



The impact of default risk on the prices of options and other derivative securities

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Abstract

This paper presents a model for valuing derivative securities when there is default risk. The holder of a security is assumed to recover a proportion of its no-default value in the event of a default by the counterparty. Both the probability of default and the size of the proportional recovery are random. The paper shows how data on bonds issued by the counterparty can be used to provide information about model parameters.

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1. Introduction

When an option or other derivative security is valued it is customary to assume that there is no risk that the counterparty will default. For an exchange-traded option or futures contract this assumption is usually a reasonable one since most

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exchanges have been very successful in organizing themselves to ensure that their contracts are always honored. The no-default assumption is far less defensible in the non-exchange-traded (or over-the-counter) market. In recent years this market has become increasingly important. Many financial institutions actively trade a variety of forward contracts, swaps, and options with other financial institutions and with corporate clients.

This paper proposes a model for reflecting default risk in the calculated prices of over-the-counter options and other derivative securities in a way consistent with the prices of other securities that are subject to default risk. The importance of considering default risk in over-the-counter markets is recognized by bank capital adequacy standards. These standards require each over-the-counter derivative security purchased by a bank from a corporation or from another financial institution to be supported by an amount of capital related to the value of the security and the creditworthiness of the counterparty.¹

Most previous research concerned with the default risk in derivative securities has focused on linear products such as forward contracts and swaps. [See for example Kane (1980), Belton (1987), Wall and Fung (1987), Hull (1989), and Cooper and Mello (1991).] An exception is Johnson and Stulz (1987). These authors examine the impact of default risk on option prices by assuming stochastic processes for the value of the assets of the option writer and the value of the asset underlying the option. They succeed in deriving closed form solutions for the prices of European options in a number of different situations and show that the comparative statics for options subject to default risk are liable to be markedly different from those for default-free options. Following Johnson and Stulz we will refer to a derivative security subject to default risk as a vulnerable derivative security.

Johnson and Stulz assume that, if the counterparty writing an option is unable to make a promised payment, the holder of a derivative security receives all the assets of the counterparty. This assumption is reasonable if there are no other claims on the assets of the counterparty that rank equally with the derivative security in the event of a default.² In this paper we extend the Johnson and Stulz model to cover situations where other equal ranking claims can exist. We assume that we know, or can estimate, the impact of default risk on bonds that have been (or could be) issued by the counterparty and rank equally with the derivative security. These are used to provide information on the impact of default risk on the derivative security itself.

The rest of this paper is organized as follows. Section 2 outlines the model and applies it to European options and American options. Section 3 considers a special

¹ See Bank for International Settlements (1988).

² The Johnson and Stulz model can be extended to cover the situation where there are claims senior to the one under consideration by defining 'assets' as total assets net of the senior claims.

case of the model that is analytically very tractable. Section 4 applies the model to derivatives such as swaps that can become either assets or liabilities to a company. Conclusions are in Section 5.

2. The model

In this section we explain the model. For ease of exposition it is initially assumed that we are interested in valuing a long position in a European option issued by a counterparty subject to credit risk. Later we show how the model can be extended to value other types of vulnerable derivative securities.

We define two securities as ranking equally in the event of a default if both pay off the same proportion of their no-default values when a default occurs.³ We assume that we know the prices of bonds that are issued by the counterparty and rank equally with the option. It is not necessary for these bonds to exist. All that is required that we be able to estimate what the prices of the bonds would be if they did exist. The bonds relevant to our analysis are zero-coupon bonds. It may be necessary for analysts to estimate yields on a number of different coupon-bearing bonds that could be issued by the counterparty and then to use approaches such as Fong and Vasicek (1982) to estimate a zero-coupon yield curve for the counterparty. This is not as onerous as it sounds. Bond traders in financial institutions regularly estimate zero-coupon yield curves for corporate bonds that have a variety of different credit ratings in order to identify those that are under- or overpriced.

Define:

f : current value of vulnerable option under consideration.

f^* : current value of the option assuming no defaults.⁴

B : current value of a vulnerable zero-coupon bond issued by the option writer that pays off \$1 at time T and ranks equally with the option in the event of a default.

³ Our definition of equal ranking securities implicitly assumes that the claim made by a security holder against the assets of a defaulting counterparty equals the no-default value of the security. This is an approximation and does not reflect all the details of the bankruptcy law. For example, in some jurisdictions, the claim made against the assets of a corporation for a risky coupon-bearing bond issued by the corporation is equal to the tax basis for the bond. Depending on how interest rates have moved since the bond was purchased this may be greater than or less than the bond's default-free market value. But, since the bond is worth less than its default-free value at the time of issue, our assumption can be expected to overstate the claim on average. It is worth noting that proportional recovery cannot be based on the market value of a security at the time of default since the latter is by definition equal to the amount recovered.

⁴ Strictly speaking f^* is not the value the security would have in a default-free world since, when we move from the vulnerable world to a default-free world, the stochastic processes followed by the underlying state variables may change. The variable f^* is the value the security has when state variables follow their vulnerable-world processes and promised payments are always made.

B^* : current value of a similar default-free zero-coupon bond.

t_0 : current time.

T : time of promised payoff.

f_T^* : value of f^* at time T .

r : the short term risk-free interest rate.

p : proportion of no-default value received when there is a default for the option under consideration and for other securities that rank equally with it in the event of a default.

θ : vector of state variables determining the f^* and r variables.

ϕ : Vector of state variables determining the occurrence of defaults and p .

The θ and ϕ variables are assumed to follow continuous time diffusion process. In the most general version of the model some of the θ variables may also be ϕ variables.

It is assumed that defaults occur at the first time, t , when

$$G(\phi, t) = 0.$$

for some function G .⁵ This defines what will be termed the ‘default boundary’ in $\{\phi, t\}$ space. The ‘payoff boundary’ is the earlier of $t = T$ and the default boundary. The payoff is pf^* if the default boundary is reached first and f^* if the $t = T$ boundary is reached first. For ease of exposition we generalize the definition of p so that it equals unity on the $t = T$ boundary. This means that the payoff is pf^* at all points on the payoff boundary. The variable p is a function of ϕ and t .

We assume that θ and ϕ are sufficiently well behaved for us to be able to use the risk neutrality approach initially developed by Cox and Ross (1976) and formalized by Harrison and Pliska (1981). This means that

$$f = \hat{E}_{\theta, \phi} \left[\exp \left(- \int_{t_0}^{t_b} r \, dt \right) p_b f_b^* \right], \quad (1)$$

where \hat{E} denotes expectations under an equivalent martingale measure over all first passage paths to the payoff boundary, the subscripts to \hat{E} indicate the vectors of variables over which expectations are taken, b denotes the payoff boundary point reached, t_b is the time at point b , f_b^* is the value of f^* at point b , and p_b is the value of p at point b .

Eq. (1) gives the value of a vulnerable option at time t_0 in terms of its no-default value on the payoff boundary. It can be simplified by noting that the

⁵ We assume that all market participants know G and the current values of the ϕ variables. Some of the results can be extended to cover the situation where there is some probability measure that encodes the beliefs of market participants about G and ϕ .

no-default value at a payoff boundary point, b , equals the expected discounted no-default value at time T ; that is

$$f_b^* = \hat{E}_\theta \left[\exp \left(- \int_{t_b}^T r \, dt \right) f_T^* \right], \tag{2}$$

where \hat{E} denotes expectations under the equivalent martingale measure over all paths for θ between times t_b and T that start at point b . Combining (1) and (2)

$$f = \hat{E}_{\theta, \phi} \left[\exp \left(- \int_{t_0}^T r \, dt \right) p_b f_T^* \right], \tag{3}$$

where the expectations operator \hat{E} is taken over all paths for θ and ϕ between times t_0 and T and b denotes the first payoff boundary point crossed by the path. Since r is defined by θ variables, Eq. (3) can be written

$$f = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T r \, dt \right) \hat{E}_{\phi | \theta} (p_b) f_T^* \right].$$

Defining

$$w(\theta, T) = \hat{E}_{\phi | \theta} (p_b)$$

this becomes

$$f = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T r \, dt \right) w(\theta, T) f_T^* \right]. \tag{4}$$

This can be compared with the equation

$$f^* = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T r \, dt \right) f_T^* \right],$$

which is the value of the security when there is no possibility of defaults.

The variable $1 - w(\theta, T)$ is the expected loss proportion on the option between time zero and time T for a particular realization of θ . Eq. (4) has a great deal of intuitive appeal. It states that a derivative security promising a payoff at time T can be valued in the usual way if the promised payoff from a particular θ outcome is multiplied by $w(\theta, T)$.⁶ It is relatively easy to see that this result holds if defaults can occur only at the maturity of the option. Our analysis shows that the result is also true when defaults can occur at any time during the life of the option.

⁶ For an immediate application of Eq. (4) consider a warrant issued by a company on its own stock. It is reasonable to assume that, in circumstances where the company has defaulted on its liabilities, the stock price is so low that the warrant has no value. With this assumption $w(\theta, T) = 1$ whenever $f_T^* \neq 0$ so that $f = f^*$. As indicated by footnote 4, this result must be interpreted carefully. It does not mean that the warrant has the same value in a no-default world as it has in the real world. This means that, once we have correctly identified the true terminal stock price distribution, we can assume that the promised payoff is always made.

Eq. (4) is true for all derivative securities that promise a nonnegative payoff at time T and rank equally with f in the event of a default. In particular it is true for a zero-coupon bond. From (4)

$$B = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T r \, dt \right) w(\theta, T) \right]. \quad (5)$$

Also

$$B^* = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T r \, dt \right) \right]. \quad (6)$$

Eqs. (5) and (6) define conditions that must be satisfied by the function $w(\theta, T)$ in (4).

2.1. Upper and lower bounds for vulnerable European option prices

Without any further assumptions it is possible to derive upper and lower bounds for the price of a European option. The lower bound is calculated by assuming that the relationship between the θ 's and ϕ 's is such that defaults take place in the worst possible states of the world, that is in those where the default-free value of the option is as high as possible. The upper bound is similarly calculated by assuming that defaults take place in states where the default-free value of the option is as low as possible.

We illustrate the approach by considering the valuation of a call option on a stock paying a continuous dividend yield in a Black–Scholes world. In this case there is one θ variable: the stock price, S .

Define:

S_T : terminal stock price.

X : strike price.

q : dividend yield.

σ : volatility of the stock price.

$g(S_T)$: probability density function for S_T in a risk-neutral world.

The vector of state variables determining the probability of defaults and proportional losses in the event of a default is as before denoted by ϕ . Consistent with the previous notation, we define

$$w(\theta, T) = \hat{E}_{\phi|\theta}(P_b),$$

where θ represents a path for the stock price, S . Since the payoff is dependent only on the terminal stock price, S_T , it is convenient to define $u(S_T)$ as the expected value of $w(\theta, T)$ over all stock price paths ending at S_T . The function $1 - u(S_T)$ is the expected loss proportion on the option for all stock price paths that end at S_T .

Since the short-term interest rate, r , is constant in the Black–Scholes model, Eq. (4) gives the value of the call option, f , as

$$f = B^* \int_0^\infty u(S_T) \max(S_T - X, 0) g(S_T) dS_T. \tag{7}$$

Similarly

$$B = B^* \int_0^\infty u(S_T) g(S_T) dS_T. \tag{8}$$

Eq. (8) defines one condition that must be satisfied by u . Another is

$$0 \leq u(S_T) \leq 1. \tag{9}$$

Since the payoff from a call option is a monotonic increasing function of S_T the maximum value of f in (7) subject to the constraints in (8) and (9) is obtained by setting $u(S_T) = 1$ for as many high values of S_T as possible. Specifically, the maximum is obtained by setting

$$u(S_T) = \begin{cases} 1, & \text{if } S_T \geq S_1, \\ 0, & \text{if } S_T < S_1, \end{cases}$$

where S_1 is chosen so that $u(S_T)$ satisfies (8); that is, so that

$$B = B^* \int_{S_1}^\infty g(S_T) dS_T. \tag{10}$$

Similarly the values of $u(S_T)$ that minimize f are

$$u(S_T) = \begin{cases} 0, & \text{if } S_T \geq S_2, \\ 1, & \text{if } S_T < S_2, \end{cases}$$

where S_2 is chosen to satisfy (8); that is, so that

$$B = B^* \int_0^{S_2} g(S_T) dS_T. \tag{11}$$

Under the Black–Scholes assumptions the function $\log(S_T)$ is normally distributed in a risk-neutral world with mean, m , and standard deviation, s , given by

$$m = \log S + (r - q - \frac{1}{2}\sigma^2)(T - t_0),$$

$$s = \sigma\sqrt{T - t_0}.$$

It follows from (10) and (11) that

$$S_1 = \exp \left[m - N^{-1} \left(\frac{B}{B^*} \right) s \right],$$

$$S_2 = \exp \left[m + N^{-1} \left(\frac{B}{B^*} \right) s \right],$$

where N^{-1} is the inverse cumulative normal distribution function.

Denote by $C(Z)$ the Black–Scholes call price when the strike price is Z and define:

$$\pi(Z) = N \left[\frac{\ln(S/Z) + (r - q - \sigma^2/2)(T - t_0)}{\sigma\sqrt{T - t_0}} \right].$$

From the above analysis and Eq. (7) the maximum value of a vulnerable call option with strike price X is given by

$$C_{\max} = B^* \int_{S_1}^{\infty} \max(S_T - X, 0) g(S_T) dS_T$$

or

$$C_{\max} = \begin{cases} C(X), & \text{if } S_1 \leq X, \\ C(S_1) + B^*(S_1 - X)\pi(S_1), & \text{if } S_1 > X. \end{cases}$$

To understand this result recall that, in order to obtain the maximum option price, we have considered a scenario where defaults are associated with the lowest possible values of S_T . When $S_1 < X$ this scenario has all defaults occurring when $S_T < X$; that is, when the option closes out of the money. The maximum vulnerable option price is therefore the same as its no-default price, $C(X)$. When $S_1 > X$, some defaults must occur when the option closes in the money and the maximum vulnerable option price is therefore less than the no-default price in this case.

The minimum value of a vulnerable call is given by

$$C_{\min} = B^* \int_X^{S_2} \max(S_T - X, 0) g(S_T) dS_T$$

or

$$C_{\min} = \begin{cases} 0, & \text{if } S_2 \leq X, \\ C(X) - C(S_2) - B^*(S_2 - X)\pi(S_2), & \text{if } S_2 > X. \end{cases}$$

To obtain these results we have looked for worst-case scenarios. When $S_2 \leq X$ the probability of default is sufficiently high that all circumstances where $S_T > X$ can be associated with defaults where there is zero recovery. Hence the minimum value of the option is zero in this case. When $S_2 > X$, defaults cannot wipe out all positive payoffs and the minimum value of the call is positive.

A similar analysis gives the following for the maximum and minimum values, P_{\max} and P_{\min} , of a vulnerable put option

$$P_{\max} = \begin{cases} P(S_2) + B^*(X - S_2)[1 - \pi(S_2)], & \text{if } S_2 \leq X, \\ P(X), & \text{if } S_2 > X, \end{cases}$$

$$P_{\min} = \begin{cases} P(X) - P(S_1) - B^*(X - S_1)[1 - \pi(S_1)], & \text{if } S_1 \leq X, \\ 0, & \text{if } S_1 > X. \end{cases}$$

Options on foreign exchange, stock indices, and futures contracts are analogous to options on a stock paying a continuous dividend yield.⁷ The results in this section can therefore be extended to cover these types of options.

2.2. American options

Vulnerable American-style derivative securities are more complicated to value than their European counterparts. This is because the holder can use the latest information about the ϕ variables, as well as the θ variables, when deciding whether to exercise early.

A general result is that a vulnerable American option should not be exercised later than its no-default counterpart. To prove this suppose that option U is the American-style option under consideration and option U^* is the no-default version of U . Define:

h : the value of U .

h^* : the value of U^* .

Q : the promised payoff.

Option U should not be exercised when $h > Q$ and option U^* should not be exercised when $h^* > Q$. Since $h^* \geq h$, the second condition is satisfied whenever the first condition is satisfied. This proves the general result.

2.3. An example

We now illustrate the results in Sections 2.1 and 2.2 by considering a particular situation where a bank has written a call option on a foreign currency. We first assume that we have a full knowledge of the model so that the prices of both bonds and options can be computed. We then assume that we only know bond prices and use the results in Section 2.1 to calculate bounds on the option prices. Define

S : foreign currency exchange rate.

A : assets of the bank.

X : strike price.

r : domestic risk-free interest rate (assumed constant).

r_f : foreign risk-free interest rate (assumed constant).

T : time to maturity of the option.

C : option price.

p : proportional recovery made in the event of a default (assumed constant in this example).

We assume that whenever A falls to a level D , the bank defaults. In this situation A is the only ϕ variable and S is the only θ variable. The default boundary is $A - D = 0$.

⁷ See Hull (1993, Ch. 11) for a discussion of this point.

We suppose that the risk-neutral processes for S and A are

$$dS = (r - r_f)S dt + \sigma_S S dz_S,$$

$$dA = rA dt + \sigma_A A dz_A,$$

where σ_S and σ_A , the volatilities of S and A , are constant and dz_S and dz_A are Wiener processes. We suppose that the instantaneous correlation between S and A is ρ .

The variables S and A can be modeled in the form of a three-dimensional tree using an approach suggested by Hull and White (1990). Define two new orthogonal variables:

$$x_1 = \sigma_A \log S + \sigma_S \log A, \quad (12)$$

$$x_2 = \sigma_A \log S - \sigma_S \log A. \quad (13)$$

These variables follow the processes

$$dx_1 = \left[\sigma_A(r - r_f - \sigma_S^2/2) + \sigma_S(r - \sigma_A^2/2) \right] dt + \sigma_S \sigma_A \sqrt{2(1 + \rho)} dz_1,$$

$$dx_2 = \left[\sigma_A(r - r_f - \sigma_S^2/2) - \sigma_S(r - \sigma_A^2/2) \right] dt + \sigma_S \sigma_A \sqrt{2(1 - \rho)} dz_2,$$

where dz_1 and dz_2 are uncorrelated Wiener processes. The variables can be modeled using two separate binomial trees where, in time Δt , x_i has a probability q_i of increasing by h_i and a probability $1 - q_i$ of decreasing by h_i . The variables h_i and q_i are chosen so that the tree gives correct values for the first two moments of the distribution of x_1 and x_2 . Since the variables are uncorrelated the two binomial trees can be combined together to form a three-dimensional tree where the probabilities of movements in x_1 and x_2 in time Δt are as follows:

$q_1 q_2$: x_1 increases by h_1 and x_2 increases by h_2 ,

$q_1(1 - q_2)$: x_1 increases by h_1 and x_2 decreases by h_2 ,

$(1 - q_1)q_2$: x_1 decreases by h_1 and x_2 increases by h_2 ,

$(1 - q_1)(1 - q_2)$: x_1 decreases by h_1 and x_2 decreases by h_2 .

At each node of the tree S and A can be calculated from x_1 and x_2 using the inverse of the relationships in (12) and (13):

$$S = \exp \left[\frac{x_1 + x_2}{2\sigma_A} \right],$$

$$A = \exp \left[\frac{x_1 - x_2}{2\sigma_S} \right].$$

At the final nodes of the tree

$$C = \begin{cases} \max(S - X, 0), & \text{if } A > D, \\ 0, & \text{if } A \leq D. \end{cases}$$

For a European option we apply the following boundary condition as we roll back through the tree

$$C = pC^* \quad \text{when } A \leq D.$$

Table 1

Percentage reduction arising from default risk in the price of one-year foreign currency call options. The strike price and current exchange rate are both 1.0000. The foreign and domestic interest rates are both 5% per annum. The volatility of the exchange rate is 15% per annum. The assets of the option writer have an initial value of 100 and a volatility of 5%. The writer defaults when the value of the assets reaches D . The coefficient of correlation between the value of the assets and the exchange rate is ρ . The option holder makes no recovery in the event of a default. The no-default prices of the European and American options are 0.0569 and 0.0575, respectively

ρ		D			
		90	92	94	96
-0.80	European	1.72	6.18	17.86	40.87
	American	0.00	0.00	0.11	1.78
-0.40	European	0.86	3.21	9.48	23.88
	American	0.02	0.11	0.72	3.72
0.00	European	0.30	1.24	4.44	13.95
	American	0.04	0.22	1.10	4.93
0.40	European	0.03	0.20	1.21	5.69
	American	0.01	0.10	0.68	3.61
0.80	European	0.00	0.01	0.13	1.81
	American	0.00	0.01	0.12	1.78
One-year bond yield (%)		5.30	6.25	9.55	20.05
Minimum percent reduction implied by bond yield		0.00	0.00	0.00	0.00
Maximum percent reduction implied by bond yield		2.83	9.56	26.73	60.31

In the case of an American option the additional boundary condition

$$C \geq S - X \quad \text{when } A > D$$

is applied.

The price, B , of a discount bond ranking equally with the option in the event of default and maturing at time T can be obtained from the same tree by setting

$$B = \begin{cases} 1, & \text{if } A \geq D, \\ 0, & \text{if } A \leq D \end{cases}$$

at the final nodes and applying the boundary condition

$$B = pB^* \quad \text{when } A \leq D$$

at earlier nodes as we roll back through the tree where B^* is the default-free value of the bond.

Table 1 considers the case where $S = X = 1$, $A = 100$, $r = r_f = 0.05$, $T = 1$, $\sigma_A = 0.05$, $\sigma_S = 0.15$, and $\rho = 0$. It provides results based on 100 time steps for the percentage reduction in the prices of European and American call options for a variety of different values of ρ and D . The no-default prices of the European and American options are 0.0569 and 0.0575, respectively. The table shows that the

impact of default risk on the prices of American options is significantly less than its impact on the prices of European options. This is because, as discussed above, the holder of an American option can alleviate the impact of defaults by exercising when A is just above D .⁸ The advantage of being able to do this is greatest for situations when the correlation between S and A is highly negative. Regardless of what subsequently happens the owner of the option is then likely to have few regrets about the early exercise decision. If A declines, the bank defaults and the gain from early exercise is significant; if A increases, there is a tendency for S to decrease and for the value of the call option to decline. This explains why the difference between the percentage price reduction from default risk for American and European call options increases as the correlation between S and A decreases. For put options the reverse is true: the difference between the percentage reduction increases as the correlation between S and A increases.

Table 2 shows how the results in Table 1 change when the proportional recovery changes from 0 to 50%. The percentage reductions in the prices of all European options are halved as might be expected. The percentage reductions in the prices of American options also reduce, but in most cases by less than 50%. This is because the incentive to exercise early in order to reduce default risk is less than before.

Tables 1 and 2 show the yields on one-year vulnerable bonds implied by the different values of D considered. As explained in Section 2.1 the yields on vulnerable bonds can be used to produce maximum and minimum values for vulnerable European option prices. Tables 1 and 2 also show the minimum and maximum percentage reductions in the option price implied by the calculated bond yields.⁹ These are independent of the model and depend only on the prices of bonds that rank equally with the option. It can be seen that actual percentage reduction in each table is always between the minimum and maximum.

3. A special case

The most general version of the model that has been presented requires data on the ϕ variables, the default boundary, and the way in which p depends on the ϕ variables. In practice this data is unlikely to be available. This means that the model can usually produce only fairly wide bounds for the price of a derivative security. In this section we consider a special case of the model where the adjustments for credit risk depend only on the prices of bonds.

⁸ Our model may overstate the difference between the impact of default risk on the values of American and European options because it assumes that the option holder has full knowledge of the default boundary and that all obligations related to option transactions are honored right up until the time when the default boundary is reached.

⁹ The minimum percentage reduction is not always zero. In this example it exceeds zero when the one-year yield is greater than 80%.

Table 2

Percentage reduction arising from default risk in the price of one-year foreign currency call options. The strike price and the current exchange rate are both 1.0000. The foreign and domestic interest rates are both 5% per annum. The volatility of the exchange rate is 15% per annum. The assets of the option writer have an initial value of 100 and a volatility of 5%. The writer defaults when the value of the assets reaches D . The coefficient of correlation between the value of the assets and the exchange rate is ρ . The option holder makes a 50% recovery in the event of a default. The no-default European and American option prices are 0.0569 and 0.0575, respectively

ρ		D			
		90	92	94	96
-0.80	European	0.86	3.09	8.93	20.43
	American	0.00	0.00	0.10	1.73
-0.40	European	0.43	1.61	4.74	11.94
	American	0.01	0.09	0.59	3.03
0.00	European	0.15	0.62	2.22	6.97
	American	0.03	0.15	0.73	3.24
0.40	European	0.01	0.10	0.61	2.84
	American	0.01	0.06	0.40	2.07
0.80	European	0.00	0.00	0.06	0.91
	American	0.00	0.00	0.06	0.89
One-year bond yield (%)		5.15	5.62	7.25	12.24
Minimum percent reduction implied by bond yield		0.00	0.00	0.00	0.00
Maximum percent reduction implied by bond yield		1.53	5.29	15.45	37.50

We suppose the state variables comprising θ are independent of the state variables comprising ϕ . In this case

$$w(\theta, T) = \hat{E}_{\phi|\theta}(p_b) = \hat{E}_{\phi}(p_b).$$

The function $w(\theta, T)$ is independent of θ and can be written as $w(T)$. Eq. (5) becomes

$$B = w(T) \hat{E}_{\theta} \exp\left(-\int_{t_0}^T r \, dt\right),$$

and combining this with (6)

$$w(T) = \frac{B}{B^*}. \tag{15}$$

Eq. (4) becomes

$$f = w(T) \hat{E}_{\theta} \left[\exp\left(-\int_{t_0}^T r \, dt\right) f_T^* \right].$$

Since the current value the derivative security equals its expected discounted value at time T this reduces to

$$f = w(T) f^*, \quad (16)$$

so that from (15)

$$f = \frac{B}{B^*} f^*$$

or

$$\frac{f}{f^*} = \frac{B}{B^*}. \quad (17)$$

The intuition behind Eq. (17) is as follows. For any path followed by the ϕ variables the proportional loss on the bond and the derivative security is the same. Since the θ and ϕ variables are independent, the expected no-default values of the bond and the derivative security are independent of the path followed by the ϕ variables. Integrating across all such paths must therefore lead to the result in Eq. (17).

Defining y and y^* as the yields on B and B^* , respectively, (17) reduces to

$$f = e^{-(y-y^*)(T-t_0)} f^*. \quad (18)$$

Eq. (18) has a great deal of intuitive appeal. It suggests that the discount rates used when a vulnerable European-style derivative security is valued should be higher than those used when a similar default-free security is valued by an amount $y - y^*$.¹⁰

For an example of the application of this result we consider the data in Table 1. When $D = 90, 92, 94$ and 96 , $y - y^*$ is 0.30% , 1.25% , 4.55% , and 15.05% , respectively. The percentage reductions in the European option price in these four cases can be calculated from Eq. (18) as $0.30, 1.24, 4.44$, and 13.97 . These agree very closely with the $\rho = 0$ results calculated numerically in Table 1.

The independence assumption, although it is clearly not perfectly true, may in practice be not too unreasonable for many of the over-the-counter options written by large financial institutions. One reason for this is that any particular option is usually only a very small part of the portfolio of the financial institution. As such its influence on the fortunes of the financial institution is minimal. Another reason is that the variables underlying the options traded over the counter by financial institutions are typically interest rates, exchange rates and commodity prices. Most financial institutions try to ensure that they are at all times reasonably well hedged against the impact of these market variables.

¹⁰ For a discussion of the relationship between yield spreads and maturity, see Cook and Hendershott (1978), Jonkhart (1979), Rodriguez (1988), and Yawitz et al. (1985).

Eq. (18) leads to the following modifications to the Black–Scholes formulas for the prices, C and P , of European calls and puts on nondividend paying stocks.

$$C = e^{-(y-y^*)\chi(T-t_0)}SN(d_1) - e^{-y(T-t_0)}XN(d_2),$$

$$P = e^{-y(T-t_0)}XN(-d_2) - e^{-(y-y^*)\chi(T-t_0)}SN(-d_1),$$

where

$$d_1 = \frac{\ln(S/X) + (y^* + \sigma^2/2)(T - t_0)}{\sigma\sqrt{T - t_0}},$$

$$d_2 = d_1 - \sigma\sqrt{T - t_0},$$

S is the current stock price, X is the strike price, σ is the volatility, and N is the cumulative normal distribution function.¹¹ It is interesting to note that the normal put–call parity relationship between C and P does not apply when there is default risk. A relationship which does hold is

$$C + X e^{-y(T-t_0)} = P + S e^{-(y-y^*)\chi(T-t_0)}.$$

Unlike the usual put–call parity relationships, this relies on the independence assumption and cannot be proved using simple arbitrage arguments.

3.1. American options

When the θ and ϕ variables are assumed to be independent, it is possible to produce lower bounds for the prices of American options. Define

$F^*(t_0, t)$: the instantaneous forward rate at time t as seen at time t_0 calculated from the risk-free yield curve.

$F(t_0, t)$: the instantaneous forward rate at time t as seen at time t_0 calculated from the yield curve corresponding to bonds that rank equally with the option (or category of options) under consideration in the event of a default.

$\alpha(t_0, t)$: the forward rate differential, $F(t_0, t) - F^*(t_0, t)$.

Appendix A proves a series of propositions that lead to two general results:

1. The percentage reduction in the price of an American option caused by default risk is always less than that for the corresponding European option.
2. A lower bound to the price of a vulnerable American option is the price of a notional no-default option, X , which is such that the holder is required to make continuous payments at rate $\alpha(t_0, t)$ times the option's value at time t until the option either expires or is exercised.

¹¹ For a sufficiently large yield differential a call option price decreases as the time to maturity increases. Johnson and Stulz (1987) show that this is also a feature of some of the models they consider.

The first result can be used to provide a lower bound for the price of the American option:

$$h < h^* \frac{f}{f^*},$$

where h and f are the values of the vulnerable American option and its European counterpart, and h^* and f^* are the values of the corresponding no-default options. The second result produces a tighter lower bound than this. Option X is the value the American option would have if the market does not change its view of the probability of the counterparty defaulting during particular time intervals. It can be valued using a Cox, Ross, and Rubinstein (1979) binomial tree in which the discount rate at time t is increased by $\alpha(t_0, t)$.¹²

4. Extension to swaps and similar contracts

The results in Sections 2 and 3 apply to contracts that are always positively valued. One of the attractive features of the analysis is that it can be extended to cover contracts such as swaps and forward contracts that can be either assets or liabilities to a company. In this section we first present the general model and then show the effect of making the independence assumption of Section 3.

We suppose that the company for which the valuation is being carried out is company A and that its counterparty is company B. We make the simplifying assumption that there is no chance of a default by company A.¹³ We assume that if company B defaults when the no-default value of the security to company A is positive, company A is able to make a claim equal to the no-default value of the security against the assets of company B. If company B defaults on its liabilities when the no-default value of the security is negative, we assume that there is no effect on the position of company A.¹⁴

¹² The use of a higher discount rate on the tree tends to cause the option to be exercised earlier. This is consistent with Section 2.2.

¹³ The model can be extended to cover two-sided default risk, but it becomes considerably more complicated. Company X makes a gain at time t if (a) it defaults (b) the no-default value of the contract is positive to X, and (c) company Y has not already defaulted at an earlier time when the no-default value of the contract to Y was positive. Company X makes a loss at time t if (a) Y defaults, (b) the no-default value of the contract is positive to Y, and (c) company X has not already defaulted at an earlier time when the no-default value of the contract to X was positive.

¹⁴ As noted in footnote 3 we are not reflecting all the details of bankruptcy law in our analysis. Many issues associated with swap defaults are being decided on a case by case basis. Litzenberger (1992) has shown that the legal position of company A in the event of a default by company B may in some circumstances be slightly better than that assumed here. The assumptions made here are a reasonable approximation to reality and are the same as those made by most researchers and bank regulators.

Using the same notation as Section 2 the company’s exposure when payoff boundary point b is reached is $\max(f_b^*, 0)$ and the credit loss at this point is

$$(1 - p_b)\max(f_b^*, 0).$$

The difference between the vulnerable and no-default value of the security is therefore given by

$$f^* - f = \hat{E}_{\theta, \phi} \left[\exp \left(- \int_{t_0}^{t_b} r \, dt \right) (1 - p_b)\max(f_b^*, 0) \right]. \tag{19}$$

This can also be termed the value of company Y’s default option. In principle it can be calculated numerically once the ϕ variables and the default boundary have been specified.

When the θ variables and ϕ variables are assumed to be independent Eq. (19) can be simplified. Suppose that the security under consideration lasts until time T . Eq. (19) can be written

$$f^* - f = \int_{t_0}^T \hat{E}_{\theta} \left[\exp \left(- \int_{t_0}^t r \, dt \right) \max(f_t^*, 0) \right] \pi(t) \, dt, \tag{20}$$

where f_t^* is the value of f^* at time t , $\pi(t)$ is the rate at which losses are experienced as a proportion of the no-default value of contracts at time t , and expectations are taken at time t_0 .

Define

B_t : price at time t_0 of a discount bond ranking equally with the security and maturing at time t ,

B_t^* : price at time t_0 of a riskless bond maturing at time t ,

y_t : yield on B_t ,

y_t^* : yield on B_t^* .

When the derivative security is a discount bond maturing at time t , Eq. (20) becomes

$$B_t^* - B_t = B_t^* \int_{t_0}^t \pi(s) \, ds$$

or

$$1 - e^{-(y_t - y_t^*) \chi^{t-t_0}} = \int_{t_0}^t \pi(s) \, ds.$$

Differentiating with respect to t

$$\pi(t) = \alpha(t_0, t) e^{-(y_t - y_t^*) \chi^{t-t_0}}, \tag{21}$$

where as before $\alpha(t_0, t)$ is the instantaneous forward rate differential as seen at time t_0 for a contract maturing at time t . Substituting (21) into (20) we obtain

$$f^* - f = \int_{t_0}^T \alpha(t_0, t) e^{-(y_t - y_t^*) \chi^{t-t_0}} v(t) \, dt, \tag{22}$$

where

$$v(t) = \hat{E} \left[\exp \left(- \int_{t_0}^t r \, dt \right) \max(f_t^*, 0) \right] dt. \quad (23)$$

The variable $v(t)$ is the value of a contingent claim that pays off the exposure at time t .

The structure of Eq. (22) can be understood by considering the situation where no recoveries are made. The term $\alpha(t_0, t)\Delta t$ is the probability of a default between times t and $t + \Delta t$ conditional on no earlier default; the term $e^{-(y_t - y_{t_0})X(t-t_0)}$ is the probability of no default between time t_0 and t ; the term $v(t)$ is the expected present value of the loss incurred by a default at time t .

In some circumstances $v(t)$ can be calculated analytically. In other circumstances a tree or Monte Carlo simulation must be used to estimate it. Once $v(t)$ has been obtained, numerical integration can be used to evaluate the expression in (22).

As illustration of Eq. (22) consider a currency swap where a fixed rate of interest in the domestic currency is received and a fixed rate of interest in a foreign currency is paid. Principals are exchanged at the end of the life of the swap and interest payments are exchanged periodically. At time t , the value of the swap is $B_d(t) - S(t)B_f(t)$ where $B_d(t)$ is the value in the domestic currency of the domestic bond underlying the swap at time t , $B_f(t)$ is the value in the foreign currency of the foreign bond underlying the swap at time t , and $S(t)$ is the exchange rate at time t .

In this case $v(t)$ is the value of an option that pays off $\max[B_d(t) - S(t)B_f(t), 0]$ at time t . Assuming interest rates are constant, the values of $B_d(t)$ and $B_f(t)$ are known and $v(t)$ is the value of a currency option.

Table 3

Cost of defaults expressed in basis points per year on a currency swap where interest and principal in the domestic currency is received and interest and principal in the foreign currency is paid. The initial exchange rate is 1.0000 and principals in the two currencies are both 100. Interest is paid at 5% once a year in both currencies. The risk free interest rate is assumed to be constant at 5% per annum (with annual compounding) in both currencies. The spread over the risk-free rate for zero-coupon bonds of maturities 1, 2, 3, 4, 5, and 10 yr issued by the counterparty is 25, 50, 70, 85, 95, and 120 basis points, respectively

Swap life (yr)	Exchange rate volatility (% per annum)			
	5.0	10.0	15.0	20.0
2	1.2	2.3	3.5	4.6
4	2.6	5.3	7.9	10.5
6	3.6	7.2	10.7	14.3
8	4.4	8.7	13.0	17.3
10	5.1	10.2	15.3	20.3

Table 3 shows the impact of defaults on the value of the swap for different exchange rate volatilities and swap lives. Both domestic and foreign interest rates are assumed to be 5% per annum with annual compounding. The principal in each currency is assumed to be 100 and interest at the rate of 5% in each currency is assumed to be exchanged annually. The values of $y_t - y_t^*$ for t equal to 1, 2, 3, 4, 5, and 10 years were assumed to be 25, 50, 70, 85, 95, and 120 basis points with intermediate values of t being calculated using linear interpolation. The Garman and Kohlhagen (1983) model was used to calculate $v(t)$ and Simpson's rule was used to evaluate the integral in Eq. (22) numerically.

Table 3 shows the payments on an annuity which lasts for the same length of time as the swap and which (using the domestic risk-free interest rate as the discount rate) has the same value as $f^* - f$. These payments are expressed in basis points on the principal and provide a guide to the spread required on the currency swap to compensate for credit risk. (For example, the 10.2 basis points shown for a volatility of 10% per annum and a swap life of 10 years was calculated from a value for $f^* - f$ of 0.79.) The table indicates that the required spread increases with both the volatility and the life of the swap.¹⁵

5. Conclusions

This paper has proposed a model for incorporating default risk into the prices of derivative securities. It shows how the value of a vulnerable security can be related in a consistent way to the no-default value of the security, the values of default-free zero-coupon bonds, and the values of vulnerable zero-coupon bonds that would be issued by the writer of the derivative security. The latter can be estimated from the yields on actively traded corporate bonds.

Upper and lower bounds for the prices of European options subject to default risk can be calculated analytically. The model can also be implemented numerically when full information about the variables affecting defaults is available. We have produced results for the situation where a bank writes a foreign currency option and the assets of the bank are correlated with the foreign exchange rate. The impact of default risk on the price of an American option is less than that on the price of a European option. In the case of call options the difference between the two is greatest when the correlation between the foreign exchange rate and the

¹⁵ Since a swap spread is not riskless, Table 3 slightly understates the required swap spread. Note that Table 3 is indicative of the spread required on a single currency swap. For this particular example, the spread on a matched pair of swaps would be twice as great. If the foreign interest rate is higher than the domestic rate, the swap in which we receive domestic is worth more than the corresponding swap where we pay domestic. The reverse is true when the domestic rate is higher than the foreign rate. However, the total spread required on a matched pair of swaps is fairly insensitive to differences between the two rates.

value of the assets of the financial institution is negative. For put options the difference is greatest when this correlation is positive.

The model leads to analytic pricing formulas for European warrants issued by a company on its own stock. It also leads to procedures for calculating the impact of default risk on contracts such as forwards and swaps that can become either assets or liabilities to a company.

A simplifying assumption that is likely to have a great deal of appeal to practitioners is that the variables concerned with defaults are independent of the variables underlying the value of the derivative security in a no-default world. This avoids the need for assumptions to be made about the variables concerned with defaults. Vulnerable European option prices can then be determined analytically. Lower bounds can be calculated for the prices of vulnerable American options. The impact of default risk on contracts such as swaps and forwards can be calculated using numerical integration.

In this analysis a vulnerable derivative security is regarded as a claim partially contingent on a similar vulnerable bond. An important issue is therefore whether vulnerable bonds are correctly priced by the market. Altman's (1989, 1990) research shows that the excess yields on corporate bonds over treasury bonds are higher than can be justified by their default experience. This may be because of market inefficiency, illiquidity or some other factor that we do not wish to reflect in the prices of derivative securities. If so, the market prices of vulnerable bonds in the analysis in this paper should be replaced by notional bond prices calculated to reflect the actual default experience on bonds with a similar credit rating.

Appendix A

In this appendix we prove a number of propositions which lead to results for American options in the situation where the θ variables are assumed to be independent of the ϕ variables.

Proposition A1. *When the θ variables are independent of the ϕ variables, the value of a vulnerable derivative security that promises a non-negative payment only at time T is equal to the value of a similar no-default security where the holder is required to make continuous payments at rate $\alpha(t_0, t)$ times the security's value at time t ($t_0 \leq t \leq T$) where $\alpha(t_0, t)$ is the forward rate differential defined in Section 3.1.*

Proof: As usual we define f as the price of the vulnerable derivative security and f^* as the price it would have in a no-default world. In addition we define g^* : price of security which is the same as the no-default security under consideration except that the holder is required to make payments at rate $\alpha(t_0, t)$ times its value at time t ($t_0 \leq t \leq T$).

If f_T^* and g_T^* are the values of f^* and g^* at time T

$$f^* = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T r \, dt \right) f_T^* \right]$$

and

$$g^* = \hat{E}_\theta \left[\exp \left(- \int_{t_0}^T [r + \alpha(t_0, t)] \, dt \right) g_T^* \right].$$

Since $f_T^* = g_T^*$

$$g^* = f^* \exp \left(- \int_{t_0}^T \alpha(t_0, t) \, dt \right).$$

Since

$$\int_{t_0}^T \alpha(t, s) \, ds = (y - y^*)(T - t_0), \tag{A.1}$$

it follows that

$$g^* = f^* e^{-(y - y^*)(T - t_0)}.$$

From Eq. (18) this in turn means that $f = g^*$, which is the desired result. Q.E.D.

This result applies to all European-style derivative securities including those where the promised payoff depends on the path followed by θ variables. It should be emphasized that f and g^* have equal values only at time t_0 . When we move to a new time t^* ($t^* > t_0$), the value of the vulnerable security is equated to the value of a new no-default security where the continuous payments at time t are at a rate $\alpha(t^*, t)$ times the security value. In general $\alpha(t_0, t)$ and $\alpha(t^*, t)$ are not equal.

The next proposition extends the result in proposition A1 to securities with a general payoff boundary in a no-default world.

Proposition A2. *The result in Proposition A1 is true for any derivative security that has a no-default payoff boundary that is a function of θ and t .*

Proof: The derivative security can be regarded as the sum of Arrow–Debreu securities each promising a payoff if one particular path in $\{\theta, t\}$ space is followed, but promising zero payoff if any other path is followed. Suppose that the path used to define a particular Arrow–Debreu security first crosses the payoff boundary at time τ . The security can be regarded as a European-style security promising a (path dependent) payoff at the time τ . As shown in Proposition A1 it is equal in value to a similar no-default Arrow–Debreu security where the option holder is required to make a continuous payment at rate $\alpha(t_0, t)$ times the option’s value at

time t until time τ . The sum of these no-default Arrow–Debreu securities is option X. The result follows. Q.E.D.

We now move on to consider the relative effects of default risk on European and American options.

Proposition A3. *In the situation where the θ variables are independent of the ϕ variables, the proportional effect of default risk on the price of an American option is always less than the proportional effect of default risk on its European counterpart.*

Proof: Define

h, h^* : the values of a vulnerable American option and its no-default counterpart.

f, f^* : the values of the vulnerable and no-default European versions of the option.

Option W: a notional option that is the same as the vulnerable American option except that the option holder is constrained to exercise the option on the early exercise boundary that is optimal for the no-default American option.

Denote the value of option W by H .

Similarly to (16)

$$H = \hat{E}_\phi(p_c) h^*, \quad (\text{A.2})$$

where the expectations operator is taken over all paths followed by ϕ , c is the first point reached on the payoff boundary for W and p_c is the value of p at point c . For any path followed by the ϕ variables, when the default boundary is reached before the early exercise boundary, the proportional payoff on W is the same as the proportional payoff on the corresponding European option; that is $p_c = p_b$. In other circumstances, $p_c = 1$. Consequently, it is always true that

$$p_c \geq p_b,$$

so that

$$\hat{E}_\phi(p_c) \geq \hat{E}_\phi(p_b).$$

From (16) and (A.2)

$$\frac{H}{h^*} \geq \frac{f}{f^*}. \quad (\text{A.3})$$

Option W is the same as the American option under consideration except that the holder is constrained to use a possibly suboptimal early exercise boundary. By

definition the optimal early exercise boundary is the one that maximizes the value of an option. It follows that $h \geq h'$ so that from (A.3),

$$\frac{h}{h^*} \geq \frac{f}{f^*},$$

which is the required result. Q.E.D.

For the next proposition, define

Option X: a notional no-default American option which provides the same payoff as option under consideration and is such that the option holder is required to make continuous payments at rate $\alpha(t_0, t)$ times the option's value at time t until the option either expires or is exercised. [Recall that $\alpha(t_0, t)$ is the spread between the risky and riskless forward rates at time t as seen at time t_0 .]

Option Y: a notional vulnerable option which is the same as the option under consideration except that the holder is constrained to use the optimal early exercise boundary for option X

Proposition A4. *In the situation where the θ variables are independent of the ϕ variables, a lower bound for an American option is the value of option X.*

Proof: Since the optimal early exercise boundary is the one that maximizes the value of an option, the American option under consideration is worth no less than option Y. From Proposition A2 the values of options X and Y are the same. The result follows. Q.E.D.

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