss on the transaction. That is, we would like to apply the exposure measures as we do currently, and so we endeavor to appropriately adjust the measures.

To generalize our computation of expected default loss (or any application of exposure measures), we must concentrate on the conditional distribution of risk factors, given that a default occurs. This will allow us to compute, for instance, the conditional expected exposure at time \( t \), given that a default occurs, or \( E_t\left[ R_t \right| \text{Def} \] . The expected default loss will then be simply \( p_{\text{def}} \cdot E_t\left[ R_t \right| \text{Def} \] , where \( p_{\text{def}} \) denotes the counterparty’s unconditional probability of default. Similarly, to estimate the worst case loss given a counterparty default, we compute the conditional maximum exposure at confidence level \( q \), that is, the level \( x \) such that \( \mathbb{P}\{ E_t(R_t) < x \| \text{Def} \} = q \).

It is tempting to specify the conditional distribution of the risk factors through stress scenarios (as suggested in the CRMPG report) stating that default occurs when the underlying rates move significantly against the counterparty. This approach ignores, however, the possibility that the counterparty defaults for reasons other than a negative move in the transaction with us. Some of the difficulty here is that it is counterintuitive to condition risk factors on a default event. A more intuitive approach is to condition defaults on the risk factors, and use Bayes formula to work backwards.

Suppose our transaction has only one underlying factor. Define \( p_t(r) \) to be the conditional probability of default given that \( R_t = r \). For example, for the swap with the leveraged counterparty above, the function
would give the dependence of counterparty default probability on the swap’s underlying interest rate; presumably, \( p_t(r) \) would be significantly higher for higher interest rate levels. By contrast, when default is independent of the underlying factor, the function \( p_t \) is constant, with \( p_t(r) = p_{\text{def}} \) for all \( r \). In order that this framework be consistent with the unconditional default probability \( p_{\text{def}} \) (that is, the probability of default over all realizations of \( R_t \)), the expected value of \( p_t(R_t) \) must be equal to \( p_{\text{def}} \). This gives us the condition

\[
\int_0^\infty dr \ f_t(r) \cdot p_t(r) = p_{\text{def}}. \tag{3}
\]

We can now express our desired conditional distribution in terms of \( f_t \) and \( p_t \). The conditional probability that \( R_t \) lies below a fixed value \( y \), given that a default has occurred, is

\[
P \{ R_t < y | \text{Def} \} = \frac{P \{ R_t < y \cap \text{Def} \}}{P \{ \text{Def} \}} = \frac{1}{p_{\text{def}}} \cdot \int_0^y dr \ f_t(r) \cdot p_t(r). \tag{4}
\]

From (4), it is clear that the conditional density \( f_t^{\text{def}} \) for \( R_t \), given that a default occurs, is given by

\[
f_t^{\text{def}}(r) = f_t(r) \cdot \frac{p_t(r)}{p_{\text{def}}}. \tag{5}
\]

Thus the conditional density \( f_t^{\text{def}} \) is just the unconditional density \( f_t \) multiplied by the relative probability of default for the given level of the risk factor. This expression illustrates that the conditional density does not depend on the absolute level of the default probability \( p_{\text{def}} \), but only on how the conditional default probability is related to the risk factor. For another counterparty with a higher unconditional default probability, but the same dependence on the risk factor\(^3\), the conditional density in (5), and thus also our conditional exposure estimate, would be identical to the values we compute here.

Our conditional expected exposure on the transaction is given by the integral

\[
\int_0^\infty dr \ f_t^{\text{def}}(r) \cdot E_t(r) = \int_0^\infty dr \ f_t(r) \cdot \frac{p_t(r)}{p_{\text{def}}} \cdot E_t(r). \tag{6}
\]

This is simply the expectation of \( \frac{p_t(R_t)}{p_{\text{def}}} \cdot E_t(R_t) \), which we may evaluate by performing Monte Carlo simulations on \( R_t \). Thus, it is not necessary to even generate scenarios according to the conditional density \( f_t^{\text{def}} \) to compute the conditional expected exposure.

\(^3\)Such as the case where \( p_{\text{def}} = 2p_{\text{def}} \) and \( p_t = 2p_t \).
The conditional maximum exposure at confidence level \( q \) is the level \( x \) satisfying

\[
q = P\{E_t(R_t) < x | \text{Def}\} = \int_{\{r : E_t(r) < x\}} dr \ f_t^{\text{def}}(r) = \int_{\{r : E_t(r) < x\}} dr \ f_t(r) \cdot \frac{p_t(r)}{p_{\text{def}}}.
\]  

(7)

The integral in (7) can also be evaluated using Monte Carlo simulations on \( R_t \). For a large number of independent realizations \( r^{(1)}, r^{(2)}, \ldots, r^{(n)} \) drawn according to the density \( f_t \), we have

\[
\int_{\{r : E_t(r) < x\}} dr \ f_t(r) \cdot \frac{p_t(r)}{p_{\text{def}}} \approx \frac{1}{n} \sum_{k=1}^{n} \frac{p_t(r^{(k)})}{p_{\text{def}}},
\]

(8)

allowing us to choose \( x \) to satisfy (7) without having to perform additional simulations.\(^4\)

Our remaining task is to specify the function \( p_t(r) \). Because we are dealing with wrong way exposures, it is sensible to stipulate that \( p_t(r) \) be monotonic in \( r \). We define a monotonic base function \( g \) such that \( g(0) = 0.5, g(-1) = 0.1, g(1) = 0.9, \) and \( g(z) \) approaches zero and one as \( z \) approaches \(-\infty\) and \( \infty \), respectively. For our examples, we will use the function

\[
g(z) = \frac{1 + \tanh(z_0 \cdot z)}{2},
\]

(9)

where \( z_0 = \tanh^{-1}(0.8) \). Given the base function and the mean (\( \mu \)) and standard deviation (\( \sigma \)) of the risk factor at \( t \), we specify

\[
p_t(r) = p_{\text{Max}} \cdot g\left(\frac{r - \mu - \beta_1 \sigma}{\beta_2 \sigma}\right),
\]

(10)

where \( \beta_1 \) and \( \beta_2 \) are parameters to be specified, and \( p_{\text{Max}} \) is determined by the normalization condition (3).\(^5\)

The specification yields the relationship presented in Figure 1. As the value of the risk factor increases, there is a transition from low to high levels of default probability. The transition is centered around a \( \beta_1 \) standard deviation move, and largely occurs over a range within \( \beta_2 \) standard deviations of this center. We remark that the resemblance of Figure 1 to a cumulative distribution function is due only to our stipulation that \( p_t(r) \) be monotonic. There is no mathematical restriction that precludes \( p_t(r) \) having one or more local maxima, though such a profile would be hard to justify intuitively.

\(^4\)Note that if \( P\{E_t(R_t) > 0 | \text{Def}\} \), the conditional probability that the exposure is positive given a default, is less than \( 1 - q \), then the conditional maximum exposure will be zero. This is actually more likely to be the case where the dependence is opposite, with defaults coinciding with negative mark to market values, and zero exposure amounts.

\(^5\)Note that since \( p_{\text{Max}} \) must be less than or equal to one, (3) imposes restrictions on the permissible values of \( \beta_1 \) and \( \beta_2 \).
4 Example

To illustrate the conditional exposure methodology, we consider an example transaction. Suppose that on May 27, 1999, we put on a five year swap in which we pay quarterly floating interest payments in Thai Baht, and the counterparty pays us fixed interest payments in US Dollars. At maturity, there is an exchange of principal; we pay the counterparty a notional amount in THB, while they pay us a notional amount in USD. The cashflows for the swap are presented in Table 1. For our example, we will concentrate on our expected exposure to the counterparty in one year, meaning that only the cashflows on or after May 27, 2000 will be relevant for our analysis.

If we can assume that the credit quality of the swap counterparty is independent of moves in the rates underlying the swap (the USD term structure, 3 month THB LIBOR, and the THB exchange rate), then we may estimate the swap exposure unconditionally, as in (1) and (2). However, if the swap counterparty happens to be an Asian bank, then the scenarios in which the counterparty defaults are likely to coincide with a depreciation in Asian currencies in general, and THB in particular. These are precisely the scenarios in which the swap is most valuable to us, and so it is imperative that we account for this correlation in our...
Table 1: Cash flows for cross currency swap. USD 100,000 notional. 6.19% fixed coupon on USD. Spot FX rate at initiation: 37.17 THB per USD.

<table>
<thead>
<tr>
<th>Date</th>
<th>Payment type</th>
<th>Pay (THB)</th>
<th>Receive (USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>27-Aug-1999</td>
<td>Interest</td>
<td>LIBOR/4 * 3.717M</td>
<td>1,548</td>
</tr>
<tr>
<td>27-Nov-1999</td>
<td>Interest</td>
<td>LIBOR/4 * 3.717M</td>
<td>1,548</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>27-May-2004</td>
<td>Interest</td>
<td>LIBOR/4 * 3.717M</td>
<td>1,548</td>
</tr>
<tr>
<td>27-May-2004</td>
<td>Principal</td>
<td>3.717M</td>
<td>100,000</td>
</tr>
</tbody>
</table>

exposure estimate.

Figure 2: Normalized one year conditional default probability as a function of THB exchange rate.

For the swap, we will consider three counterparties, all of which with the same rating (and same unconditional default probability). The first is an American corporate who has taken on the exposure to hedge an obligation in Thai Baht, but who has little other exposure to Asian currencies. For this counterparty, a default is not likely to be brought about by a depreciation of the Baht; the conditional probability of default is independent of the THB rate, corresponding to the unconditional case in Figure 2. The second counterparty is exposed to Asian currencies generally. For this counterparty, a default is likely to occur in depreciation scenarios,
but there is not a critical level of the THB rate at which the likelihood of default rises drastically. We set $\beta_1 = -3$ to specify that the transition to a high default probability occurs with about a three standard deviation depreciation, and set $\beta_2 = 2$ to stipulate that the transition is mild. The third counterparty has a significant Thai Baht exposure, but has purchased hedges on this exposure. The hedges, however, only cover depreciations to about 45 THB per USD; this level is thus more critical than for the previous counterparty, and the transition from low to high default likelihoods around this level is rapid. We set $\beta_1 = -3$ and $\beta_2 = 1$ for this case. While the specification of $\beta_1$ and $\beta_2$ here may seem somewhat arbitrary, the values agree with the more rigorous calibration we present in the next section.

Figure 3: Conditional densities for THB in one year.

Given the conditional default profiles, we may now obtain the conditional distribution for the THB exchange rate through (5). We first specify the unconditional\textsuperscript{6} distribution for all of the underlying factors according to the standard RiskMetrics assumptions.\textsuperscript{7} The conditional densities associated with the three default profiles

\textsuperscript{6}Throughout this article, we mean by unconditional that we have not incorporated any information about counterparty default. The distribution of exchange rates is still conditional in the sense of the RiskMetrics Technical Document, in that we have adjusted our volatility parameters to reflect the most recent movements in the factors.

\textsuperscript{7}One technical note is that we assume here that yields follow geometric Brownian motions, whereas in the RiskMetrics Technical Document, it is assumed that discount bond prices follow these processes.
appear in Figure 3; summary statistics for the three distributions appear in Table 2. For our first counterparty, since there is no dependency between default and the exchange rate, the conditional distribution is the same as our unconditional distribution of the THB rate. For the second and third counterparties, the occurrence of a default suggests that the currency has depreciated, and our distributions of THB rates that accompany default are adjusted from the unconditional case.

Table 2: Statistics for THB rate conditional distributions given default.

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th>$\beta_2 = 2$</th>
<th>$\beta_2 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>37.25</td>
<td>39.98</td>
<td>42.70</td>
</tr>
<tr>
<td>SD</td>
<td>2.79</td>
<td>2.82</td>
<td>2.65</td>
</tr>
<tr>
<td>95th percentile</td>
<td>42.01</td>
<td>44.73</td>
<td>46.98</td>
</tr>
<tr>
<td>99th percentile</td>
<td>44.21</td>
<td>46.83</td>
<td>48.76</td>
</tr>
</tbody>
</table>

With the conditional distributions defined, we may now compute conditional exposure measures for the cross currency swap. We first generate scenarios for the USD term structure, THB LIBOR rate, and THB exchange rate according to the unconditional distributions. For each scenario, we value the remaining cashflows for the swap, obtain the swap exposure, and evaluate $p_t(r)/p_{Def}$, the ratio of the conditional to unconditional default probabilities, for our two conditional cases. A set of sample scenarios is presented in Table 3. Observe that in the scenarios where a significant THB depreciation has occurred, the ratio $p_t(r)/p_{Def}$ is at its highest. For instance, in the fourth scenario listed, the likelihood of a counterparty default is 5.6 times higher than average for our $\beta_2 = 2$ counterparty. In our exposure computations, this scenario will be weighted approximately seven times more than either the second or third scenario.

To compute the expected exposure in the unconditional case, we simply average the values (albeit over more scenarios) in the exposure column of Table 3. For the two conditional exposure measures, we take the average of the exposures, weighted in each scenario by the ratio $p_t(r)/p_{Def}$. Similarly, we apply (8) to obtain the conditional maximum exposure measures. We present the results in Table 4. The results are as expected, with increased exposure estimates for the two conditional cases, particularly the $\beta_2 = 1$ case, where the transition from low to high default likelihood is most severe.
Table 3: Scenarios for cross currency swap exposure computation.

<table>
<thead>
<tr>
<th>USD 3m</th>
<th>USD 2y</th>
<th>THB 3m</th>
<th>THB per USD</th>
<th>Swap MTM</th>
<th>Swap Exposure</th>
<th>$\beta_2 = 2$</th>
<th>$\beta_2 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2%</td>
<td>7.3%</td>
<td>3.4%</td>
<td>39.4</td>
<td>3,582</td>
<td>3,582</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>5.7%</td>
<td>5.4%</td>
<td>3.9%</td>
<td>37.8</td>
<td>5,574</td>
<td>5,574</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>6.2%</td>
<td>7.9%</td>
<td>12.7%</td>
<td>37.8</td>
<td>(4,711)</td>
<td>0</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>6.1%</td>
<td>7.1%</td>
<td>2.8%</td>
<td>43.3</td>
<td>12,932</td>
<td>12,932</td>
<td>5.6</td>
<td>16.8</td>
</tr>
<tr>
<td>5.9%</td>
<td>7.0%</td>
<td>2.2%</td>
<td>33.8</td>
<td>(11,222)</td>
<td>0</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4: Conditional exposure measures for 5 year THB/USD cross currency swap. Counterparty pays USD.

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th>$\beta_2 = 2$</th>
<th>$\beta_2 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected</td>
<td>3,370</td>
<td>7,430</td>
<td>12,150</td>
</tr>
<tr>
<td>Maximum (95%)</td>
<td>12,900</td>
<td>18,170</td>
<td>22,650</td>
</tr>
<tr>
<td>Maximum (99%)</td>
<td>17,540</td>
<td>22,560</td>
<td>26,370</td>
</tr>
</tbody>
</table>

In practice, the use of conditional exposure measures induces exactly the type of counterparty risk management desired. Consider the same THB/USD swap, but where the counterparty pays THB and we pay USD. The conditional exposure measures for this transaction are presented in Table 5. Suppose we use maximum exposure at 99% for limit purposes, and that we are allowed equal exposure limits to the three counterparties. With the counterparty paying THB, we can do more transactions with the Asian parties than the uncorrelated party before hitting our exposure limits. On the other side, with the counterparty paying USD, we can do roughly the same number of transactions with the uncorrelated party, and significantly fewer with the parties whose defaults coincide with a strong dollar. Thus, the exposure measures imply little incentive\(^8\) for which side of the transaction to take on with the uncorrelated party, and a strong incentive to pay THB when doing business with the correlated parties. Presumably, such an incentive structure would result in a better matching between counterparties and positions, or at least, in the requirement of greater compensation for the riskier wrong-way trades.

\(^8\)The upward-sloping USD yield curve, and to a lesser extent, our use of the lognormal distribution, explain why the exposures in the independent case are slightly different for the two sides of the transaction.
Table 5: Conditional exposure measures for 5 year THB/USD cross currency swap. Counterparty pays THB.

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th>$\beta_2 = 2$</th>
<th>$\beta_2 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected</td>
<td>2,860</td>
<td>635</td>
<td>73</td>
</tr>
<tr>
<td>Maximum (95%)</td>
<td>12,500</td>
<td>5,090</td>
<td>0</td>
</tr>
<tr>
<td>Maximum (99%)</td>
<td>18,470</td>
<td>10,580</td>
<td>2,980</td>
</tr>
</tbody>
</table>

5 Calibrating the model

In order to specify $\beta_1$ and $\beta_2$, we may rely on counterparty information and intuition, as in the example. Alternately, we may calibrate the parameters to some known characteristics of the conditional distribution. In this section, we present one such calibration technique.

Since there are two parameters to fit, a natural method to calibrate is to specify the mean ($\mu_{\text{Def}}$) and standard deviation ($\sigma_{\text{Def}}$) of the conditional distribution, and find values for $\beta_1$ and $\beta_2$ such that (recall that $f_t^{\text{Def}}$ depends on both $\beta_1$ and $\beta_2$)

$$\int_0^{\infty} dr f_t^{\text{Def}}(r) \cdot r = \mu_{\text{Def}}, \quad (11)$$

and

$$\int_0^{\infty} dr f_t^{\text{Def}}(r) \cdot r^2 = \sigma_{\text{Def}}^2 + \mu_{\text{Def}}^2. \quad (12)$$

It is possible to solve this system of equations numerically. The difficult task is the specification of $\mu_{\text{Def}}$ and $\sigma_{\text{Def}}$.

JP Morgan (1999) presents a procedure to obtain the "residual currency value" upon counterparty default. In this context, residual value refers to the proportion of currency value remaining after a default; thus, a residual value of 70% implies that a default is accompanied by a 30% currency depreciation. The authors consider two possibilities for a (non-sovereign) counterparty default: the first is the default of the sovereign in which the counterparty is domiciled, which forces the default of the counterparty itself; the second is the default of the counterparty without the default of the sovereign. The residual value of the currency in the
case of counterparty default is then the average of the residual value under these two scenarios, weighted by the scenarios’ relative probabilities.

For the first case, the sovereign default, the authors present the results of a study of currency depreciations that accompanied 92 sovereign defaults. In general, the authors found that the currency depreciation was less severe when the defaulting sovereign already carried a low rating, and more severe when the sovereign appeared more sound. They rationalize that for a highly rated sovereign to default, a larger economic shock is needed than for a lower rated sovereign default. The residual currency values by sovereign rating are reproduced in Table 6.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Residual value</th>
<th>Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>17%</td>
<td>83%</td>
</tr>
<tr>
<td>AA</td>
<td>17%</td>
<td>83%</td>
</tr>
<tr>
<td>AA</td>
<td>22%</td>
<td>78%</td>
</tr>
<tr>
<td>BBB</td>
<td>27%</td>
<td>73%</td>
</tr>
<tr>
<td>BB</td>
<td>41%</td>
<td>59%</td>
</tr>
<tr>
<td>B</td>
<td>62%</td>
<td>38%</td>
</tr>
<tr>
<td>CCC</td>
<td>62%</td>
<td>38%</td>
</tr>
</tbody>
</table>


For the second scenario, where the counterparty but not the sovereign defaults, the authors present a simple framework based on the Merton bankruptcy model. Default is assumed to occur when the counterparty’s (stochastic) assets fall below its (fixed) liability level. The link to the currency depreciation is built by assuming that the assets are correlated with changes in the exchange rate. Let $\sigma_{FX}$ denote the exchange rate volatility, and $\rho$ the correlation between the exchange rate and the counterparty’s assets. The residual currency value conditional on counterparty but not sovereign default is given by:

$$ R_{VC} = \left[ 1 + \rho \sigma_{FX} \Phi^{-1}(p_C(h)/2) \sqrt{h} \right] \cdot \frac{1 - p_S(h)}{1 - p_S(h)} R_{VS}, $$

where $h$ is the risk horizon, $R_{VS}$ is the residual currency value given sovereign default, $\Phi$ is the standard

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9 See Merton (1990).
normal cumulative distribution function, and $p_C(h)$ and $p_S(h)$ are the probabilities that the counterparty and sovereign, respectively, default before the risk horizon. The authors remark that $RVC$ is the product of two terms: the first accounts for the expected depreciation given the counterparty default, while the second accounts for the (small) expected appreciation given the non-default of the sovereign. In the first term, observe that $\Phi^{-1}(p_C(h)/2)\sqrt{h}$ is the number of standard deviations the counterparty’s assets must decline in order to produce a default, while $\rho$ times this quantity is the expected number of standard deviations the exchange rate moves, given that the assets have declined to the default level. The second term is derived from the condition that, averaged over default and non-default of the sovereign, there should be no expected move (other than that toward the forward rate) of the exchange rate.

Following our earlier example, we consider a B-rated counterparty domiciled in a BB-rated sovereign.\footnote{At this writing, Thailand carried a long term rating of Ba1 from Moody’s and BBB- from Standard & Poor’s.} The historical one year default rates for these ratings are 6.50% and 1.34%.\footnote{Keenan et al (1999). Note that JP Morgan (1999) utilizes risk neutral, rather than historical, default rates.} From Table 6, the residual currency value upon sovereign default, $RV_S$, is 41%. To compute the residual value upon counterparty but not sovereign default, we take the same THB exchange rate volatility (7.5% annual) as in our prior example, and the same asset-exchange rate correlation (40%) as in the JP Morgan article. Applying (13) yields a residual value, $RV_C$, of 95.2%. Since we have assumed that sovereign default implies counterparty default, the probability of counterparty but not sovereign default is the difference of the two default rates, or 5.16%. We then take the average of the two residual values, weighted by the probabilities of the two default events, obtaining a residual currency value, conditional on counterparty default, of $(1.34\%RV_S + 5.16\%RV_C)/6.50\%$, or 84.06%.

The residual value implies a 15.94% depreciation, corresponding to $\mu_{def} = 42.95$ THB per USD. Again following the JP Morgan article, we assume the same standard deviation for the conditional distribution as for the unconditional; from Table 2, we obtain $\sigma_{def} = 2.79$. Solving (11) and (12) gives $\beta_1 = -3.37$ and $\beta_2 = 0.985$, comparable to our example with $\beta_1 = -3$ and $\beta_2 = 1$. We repeat this exercise for all pairs of sovereign and counterparty ratings. The resulting currency depreciations and $(\beta_1, \beta_2)$ values are presented in Tables 7 and 8.
Table 7: Expected currency depreciations conditional on counterparty default.

<table>
<thead>
<tr>
<th>Sovereign rating</th>
<th>Default rate</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.01%</td>
<td>47.06%</td>
<td>28.69%</td>
<td>14.39%</td>
<td>7.96%</td>
<td>5.63%</td>
<td>3.38%</td>
</tr>
<tr>
<td>AA</td>
<td>0.02%</td>
<td>-</td>
<td>46.79%</td>
<td>19.29%</td>
<td>8.51%</td>
<td>5.74%</td>
<td>3.40%</td>
</tr>
<tr>
<td>A</td>
<td>0.04%</td>
<td>-</td>
<td>-</td>
<td>27.74%</td>
<td>9.48%</td>
<td>5.94%</td>
<td>3.44%</td>
</tr>
<tr>
<td>BBB</td>
<td>0.15%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>14.65%</td>
<td>6.98%</td>
<td>3.65%</td>
</tr>
<tr>
<td>BB</td>
<td>1.34%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>15.94%</td>
<td>5.47%</td>
</tr>
<tr>
<td>B</td>
<td>6.50%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10.05%</td>
</tr>
<tr>
<td>CCC</td>
<td>26.16%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 8: Values of \((\beta_1, \beta_2)\), calibrated to conditional depreciations.

<table>
<thead>
<tr>
<th>Sovereign</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>(-5.42, 0.27)</td>
<td>(-3.24, 1.08)</td>
<td>(-2.73, 1.81)</td>
<td>(-2.54, 2.42)</td>
<td>(-2.22, 3.62)</td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>(-6.01, 0.38)</td>
<td>(-3.66, 0.83)</td>
<td>(-2.77, 1.71)</td>
<td>(-2.55, 2.39)</td>
<td>(-2.22, 3.60)</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>(-4.48, 0.61)</td>
<td>(-2.85, 1.56)</td>
<td>(-2.57, 2.32)</td>
<td>(-2.23, 3.57)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBB</td>
<td>(-3.26, 1.06)</td>
<td>(-2.66, 2.03)</td>
<td>(-2.28, 3.41)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BB</td>
<td>(-3.37, 0.98)</td>
<td>(-2.35, 2.48)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>(-2.89, 1.48)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 Conclusions

A key deficiency in the management of counterparty exposure is the failure to account for the dependency between exposure size and counterparty default. While stress tests are often proposed to address this issue, it is difficult to utilize stress test exposure measures within the frameworks in which standard exposure measures are used currently. In this article, we present a methodology which accounts for the dependency between exposure size and counterparty default, while producing exposure measures that can be applied in the same way as the existing measures. The methodology is flexible enough to utilize any forecasted distribution of risk factors, and requires the specification of only two parameters related to the dependency in question. We show through an example that the new exposure measures provide more prudent incentives for counterparty
exposure management.

References


