THRESHOLDS FOR RATINGS' FORECAST DEFAULT PROBABILITIES: A MEAN SQUARED ERROR BASED APPROACH

by G. Bichisao[†], E. Grillo, M. Marchesi[*], C. Zucca[†]

Preliminary version.– December 12th 2005

Abstract:

Given the increased importance rating agencies have assumed in the determination of credit institutions' capital requirements according to the Standardised Approach in the Basel II framework, the Basel Committee for Banking Supervision has proposed a procedure to "map" the ratings of different rating agencies into the risk weights that determine credit institutions' capital requirements. This procedure is built upon Cumulative Default Rates (CDRs) and is based for each rating class on reference values and upper admissible thresholds for observed CDRs. In fact, whenever the upper thresholds are exceeded, supervisors are encouraged to enquiry over the rating agencies' rating assignment standards. Thresholds are calculated on the basis of Monte Carlo simulations and assigned using the 99th and the 99.9th percentile of the distribution so obtained.

Some observers notice that this procedure seems to lead to very broad ranges, and worry that this could prevent the prompt detection of on-going weaknesses in the rating assignment standards followed by some rating agencies. In this paper, an approach to determine thresholds for ratings' forecast default probabilities using a test statistic based on the Mean Squared Error (MSE) over realized CDRs is presented. Such approach could represent a potential improvement of the "mapping" procedure suggested by the Basel Committee as the MSE can take directly into account the reference value suggested by the Basel Committee for each rating class, and it can also test the hypothesis of that reference value being correct.

[†] European Investment Bank.
[*] European Commission.

Corresponding Author: C. Zucca: email: zucca@eib.org
Tel.: +35224856033

The views expressed in the paper are those of the authors and do not necessarily correspond to the views of the European Investment Bank or of the European Commission.

1. Introduction

The Basel II framework has put credit risk measurement at the heart of credit institutions' capital requirements: the credit risk embedded in each credit institution exposure is, in fact, measured. And such measure, after some calculations, determines the credit institution's capital requirement stemming from that exposure.

In the Standardised Approach, capital requirements depend on risk weights that are determined on the basis of rating agencies' ratings. In order to link rating agencies' ratings with risk weights, the Basel Committee on Banking Supervision ('Basel Committee') has issued as guidance a "mapping" procedure that sets a common ground for comparing ratings coming from different rating agencies and possibly assigned on the basis of different rating philosophies.

Ratings can in fact be assigned mainly on the basis of two alternative rating philosophies. "Through-the-cycle" (TTC) ratings tend to remain more-or-less constant as macroeconomic conditions change over time. "Point-in-time" (PIT) ratings tend instead to adjust quickly to the changing economic environment. Between these two extreme cases lie hybrid rating systems that embody characteristics of both TTC and PIT rating philosophies.

The main practical consequence of this difference in the rating philosophy is the fact that TTC ratings tend to be associated with a constant (through-the-cycle) default rate for each class of obligors (so that obligors normally move between rating classes). PIT ratings tend instead to be associated with a default rate for each class of obligors that changes by virtue of changes in the current business conditions (so that obligors do not normally move between rating classes).

The "mapping" procedure of the Basel Committee is built upon Cumulative Default Rates ("CDRs") and based for each rating class on reference values and upper admissible thresholds for observed CDRs. In fact, whenever the upper thresholds are exceeded, supervisors are encouraged to enquiry over the rating agencies' rating assignments standards.[1] Thresholds are calculated on the basis of Monte Carlo Simulations, and assigned using the 99th and the 99.9th percentile of the distribution so obtained.

Some observers notice that this procedure seems to lead to very broad ranges, and worry that this could prevent the prompt detection of on-going weaknesses in the rating assignment standards followed by some rating agencies. In this paper, an approach to determine thresholds for ratings' forecast default probabilities using a test statistic based on the Mean Squared Error ("MSE") over realized CDRs is presented. Such approach could represent a potential improvement of the "mapping" procedure suggested by the Basel Committee as the MSE can take directly into account the reference value suggested by the Basel Committee for each rating class, and it can also test the hypothesis of that reference value being correct.

The presented approach follows an idea proposed by Rauhmeier and Scheule (2005). The idea to use MSE to test the accuracy of forecasts had, however, already been proposed in previous some articles of statistical medicine regarding clinical forecasts. In particular Spiegelhalter (1986) suggested the test statistic used throughout this paper, and some ideas are also due to Redelmeier, Bloch and Hickam (1991). The Basel Committee has finally also considered that the MSE, or Brier Score, is a measure of discriminatory power in the validation of banks' internal rating systems[2].

The paper is organized as follows: Section 2 describes the "mapping" procedure suggested by the Basel Committee and its potential issues. Section 3 presents the MSE based "mapping" approach and the related test statistic. The simulations and their results for each rating class are reported in Sections 4a and 4b. Section 5 concludes providing a few comments on the results obtained and some suggestions for further research.

[1] Therefore, even if without any explicit reference to it, the Basel Committee seems to have in mind a system of TTC ratings.
[2] Basel Committee on Banking Supervision (2005).

2. The Basel Committee approach to evaluate ratings’ CDRs

In order to “map” the credit assessments of different rating agencies (referred to in Basel II as External Credit Assessment Institutions – ECAIs) into the risk weights that determine credit institutions’ capital requirements, the Basel Committee has issued as guidance a “mapping” procedure attached to the Basel II framework (Annex II)[3].

The “mapping” procedure suggested by the Basel Committee is based, among other things, on two different quantitative indicators for each rating class:

- the ten year average of three year Cumulative Default Rates (“CDRs”) in the rating class;
- the two most recent three year CDRs in the rating class.

The Basel Committee has proposed for these two measures the benchmarks reported in Table I and Table II.

Table I

Long-run “reference” three year CDRs benchmarks

S&P assessment (Moody’s)	**AAA-AA (Aaa-Aa)**	**A (A)**	**BBB (Baa)**	**BB (Ba)**	**B (B)**
10 year average of three year CDR	0.10%	0.25%	1.00%	7.50%	20.00%

Table II

Three year CDR benchmarks

S&P assessment (Moody’s)	**AAA-AA (Aaa-Aa)**	**A (A)**	**BBB (Baa)**	**BB (Ba)**	**B (B)**
Monitoring Level	0.8%	1.0%	2.4%	11.0%	28.6%
Trigger Level	1.2%	1.3%	3.0%	12.4%	35.0%

[3] Basel Committee on Banking Supervision (2004).

The benchmark for the first indicator (ten year average of three year CDRs) shows the expectation the Basel Committee has regarding the long-run CDR which should be observable in each rating class of any eligible ECAI. Since an ECAI's own long-run CDR will generally not exactly match the benchmark for the first indicator, two different upper thresholds (Monitoring Level and Trigger Level) have been proposed as benchmarks for evaluating any upward deviations in the two most recent three year CDRs from the long-run expected average.

The Monitoring and Trigger Levels are in particular used in the following way:

- if the last CDR exceeds neither the Monitoring nor the Trigger Level, the "mapping" of the rating class is considered correct and no further enquiry is carried on;
- if the last CDR exceeds the Monitoring Level, but does not exceed the Trigger Level, supervisors should evaluate whether the ECAI correctly assesses the credit risk;
- if the last (and a fortiori the last two) CDRs exceed the Trigger Level, supervisors should conduct an investigation to check whether ratings are correctly assigned in the ECAI. Supervisors are generally expected from the Basel Committee to revise in a more conservative way the "mapping" of ECAIs' ratings unless they have solid reasons to still maintain the existing "mapping".

It should be noted that for each rating class the Monitoring and even more the Trigger Level are set by the Basel Committee much higher than the long-run reference CDR for each rating class. This means that the last two three year CDRs are possibly allowed to exceed long-run benchmarks by a large amount without any investigation on the ECAI's credit risk measurement standards taking place on the initiative of a supervisor. For example, the three year CDR for a rating A could go very close to 1% (i.e. very close to the BBB long-run benchmark CDR) without possibly triggering any supervisor's enquiry to check the soundness of credit risk measurement standards.

Some observers seem to consider this as a potential problem, i.e. they seem to think that the Monitoring and Trigger Levels set by the Basel Committee for the various rating classes might be so high that it might in fact be easy to overlook and/or react only too late to problems in the on-going performance of an ECAI's rating system.

The Monitoring and Trigger Levels suggested by the Basel Committee are produced with Monte Carlo simulations and derived respectively from the 99th and the 99.9th percentile of the distributions so obtained.

We now present an approach to calculate - via Monte Carlo simulations - thresholds for realized CDRs, which represents an attempt to contribute to a possible future improvement of the existing Monitoring and Trigger Levels. In fact the presented approach, based on the Mean Squared Error ("MSE") or Brier Score, could represent an improvement of the procedure suggested by the Basel Committee for setting rating thresholds as the MSE can take directly into account the reference value of the long-run average CDR suggested by the Basel Committee for each rating class, and it can also test the hypothesis of that reference value being correct.

3. The MSE approach

If we define $\hat{\pi}_k$ as the reference (forecast) CDR that a certain ECAI assigns to the k^{th} rating class, the proposed approach tests whether the realized CDR within the k^{th} rating class is coherent with the forecast CDR assigned to it by the ECAI or, to put it in another way, whether the observed CDR is likely to have an expected value (π_k) equal to $\hat{\pi}_k$.

The MSE for a rating class k, where N_k issuers are rated, is defined as[4]:

$$MSE_k = \frac{1}{N_k} \sum_{i=1}^{N_k} (y_{ik} - \hat{\pi}_k)^2 \qquad (1)$$

where y_{ik} are binary variables, called default indicators, which assume value 1 if the i^{th} borrower in the k^{th} rating class defaults, 0 otherwise.

Let us assume as null hypothesis that the forecast CDR is correct, i.e.:

---

[4] In the following, it is assumed that: Cov(yi,yj)=0 for i≠j, i.e. that default events are independent. More general correlation assumptions could be the argument for further research: however, it has to be kept in mind that the zero correlation assumption is likely to prove to be a conservative assumption when considering the results of the analysis.

H0: $\hat{\pi}_k = \pi_k = E(y_{ik})$ for i=1,2,…,N_k (2)

It is also possible to write down the Expected Value and the Variance of the MSE:

$$E(MSE)_k = \frac{1}{N_k}\sum_{i=1}^{N_k}\left[\pi_k(1-\pi_k)\right] \quad (3)$$

$$Var(MSE)_k = \frac{1}{N_k^{\ 2}}\sum_{i=1}^{N_k}\left[\pi_k(1-\pi_k)(1-2\pi_k)^2\right] \quad (4)$$

Under the null hypothesis that the reference CDR is correct, (3) and (4) can be rewritten as:

$$E(MSE_{\hat{\pi}=\pi})_k = \frac{1}{N_k}\sum_{i=1}^{N_k}\left[\hat{\pi}_k(1-\hat{\pi}_k)\right] \quad (5)$$

$$Var(MSE_{\hat{\pi}=\pi})_k = \frac{1}{N_k^{\ 2}}\sum_{i=1}^{N_k}\left[\hat{\pi}_k(1-\hat{\pi}_k)(1-2\hat{\pi}_k)^2\right] \quad (6)$$

Recalling that the MSE is a sum of N_k i.i.d. random variables, under the null hypothesis, the Central Limit Theorem assures that

$$Z_k = \frac{MSE_k - E(MSE_{\hat{\pi}=\pi})_k}{\sqrt{Var(MSE_{\hat{\pi}=\pi})_k}} \quad (7)$$

has an asymptotic standard normal distribution.

The MSE-based Z statistic can now be used to test the correctness of the reference (forecast) CDR. In fact, if the forecast CDR is correct, π_k can be replaced with $\hat{\pi}_k$ and statistical theory assures that Z is asymptotically standard normally distributed. And, if the realised CDR does not match the forecast CDR, only values within certain thresholds - calculated via standard hypothesis testing statistical techniques - will be compatible, with a certain degree of confidence, with the fact that the Z statistic is standard normally distributed.

4a. The simulations: first set of results

Thresholds for acceptable CDRs have been calculated via Monte Carlo simulations: for each rating class, a size of 5,000 issuers has been assumed and their default times have been generated using a standard intensity default process[5], using as long-run CDRs the same long-run CDRs proposed by the Basel Committee. Default indicators have then been generated assuming (consistently with the Basel Committee proposal) a period of 3 years: hence a default indicator is equal to 1 if the default time is equal or lower than 3, equal to 0 otherwise.

Average number of defaults generated following this procedure is reported in Table III: as one can see, the model leads to default rates very close to the CDRs associated to the rating class. For each rating class the procedure has been repeated 50 times to obtain robust results[6].

Table III

Three year default rates obtained with the simulations

Rating class	Average number of defaults	Default rate	Basel II's long-run reference CDR
AAA-AA	5.27	0.11%	0.10%
A	12.05	0.24%	0.25%
BBB	49.69	0.99%	1.00%
BB	375.25	7.51%	7.50%
B	1000	20%	20.00%

[5] For an introduction to this class of models see, for example, Jarrow and Turnbull (1995)

[6] When different and higher number of repetitions have been tried the average results does not significantly change. Hence, on computational time grounds, it has been decided to limit the number of the repetitions to 50.

Recalling that the Z statistic is standard normal distributed under H0, ranges for realized three year CDRs have been calculated. In fact, if the forecast CDR is not close in a statistical sense with the real CDR (i.e. the one which default indicators have been generated with), the values the Z statistic assumes will be too extreme to suppose, with a certain level of confidence, a standard normal distribution for Z.

Following this reasoning, both the upper and the lower thresholds compatible with confidence levels of 95%, 99%, and 99.9% have been calculated. In fact, from the point of view of a supervisor both these measures could be interesting: while the upper threshold gives a measure of the highest acceptable underestimation of credit risk (a measure which is therefore comparable to the Monitoring and Trigger Levels suggested by the Basel Committee), the lower threshold gives a measure of the maximum acceptable overestimation of credit risk.

The results of the simulations for each rating class are reported in Table IV.

Table IV

Thresholds for ratings' CDRs obtained via the MSE approach

Rating class	Basel II long-run CDR	Basel II Monitoring and Trigger Levels	Confidence level	Lower threshold	Upper threshold
AAA-AA	0.10%	Monitoring Level 0.8% Trigger Level 1.2%	95% 99% 99.9%	0% 0% 0%	0.19% 0.22% 0.24%
A	0.25%	Monitoring Level 1.0% Trigger Level 1.3%	95% 99% 99.9%	0.13% 0.09% 0%	0.38% 0.42% 0.50%
BBB	1%	Monitoring Level 2.0% Trigger Level 3.00%	95% 99% 99.9%	0.77% 0.68% 0.59%	1.27% 1.31% 1.47%
BB	7.50%	Monitoring Level 11.0% Trigger Level 12.4%	95% 99% 99.9%	6.83% 6.67% 6.35%	8.16% 8.34% 8.68%
B	20.00%	Monitoring Level 28.6% Trigger Level 35.0%	95% 99% 99.9%	19.14% 18.59% 18.25%	21.03% 21.28% 21.82%

As it can be seen from Table IV, the thresholds obtained following this approach define ranges stricter than the ones suggested by the Basel Committee (Table II). For example, whereas the Basel II Monitoring Level potentially allows a CDR very close to 1% for obligors classified in rating A without any check of the credit risk measurement standards being performed, the proposed approach would already trigger some supervisory control when the realized CDR is somewhere between 0.42% and 0.50%. A similar result is valid also for all other rating classes.

The stricter ranges obtained via the MSE approach could be useful to overcome the potential problem perceived by some observers with the "mapping" procedure suggested in the Basel framework, i.e. the excessive tolerance for realized CDRs being higher than the long-run benchmarks.

Robustness of results have been checked with a smaller sample in the Monte Carlo analysis: when assuming fewer obligors for each rating class, the obtained thresholds resulted slightly wider than those derived from the larger sample Monte Carlo analysis, but still significantly lower than the Monitoring and Triggering Levels proposed by the Basel Committee.

It is finally worth mentioning that a specific analysis has been carried out to check under what conditions the Monitoring or the Trigger Levels proposed by the Basel Committee would be admissible on the basis of the MSE-based statistic test. Such thresholds would be admissible only with a confidence level extremely close to 100% and such a level would prevent any rejection of the null hypothesis. In Table V we also report in the Annex the value the Z statistic should, on average, assume in order to determine a threshold equal to either the Trigger or the Monitoring Level.

4b. The simulations: second set of results

In order to get a sense of the discriminatory power of this procedure, for each rating class a given percentage of obligors has been assumed to have the default probability of a different rating class. In this way, it has therefore been simulated the possibility that an ECAI assigns some obligors to the wrong rating class.

Three different partitions of borrowers have been tested: in each partition 1/3, 1/2 and 2/3 of the borrowers of any rating class have a default probability not equal to the long-run CDR of the rating class they belong (CDR1), but equal to the long-run CDR of another rating class (CDR2). The experiment has been repeated 50 times and the results obtained are reported in Table VI.

As the last column of Table VI shows, in nearly all cases the Z statistic assumes a value too extreme to be likely for a standard Normal distribution, leading consequently to the rejection of the null hypothesis. Hence, the procedure based on the Z statistic "recognizes" that not all the borrowers are correctly assigned and signals this problem by rejecting H0.

The hypothesis of correct forecast, however, is not refused when the two highest rating classes (AAA-AA and A) are considered. This particular result is probably due to the closeness of the long-run CDRs associated with these two classes (0.10% and 0.25%). However, since the difference in the CDRs of these two classes is also small, this should not create excessive problems from a supervisory point of view.

Table VII reports the maximum percentage of observations generated with CDR2 that, following this same procedure, the Z statistic allows. To put it in another way, if the percentage of borrowers wrongly assigned exceeds these percentages, the test statistic Z will reject the null hypothesis of correct forecast. The analysis has been carried out for three different levels of confidence: 95%, 99% and 99.9%.

Also under this perspective, the test statistic Z shows a good discriminatory power: high percentages of incorrectly assigned borrowers are allowed only for the two highest classes, while percentages for the other classes vary between 4% and 14%, preventing the number of borrowers wrongly assigned to better rating classes from becoming material.

The analysis has finally been repeated to find the CDRs upper thresholds in case of samples smaller than the ones initially considered. In particular, a number of borrowers in each rating class equal to the number of issuers rated by S&Ps in 2004[7] has been assumed. Also in this case the simulation has been repeated 50 times for each rating class so as to have a sense of the distribution of the results in case of smaller samples. Results are presented in Table VIII.

The ranges defined using smaller samples are, as expected, broader than the one established with the initial simulations, but they are still narrower than the ones proposed

[7] Standard & Poor's (2005).
[8] Standard & Poor's (2005).

by the Basel Committee. Nevertheless, the issue of the size of the sample is very relevant for the proposed approach to CDRs' thresholds: the bigger the sample, the smaller the range of CDR thresholds that can be expected.[9]

5. Conclusion

In the Standardised Approach of the Basel II framework, capital requirements depend on risk weights that are determined on the basis of rating agencies' ratings. In this paper we present an approach based on a recent article by Rauhmeier and Scheule (2005) to determine thresholds for ratings' forecast default probabilities using a test statistic based on the Mean Squared Error (MSE) over realized Cumulative Default Rates (CDRs). Such approach could represent an improvement of the "mapping" procedure suggested by the Basel Committee as the MSE can take directly into account the reference value suggested by the Basel Committee for each rating class, and it can also test the hypothesis of that reference value being correct.

This approach has been tested via Monte Carlo simulations on samples of 5,000 borrowers for each rating class and the CDRs' thresholds so obtained result to be narrower than the ones proposed by the Basel Committee. Furthermore it has been shown that the CDRs chosen as Monitoring and Trigger Levels by the Basel Committee could not be reached in this test unless allowing for a 100% confidence interval.

It has been checked that results do not depend much on the very large sample size used to implement the simulation. In particular, ranges stricter than the Basel II ones have been obtained also using, for each rating class, samples whose size is equal to the number of issuers rated by Standard & Poor's in 2004.

The proposed test is furthermore able to "recognize" small sub-samples of observations associated with higher default rates in nearly all cases. Some problems have been noted only with the two lowest risk rating classes, but such problem should probably not be considered as material.

[9] It is however also important to mention that the thresholds calculated in the paper are just the result of a statistical procedure, while the way in which they might be used for supervisory purposes can be very different according to the specific procedures in use across supervisors.

Further and deeper researches and investigations could be carried out in order to explore and overcome the limitations of the approach presented. In particular throughout this paper, independence of defaults has been assumed to benefit from using the Central Limit Theorem[10]. The assumption of defaults' independence could of course be relaxed in several ways; however, the authors believe that its assumption is likely to produce more conservative CDRs' thresholds.

[10] Evidence exists, however, on the fact that defaults are correlated See, for example, Das, Freed, Geng and Kapdia (2004) for empirical results and Basel Committee on Banking Supervision (2005) pag. 47-52 for an analysis of the consequences of defaults' correlation.

Annex

Table V

Confidence Level to obtain Basel II thresholds with MSE approach

Rating class	Long-run CDR	Monitoring Level	Average Value of test statistic Z to obtain Monitoring Level	Associated confidence level	Trigger Level	Average Value of test statistic Z to obtain Trigger Level	Associated confidence level
AAA-AA	0.10%	0.8	-5.6119	100%	1.2%	-7.1602	100%
A	0.25%	1.0%	-5.3564	100%	1.3%	-6.5206	100%
BBB	1%	2.4%	-5.0745	100%	3.0%	-11.888	100%
BB	7.50%	11.0%	-8.1918	100%	12.4%	-11.258	100%
B	20%	28.6%	-16.3006	100%	35.0%	-33.504	100%

Table VI

Z values for samples with a sub-sample with higher or lower CDR

Rating Class	Number of observations with CDR 1	Number of observations with CDR 2	CDR 1	CDR 2	Average value for the Z statistic
AAA-AA	3750	1250	0.10%	0.25%	0.7338
	2500	2500	0.10%	0.25%	1.4885
	1250	3750	0.10%	0.25%	2.5146
A	3750	1250	0.25%	1.00%	2.5771
	2500	2500	0.25%	1.00%	5.1712
	1250	3750	0.25%	1.00%	8.1844
	3750	1250	0.25%	0.10%	-0.2775
	2500	2500	0.25%	0.10%	-1.0138
	1250	3750	0.25%	0.10%	-1.3084
BBB	3750	1250	1.00%	7.50%	11.442
	2500	2500	1.00%	7.50%	22.758
	1250	3750	1.00%	7.50%	34.638
	3750	1250	1.00%	0.25%	-1.4043
	2500	2500	1.00%	0.25%	-2.5783
	1250	3750	1.00%	0.25%	-3.9257
BB	3750	1250	7.50%	20.00%	8.2869
	2500	2500	7.50%	20.00%	16.792
	1250	3750	7.50%	20.00%	25.303
	3750	1250	7.50%	1.00%	-4.3459
	2500	2500	7.50%	1.00%	-8.8818
	1250	3750	7.50%	1.00%	-13.155
B	3750	1250	20.00%	7.50%	-5.5713
	2500	2500	20.00%	7.50%	-11.261
	1250	3750	20.00%	7.50%	-16.564

Table VII

Maximum number of observations with a higher or lower CDR without our test rejecting the null hypothesis

Rating class	CDR 1	CDR 2	Maximum number of observations generated with CDR 2 and test not rejecting null hypothesis	Confidence level
AAA-AA	0.10%	0.25%	54%	95.00%
	0.10%	0.25%	71%	99.00%
	0.10%	0.25%	80%	99.90%
A	0.25%	1.00%	19%	95.00%
	0.25%	1.00%	23%	99.00%
	0.25%	1.00%	31%	99.90%
	0.25%	0.10%	74%	95.00%
	0.25%	0.10%	90%	99.00%
	0.25%	0.10%	99%	99.90%
BBB	1.00%	7.50%	4%	95.00%
	1.00%	7.50%	5%	99.00%
	1.00%	7.50%	7%	99.90%
	1.00%	0.25%	32%	95.00%
	1.00%	0.25%	43%	99.00%
	1.00%	0.25%	55%	99.90%
BB	7.50%	20.00%	5%	95.00%
	7.50%	20.00%	7%	99.00%
	7.50%	20.00%	9%	99.90%
	7.50%	1.00%	10%	95.00%
	7.50%	1.00%	14%	99.00%
	7.50%	1.00%	17%	99.90%
B	20.00%	7.50%	7%	95.00%
	20.00%	7.50%	11%	99.00%
	20.00%	7.50%	14%	99.90%

Table VIII

Rating class	Number of observations	Long-run CDR	Monitoring and Trigger Level	Upper threshold for S&Ps sample	Upper threshold for larger sample	Confidence level	Standard deviation
AAA-AA	531	0.10%	0.80% 1.20%	0.35% 0.55% 0.55%	0.19% 0.22% 0.24%	95% 99% 99.9%	0.00252 0.00325 0.00325
A	1209	0.25%	1.00% 1.30%	0.49% 0.61% 0.70%	0.38% 0.42% 0.50%	95% 99% 99.9%	0.00211 0.00223 0.00236
BBB	1474	1%	2.00% 3.00%	1.44% 1.55% 1.77%	1.27% 1.31% 1.47%	95% 99% 99.9%	0.00323 0.00336 0.00375
BB	983	7.50%	11.00% 12.40%	9.01% 9.52% 10.22%	8.16% 8.34% 8.68%	95% 99% 99.9%	0.00917 0.00960 0.10615
B	854	20%	28.60% 35.00%	22.45% 23.51% 24.30%	21.03% 21.28% 21.82%	95% 99% 99.9%	0.01305 0.01337 0.01476

References

Basel Committee on Banking Supervision (2005), "Studies on the Validation of Internal Rating Systems", *Working Paper*, No. 14

Basel Committee on Banking Supervision (2004), "International Convergence of Capital measurement and Capital Standards, a Revised Framework"

Das S., Freed L., Geng G., Kapdia N. (2004), "Correlated Default Risk"

Jarrow R., Turnbull S. (1995), "Pricing derivatives on financial securities subject to credit risk", *Journal of Finance*, 50 (1)

Rauhmeier R., Scheule H. (2005), "Rating Properties and their Implications for Basel II capital"*, Risk Magazine*, March 2005

Redelmeier D, Bloch D., Hickman D. (1991), "Assessing Predictive Accuracy: how to compare Brier Score", *Journal of Clinical Epidemiology*, 44, 1141-1146

Spiegelhalter D. (1986) "Probabilistic Prediction in Patient Management and Clinical Trials", *Statistic in Medicine,* 5, 421-43

Standard & Poor's (2005), "Annual Global Corporate Default Study"