Risk Aversion, Liquidity, and Endogenous Short Horizons

Craig W. Holden
Indiana University

Avanidhar Subrahmanyam
University of California, Los Angeles

We analyze a competitive model in which different information signals get reflected in value at different points in time. If investors are sufficiently risk averse, we obtain an equilibrium in which all investors focus exclusively on the short term. In addition, we show that increasing the variance of informationless trading increases market depth but causes a greater proportion of investors to focus on the short-term signal, which decreases the informativeness of prices about the long run. Finally, we also explore parameter spaces under which long-term informed agents wish to voluntarily disclose their information.

We contribute to the literature on information aggregation in financial markets by analyzing trading behavior and information acquisition in a competitive model in which different signals get reflected in value at different points in time (in the short-term and in the long-term). In our framework, a continuum of risk averse agents can choose to specialize either in collecting short-term information or in collecting

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We are grateful to an anonymous referee and Franklin Allen (the editor) for insightful and constructive feedback. We also thank Hank Bessembinder, David P. Brown, Jeff Coles, Nick Crew, Francis Longstaff, Steve Manaster, John McConnell, George Pennacchi, Marie Sushka, Sheridan Titman, Joe Williams, and participants at the Arizona State University and the University of Utah seminar series, the Illinois-Indiana-Purdue Finance Symposium, and the 1995 Winter Meetings of the Econometric Society for valuable comments and/or discussions. Address correspondence and requests for reprints to Craig W. Holden, School of Business, Indiana University, Bloomington, IN 47405.

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long-term information. They then trade in a market which is semistrong efficient due to the existence of a competitive, risk-neutral market-making sector, and of uninformed liquidity traders whose share demands are unmodeled. We impose no exogenous restrictions on informed agents’ trading behavior so that all their trades and their ex ante gains from trade are determined as part of a rational expectations equilibrium. Our study of the incentives to be short-run oriented in information acquisition decisions is important because there have been arguments that excessive short-term behavior by investors leads to reduced price informativeness about long-run fundamentals, and myopic and nonvalue maximizing investment decisions on the part of corporate managers.\textsuperscript{1,2}

One of our primary goals is to explore the comparative statics associated with the equilibrium proportion of agents who collect short-term information. Under an identified linear equilibrium in the trading stage, if the risk aversion of agents is low, we obtain an interior equilibrium in which a certain proportion of agents chooses the short-term signal and the complementary proportion chooses the long-term one. However, as the risk aversion parameter is increased for all agents, the interior equilibrium disappears and the only remaining equilibrium is one in which all agents collect short-term information. This is because agents suffer disutility as their positions are buffeted by public information shocks unrelated to their private information; longer horizons accumulate more shocks. Thus, if agents are sufficiently risk averse, they all choose to collect short-term information.

It is standard in studies of security and market design to analyze the liquidity of alternative trading mechanisms. A common viewpoint of practitioners is that increasing the liquidity of financial markets lowers the cost/benefit ratio for short-term behavior relatively more than for long-term behavior, and hence increases the degree of "short-termism" in the economy. One manifestation of this argument is a proposal by the Clinton administration to tax short-term security holdings (less than 6 months) at a higher tax rate than long-term security holdings. This proposal is implicitly based on the premise that short-term in-

\textsuperscript{1} See Jacobs (1991, chap. 1 and 2) for an excellent informal elucidation of these arguments, and for anecdotal evidence on myopia on the part of investors. In particular, he states the following on p. 38: "A popular way to make money on stocks . . . is to predict whether reported results will be higher or lower than the reported forecasts. Not surprisingly, the analysts that serve stock traders want to cater to their informational needs, so they focus greater attention on predicting quarterly earnings accurately and less on becoming knowledgeable about a company's competitive forecasts." There is also empirical evidence documenting evidence of short-termism by investors: Lang and McNichols (1992) show that institutional investors are prompt in selling firms that exhibit poor earnings performance.

\textsuperscript{2} The link between the myopic behavior of corporate managers and short horizons of institutional investors is formalized in a recent paper by Coles and Suay (1994).
vestor behavior is undesirable because it adversely affects long-term price efficiency, which is important for efficient corporate investment decisions.

We find that the fraction of traders who choose to acquire the short-term signal increases as we increase the variance of informationless liquidity trading; this leads to greater market liquidity or depth in equilibrium, but prices reflect less long-term information and more short-term information. These results obtain because a marginal increase in the variance of liquidity trading has a greater positive effect on the utility of short-term agents than on long-term agents, since short-term agents take more aggressive positions based on their information. This tends to promote collection of the short-term signal, and decrease investor interest in the long-term one.

Our model contributes to the literature on the dynamic trading behavior of informed agents. In particular, we analyze the optimal trading behavior of a risk averse agent with long-lived information that is unrelated to an impending public announcement. Intuitively, if the trader takes a large position before such an announcement, then he suffers substantial disutility by bearing the risk of the price reaction to the announcement. Therefore, the agent optimally chooses to postpone heavy trading until just after the announcement, which is consistent with the empirical evidence of volume increases following public announcements documented by several researchers.

The result that all agents may choose to shorten their horizons by choosing to specialize in short-term information is obtained in a setting where informed agents cannot announce their private information credibly, which is a reasonable stipulation, given the clear manipulation incentives that arise otherwise [see Hirshleifer (1971)]. However, if credible disclosure is possible, another means by which agents can shorten their horizons is by voluntary revelation of their (long-term) private information in a dynamic setting.\(^3\) The latter part of our analysis allows for such voluntary (credible) disclosure and shows that the incentive to reveal long-term information after an initial round of trade is strong if the holding period risk is large relative to the value of the private information.\(^4\)

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\(^3\) There is a large literature (see, e.g., Diamond [1985] and Kim [1993]), which focuses on the optimal disclosure policy of firms. The focus in these articles is on the tendency of disclosure policy to affect incentives to acquire private information. In contrast, our focus is on whether informed agents themselves have an incentive to disclose their information.

\(^4\) Fishman and Hagerty (1995) analyze whether a strategic insider would wish to disclose his trade in a two-date model like ours. However, in that model, the insider may or may not be informed and if he is informed, he knows the realization of the security value exactly. Disclosure may be optimal because an uninformed insider wishes to manipulate the market by moving the price upon the disclosure. In contrast, in our model, all agents are competitive and there is no possibility of market manipulation. We focus on the aversion to holding risky positions for a long time as
Due to the dynamics and the complex procedures involved in the computation of the informed agents' ex ante utility, we are unable to obtain analytical results, and therefore rely on numerical simulations. While this limitation should be noted, we believe our model offers useful insights without imposing excessive structure or exogenieties in trading behavior. From a modeling standpoint, our analysis is related to Brown and Jennings (1989), Grundy and McNichols (1989), and Hirshleifer, Subrahmanyam, and Titman (HST) (1994). All these articles present two-period competitive trading models with asymmetric information. In Brown and Jennings (1989) and Grundy and McNichols (1989), groups of agents simultaneously receive a set of diverse information signals prior to trading at each of two trading dates, while in HST, agents receive the same information at stochastically different times. In contrast to these articles, we present a model in which different groups of risk averse agents receive information which gets reflected in value at different points in time; our goal is to explore risk sharing between the short-term and the long-term informed agents.

Our model is related to another strand of research which assumes the existence of a class of agents with exogenous short horizons, that is, who exogenously optimize over a single period in a multiperiod setting. This literature includes Bhushan, Brown, and Mello (1991), Dow and Gorton (1994), Froot, Scharfstein, and Stein (1992), McNichols and Trueman (1994), and Vines (1995). In contrast to these articles, we endogenize the choice of horizon by an agent and are therefore able to explore comparative statics associated with the proportion of agents in the economy with short horizons.

Vives (1995) proves existence and uniqueness of a linear equilibrium in a dynamic model similar to ours, but with some significant differences. Thus, while both models possess a risk-neutral market-making sector, the nature of the uncertainty faced by informed agents differs. Specifically, in Vives, each of a continuum of informed agents observes a signal about the true value with an independent noise term; thus, the signal noise in his model aggregates to zero across agents. In our model, however, all agents of a particular type (short-term informed or long-term informed) observe the same signal, but

the motive for voluntary disclosure.

Admati and Pfleiderer (1988a), Fishman and Hagerty (1993), and Vishny (1985) present explanations for why a strategic informed agent may prefer to sell information to others than trade on it as a monopolist. Our focus is on whether informed agents have an incentive to publicly disclose their information without the possibility of obtaining any remuneration for doing so.

Demski and Feltham (1994) and Kim and Verrecchia (1991) also present rational expectations models with two rounds of trade, with public revelation of noisy private signals in the second round. The focus in these articles is on the behavior of volume and trading costs around public announcements.
the final value of the security is subject to public information shocks that are uncorrelated with their private information. (This is equivalent to a model in which the signal noise is correlated across agents.) The latter feature causes the nature of our equilibria to differ from that of Vives (1995). In particular, unlike in his article, dynamic trade can occur even if agents receive only one private signal prior to commencement of any trading, as risk averse agents wish to reduce their exposure to public informational events unrelated to their private information. Thus, our model underscores the point that the timing of public information shocks plays an important role in determining the trading behavior of risk averse informed agents. It is worth observing here, however, that Vives’ structure allows him to obtain existence and uniqueness results, while such results are only available for special cases in our article.

The structure of this article is as follows. In Section 1 we describe the economic setting. Section 2 describes the equilibrium. Section 3 discusses endogenous information acquisition. Section 4 investigates the incentives for long-term informed traders to publicly disclose their information. Section 5 summarizes. All proofs, unless otherwise stated, appear in the appendix.

1. The Economic Setting

Consider a risky security that is exchanged for a riskless security at two trading dates, 1 and 2, with its liquidation value subsequent to date 2 (at date 3) given by

$$F = \delta + \eta + \theta + \epsilon,$$

where $\delta$, $\eta$, $\theta$, and $\epsilon$ are mutually independent, normally distributed random variables, each with a mean of zero. The variables $\delta$ and $\eta$ are publicly revealed at date 2, while $\theta$ and $\epsilon$ are publicly revealed at date 3. The variable $\epsilon$ is the imprecision in the long-term signal, while $\eta$ is the imprecision in the short-term signal.

In order to investigate the gains to collecting information about the long-term signal $\theta$ versus the short-term signal $\delta$, we assume that there are two types of informed traders: long-term informed traders learn precisely the realization of $\theta$ just prior to trade at date 1, while short-term informed traders learn the realization of $\delta$ just prior to trading.

Note that $F$ has an unconditional mean of zero. This is merely a normalization for expositional convenience; one could add a positive ex ante mean $\bar{F}$ to the right-hand side of Equation (1) without altering any part of the analysis.
at date 1. Each informed trader has an endowment of $B_0$ units of the riskless bond. The riskfree rate is normalized to zero.

We consider a competitive framework and thus assume that there is a continuum of each type of informed trader. The mass of long-term informed traders is $M$, while the mass of short-term informed traders is $N$. Since $M$ and $N$ can be interpreted as the proportions of each type of informed trader in the economy, we normalize $M + N = 1$. While $M$ and $N$ are exogenous in Section 2, they are determined endogenously in Section 3.

Both groups of informed traders have negative exponential utility with a common risk aversion coefficient $R$. For now, we assume that long-term informed agents cannot publicly disclose their private information at date 2. This assumption has been made by several other articles which use dynamic frameworks, for example, Brown and Jennings (1989), Froot, Scharfstein, and Stein (1992), Grundy and McNichols (1989), and HST (1994), and appears to be reasonable because an agent has no incentive to be truthful if disclosure is allowed, so that uninformed agents may disclose pure noise and pretend that they have information. Nevertheless, in Section 4, we explore the implications of allowing long-term informed traders to publicly disclose their information, assuming they can do so credibly.

Liquidity demand shocks of $z_1$ and $z_2$, each normally distributed with zero mean, arrive at the market at dates 1 and 2, respectively. These shocks are independent of each other and of $\theta$, $\delta$, $\epsilon$, and $\eta$. There is also a group of risk-neutral market makers, who possess no information about the fundamental value of the risky security. These agents represent a competitive fringe of traders (e.g., floor brokers, scalpers, or institutions who monitor trading floor activities) who are willing to absorb the net demands of the other traders at unbiased prices.

At both dates 1 and 2, informed investors submit demands as a function of their information and the past and current market prices. The market makers observe the net (combined) demands of the informed and the liquidity traders at each date. The net demand in period $i$ is denoted by $\omega_i$ ($i = 1, 2$). Since the informed traders submit price-contingent demands, the order flow in our context can be

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8 Unfortunately, we find that the modeling of strategic behavior is intractable in our dynamic framework.

9 As in much of the related literature (e.g., Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994), and Paul (1993)), we assume that specializing in one type of information precludes obtaining information of the other type. This assumption is in the spirit of Hayek’s (1945) argument that “the data from which the economic calculus starts are never for the whole society given to a single mind which could work out all implications, and can never be so given.”
interpreted as the “limit order book.” Because market makers are risk neutral and competitive, the price at each date equals the expectation of the final value of the security, conditional on the information set of the market makers.

To clarify our structure further, Figure 1 provides a time line illustrating the various types of investors and the times at which the random variables are publicly revealed.

**1.1 Equilibrium in the trading stage**

Our equilibrium in the trading stage can be defined as follows:\(^\text{10}\)

1. For each price-taking informed trader, the trades specified by his demand function at a given date maximizes his expected utility conditional upon his information at that date.
2. Prices at each date are determined by the condition that, due to competition, market makers earn zero expected profits conditional on their information set. Thus, at each date, prices equal the expected value of the security \(F\), conditional on the public information to date, that is,

\[
P_1 = E(F|\omega_1),
\]

\[
P_2 = E(F|\omega_1, \omega_2, \delta, \eta).
\]

**1.2 Equilibria under endogenous information acquisition**

We now characterize equilibria in which the choice between short-term and long-term information is endogenous. Let \(EU_s(N)\) and \(EU_l(N)\) denote the ex ante utilities of each short-term informed agent and

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\(^\text{10}\) See Boot and Thakor (1993), Hirshleifer, Subrahmanyam, and Titman (1994), and Vives (1995) for identical definitions of equilibrium.
long-term informed agent as a function of \( N \), respectively (we show how to calculate these expected utilities in Section 3). Then, if for some \( N^* \in (0, 1) \), the condition \( EU_s(N^*) = EU_l(N^*) \) is satisfied, there exists an equilibrium in which \( N^* \) is the equilibrium proportion of agents who collect short-term information and \( 1 - N^* \) is the equilibrium proportion of agents who collect long-term information. We term this equilibrium an \textit{interior equilibrium}.

It is evident that there can exist “corner” equilibria as well. For example, if \( EU_s(1) > EU_l(1) \), then it is an equilibrium for 100 percent of the agents to collect the short-term signal \( \delta \). To see that this is an equilibrium under the stated condition, notice that if the entire mass of agents analyzes the short-term signal, it does not pay for any zero-mass agent to deviate and collect information about the long-term, as the utility from doing so \([EU_l(1)]\) is less than the utility from collecting information about the short-term signal \([EU_s(1)]\). Note that if \( EU_s(N) > EU_l(N) \) for all \( N \in [0, 1] \), then in fact the only equilibrium (fixing the equilibrium in the trading stage) is one in which the entire mass of agents collect short-term information.

2. Equilibrium Trades and Prices

In this section, we fix \( M \) (and therefore \( N \)) and characterize linear equilibria in the trading stage. (We discuss the endogenous \( M \) case in Section 3.)

2.1 The optimal demands of the informed

Let \( P_1 \) and \( P_2 \) denote the date 1 and date 2 equilibrium prices for the security. Also, let \( x_{l1} \) and \( x_{l2} \) denote the demands (holdings) of the long-term informed agents and \( x_{s1} \) and \( x_{s2} \) denote the holdings of the short-term informed agents at dates 1 and 2, respectively. Further, let \( \phi_{l1} \) and \( \phi_{l2} \) denote the information set of the long-term informed traders at dates 1 and 2 and \( \phi_{s1} \) and \( \phi_{s2} \) denote the information set of the short-term informed traders at the two dates. We begin by conjecturing that all informed trader demands and both date 1 and date 2 prices are normally distributed. In the linear equilibrium we derive, this conjecture is confirmed to be correct.

At date 2, long-term informed agents maximize

\[
E[-\exp(-RW_{L2}^f)|\phi_{l2}],
\]

where \( W_{L2}^f = x_{l2}(\theta + \delta + \epsilon + \eta) - x_{l1}P_1 - (x_{l2} - x_{l1})P_2 + B_0 \), and short-term informed agents maximize

\[
E[-\exp(-RW_{S2}^f)|\phi_{s2}],
\]
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where $W_S^2 = x_{s2}(\theta + \delta + \epsilon + \eta) - x_{s1}P_1 - (x_{s2} - x_{s1})P_2 + B_0$. Since the conditional date 2 wealth is normally distributed, one can use the mean-variance framework to show that the optimal risky holdings of the long-term agents at the end of date 2 are given by

$$x_{l2} = \frac{\theta + \delta + \eta - P_2}{R\sigma^2_\epsilon},\quad (4)$$

and that the date 2 holdings of the short-term traders are given by

$$x_{s2} = \frac{E(\theta|\phi_{s2}) + \delta + \eta - P_2}{R[\sigma^2_\epsilon + \text{var}(\theta|\phi_{s2})]} \quad (5)$$

But note that $\phi_{s2}$ is identical to the information set of the market makers, so that the numerator in Equation (5) is the difference between the date 2 price and the date 2 conditional expected value of the security, and is therefore zero [from Equation (3)]. We thus have

$$x_{s2} = 0 \quad (6)$$

in equilibrium, so that the terminal wealth of the short-term trader is $x_{s1}(P_2 - P_1) + B_0$. Thus, the conditional date 1 wealth of the short-term agents is normally distributed, so that $x_{s1}$ can again be calculated using the mean-variance approach, and is given by the simple expression

$$x_{s1} = \frac{E(P_2|\phi_{s1}) - P_1}{R\text{var}(P_2|\phi_{s1})} \quad (7)$$

Equations (6) and (7) demonstrate that the short-term informed traders take a position to exploit the expected price change across dates 1 and 2, and (endogenously) reverse their positions at date 2. The latter happens simply because the equilibrium date 2 price is set by risk-neutral agents who have the same information as the short-term informed agents, and therefore does not offer a conditional risk premium to the risk averse short-term agents. The size of the position the short-term agents take is determined by the variances $\sigma^2_\epsilon$, $\sigma^2_\theta$, and $\sigma^2_\eta$. Note that the variance $\sigma^2_\epsilon$ does not contribute directly to their trading aggressiveness, as they do not bear this risk.

A more complex problem is the calculation of $x_{l1}$, the date 1 demands of the long-term traders. At date 1, the long-term traders maximize the derived expected utility of their date 2 wealth

$$E[-\exp\{-R(B_0 - x_{l1}P_1 + x_{l1}P_2 + (\theta + \delta + \eta - P_2)^2/(2R\sigma^2_\epsilon))\}|\phi_{l1}]$$

The appendix shows that the optimal date 1 demand of each long-
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term informed trader can be written as

\[ x_{t1} = \frac{E(P_2|\phi_{t1}) - P_1}{RS_t} + \rho E(x_{t2}|\phi_{t1}) \]  

(8)

where \( \rho \) and \( S_t \) are nonstochastic quantities defined in the appendix. Note that the demand represented by Equation (8) consists of two components, one to exploit the expected price appreciation across dates 1 and 2, and another to hedge in advance the anticipated demand at date 2. The constant \( \rho \) represents the degree to which the long-term traders hedge their expected date 2 demands in advance. If the date 2 risk is small (i.e., if the variance of \( P_2 \) conditional on \( \phi_{t1} \) is small), \( \rho \) is large. In fact, it can easily be shown that the hedge coefficient \( \rho \) lies between 0 and 1, and equals 1 if the only source of the date 2 risk is the date 2 liquidity trading, that is, if \( \sigma^2_\delta = \sigma^2_\eta = 0 \). It can also be shown that in this case, there exists a fairly simple closed-form solution to the partially revealing linear equilibria of the model (see Proposition 1 below). However, in the general case, the presence of these hedging demands complicates the calculation of the equilibrium considerably.

2.2 Equilibrium

We will consider linear equilibria implied by the model. Thus, let us postulate that \( P_1 \) and \( P_2 \) are linearly related to the observables at each date such that

\[ P_1 = a\theta + b\delta + cz_1, \]  

(9)

\[ P_2 = \delta + \eta + e\theta + fz_1 + gz_2. \]  

(10)

Note that the variables \( \delta \) and \( \eta \) have coefficients of unity in the conjectured form for \( P_2 \). This is an intuitive conjecture as both \( \delta \) and \( \eta \) are publicly revealed at date 2. In the ensuing analysis we verify that this conjecture is consistent with the equilibrium we derive.

We first use standard multivariate normal distribution theory [see, for example, Anderson (1984, chap. 2)] to calculate the various conditional expectations in the informed investors’ demands of Equations (4) through (8) in terms of the constants \( a, b, c, e, f, \) and \( g \). Note that \( \phi_{s1} = [\delta, P_1] \) and \( \phi_{l1} = [\theta, P_1] \). Since conditional expectations involving normal random variables are linear in the conditioning variable, we can write

\[ E(P_2|\phi_{s1}) = E(P_2|\delta, P_1) = \delta + k_1\theta + k_2z_1, \]  

(11)

\[ E(P_2|\phi_{l1}) = E(P_2|\theta, P_1) = \theta - \rho \delta + k_3\theta + k_4z_1, \]  

(12)

\[ E(x_{t2}|\phi_{s1}) = E(x_{t2}|\delta, P_1) = k_5\theta + k_6z_1, \]  

(13)

\[ E(x_{t2}|\phi_{l1}) = E(x_{t2}|\theta, P_1) = k_7\theta + k_8z_1. \]  

(14)

Note that in Equation (11), \( \delta \) has a coefficient of unity because the short-term informed trader’s date 1 information set \( \phi_{s1} \) contains \( \delta \).
\[ \begin{align*}
E(P_2|\phi_1) &= E(P_2|\theta, P_1) = m_1\delta + m_2\theta + m_3z_1, \quad (12) \\
E(\delta|\phi_1) &= E(\delta|\theta, P_1) = l_1\delta + l_2z_1. \quad (13)
\end{align*} \]

where the values of the (subscripted) constants \( k, m, \) and \( l \) in terms of the price coefficients can be calculated from standard results on the conditional moments of multivariate normal distributions, and are provided in the Appendix. Further,

\[ \begin{align*}
\text{Var}(P_2|\phi_1) &= \text{Var}(P_2|\theta, P_1) = \sigma^2_0 + \sigma^2_1 + \sigma^2_2 (14) \\
- &\frac{c(2a\sigma^2_0 + c(f^2\sigma^2_1 - e^2\sigma^2_0))}{a^2\sigma^2_0 + c^2\sigma^2_1}.
\end{align*} \]

Now, the market makers observe \( \omega_1 = Ns_1 + M\chi_1 + z_1 \) at date 1 which, from Equations (7) and (8), is observationally equivalent to

\[ \tau_1 \equiv N \frac{E(P_2|\delta, P_1)}{R\text{Var}(P_2|\delta, P_1)} + M \times \left( \frac{E(P_2|\theta, P_1)}{RS_1} + \frac{\theta + E(\delta - P_2)|\theta, P_1|S_1 - S_2}{R\sigma^2_\epsilon} \right) + z_1, \quad (15) \]

and at date 2 they observe, in addition to \( \tau_1 \), the date 2 holdings, which, from Equation (4), is observationally equivalent to\(^{12}\)

\[ \tau_2 \equiv \frac{M\theta}{R\sigma^2_\epsilon} + z_1 + z_2. \quad (16) \]

We can then write

\[ \begin{align*}
P_1 &= E(\theta + \delta + \eta + \epsilon|\tau_1), \quad (17) \\
P_2 &= E(\theta + \delta + \eta + \epsilon|\tau_1, \tau_2, \delta, \eta). \quad (18)
\end{align*} \]

Since \( \tau_1 \) and \( \tau_2 \) involve linear combinations of either normal random variables, or conditional expectations of normal random variables, they are normally distributed. Thus, \( P_1 \) and \( P_2 \) are normally distributed.

Solving for the equilibrium involves determining values for the price coefficients \( a, b, c, e, f, \) and \( g \) that satisfy the zero profit condition for the market makers and optimality of demands for the informed traders. The specifics involve first substituting for the conditional expectations and variance in the expressions for \( \tau_1 \) and \( \tau_2 \) in Equations

\(^{12}\) Note that jointly observing the total date 1 and date 2 holdings is observationally equivalent to jointly observing the total date 1 and date 2 order flows since the date 2 order flow is merely the difference between the date 2 and the date 1 holdings.
and (16) from Equations (11) through (15). We can then calculate the conditional expectations in Equations (17) and (18) to obtain implicit expressions for the constants $a, b, c, e, f,$ and $g$. We thus obtain a system of six nonlinear equations in six unknowns which can be solved using standard numerical techniques. This procedure verifies the functional forms of Equations (9) and (10) and confirms the consistency of the postulated linearity in Equations (9) and (10), and the normality of prices in the linear equilibria. Since the demands in Equations (4) through (8) involve either prices or expectations of prices conditional on normal random variables, the demands are normally distributed as well, confirming the postulated normality of both prices and demands.

While we were able to obtain numerical solutions to the equation system described above in many (though not all) cases, we have not been able to prove existence of linear equilibria in general. Problems in formally proving existence of linear equilibria in dynamic rational expectations models were also encountered by Brown and Jennings (1989). However, just as in Brown and Jennings, closed-form solutions for linear equilibria do exist in special cases. We use one such special case as the benchmark for identifying linear equilibria of our model numerically. The existence of the closed-form solution in this special case is stated as a proposition (the proof is straightforward but tedious and is available from the authors on request).

**Proposition 1.** Suppose that $\sigma_z^2 = \sigma_\eta^2 = 0, \eta \equiv 0, \sigma_{z1}^2 = \sigma_{z2}^2 = \sigma_z^2,$ and $M = 1$. Then two linear noisy rational expectations equilibria with $P_1 \neq P_2$ can exist. The closed-form solution to these equilibria is given by

\begin{align}
    a &= \frac{\sigma_\theta^2}{\sigma_\theta^2 + k^2 \sigma_z^2} \\
    b &= 1 \\
    c &= ka \\
    e &= \frac{\sigma_\xi^2((k - R \sigma_\xi^2)^2 + R^2 \sigma_\xi^4)}{D} \\
    f &= \frac{kR^2 \sigma_\xi^4 \sigma_\theta^2}{D} \\
    g &= \frac{kR \sigma_\xi^2 \sigma_\theta^2 (k - R \sigma_\xi^2)}{D},
\end{align}

where

\[ D \equiv \sigma_\theta^2(k - R \sigma_\xi^2)^2 + R^2 \sigma_\xi^4(\sigma_\theta^2 + k^2 \sigma_z^2). \]
The variable $k$ solves the quadratic equation, the variable $k$ solves the quadratic equation

$$k^2 \left[ \sigma_\theta^2 + R^2 \sigma_\epsilon^2 \sigma_z^2 (\sigma_\epsilon^2 + \sigma_\theta^2) \right] - kR \sigma_\theta^2 \sigma_\epsilon^2 (2 + R^2 \sigma_\epsilon^2 \sigma_z^2) + 2R^2 \sigma_\epsilon^4 \sigma_\theta^2 = 0. \quad (26)$$

These linear equilibria exist if and only if the discriminant of the quadratic equation (26) is positive, that is, if

$$\frac{4R^2 \sigma_\epsilon^2 \sigma_z^2 (2\sigma_\epsilon^2 + \sigma_\theta^2) + 4\sigma_\theta^2}{R^3 \sigma_\epsilon^4 \sigma_z^2 \sigma_\theta^2} < 1. \quad (27)$$

Since it can easily be verified that the second-order condition is satisfied whenever a real root of Equation (27) exists, both roots of this equation make economic sense. To obtain more intuition, we now describe the period 1 and period 2 demands of the long-term informed investors under the special case described in Proposition 1:

$$x_{l2} = \frac{\theta + \delta - P_2}{R \sigma_\epsilon^2} \quad (28)$$

$$x_{l1} = \frac{\theta + \delta - P_1}{R \sigma_\epsilon^2} + \frac{E[P_2|\phi_{l1}] - P_1}{R \text{ var}(P_2|\phi_{l1})}. \quad (29)$$

Thus, while the date 2 demand of the long-term informed traders has the form obtained from a standard one-period exponential-normal model, their date 1 demand contains two components, one to exploit the expected price appreciation across dates 1 and 2, and one to exploit the expected price appreciation across dates 1 and 3. In fact, it can easily be shown that

$$\text{Cov} (x_{l2} - x_{l1}, P_2 - P_1) < 0$$

in equilibrium, so that the traders, “on average” partially reverse their positions at date 2. The intuition is simply that agents conjecture that a nonnull price move will occur across dates 1 and 2, and when it does, so that the price moves more in the direction of $\theta$, agents partially reverse positions to reduce their exposure to the $\epsilon$ shock which buffets their final position. This equilibrium exists if and only if the parameter values are such that the conjecture of a price move across dates 1 and 2 can be self-fulfilling. Similar scenarios involving multiple equilibria (characterized by the roots of a quadratic equation) and the possibility of a nonexistence of equilibrium occur in Grundy and McNichols (1989, pp. 501–504).

At this point it is perhaps worth comparing our model to that of Vives (1995). He proves existence and uniqueness of equilibrium in
a dynamic model with a competitive, risk-neutral market-making sector in which a continuum of competitive, risk-averse agents receive diverse signals every period. The signal errors are, however, uncorrelated across agents in every period, so that there is no aggregate uncertainty about the final value of the asset in any period. This enables him to obtain a simple sufficient statistic for the aggregate information in the market. Vives thus finds (see his Proposition 4.1) that the optimal demand of an agent in any given period has the simple form

\[ X_t = a_t (s_t - P_t) \]  

(30)

where \( a_t \) is nonstochastic, \( s_t \) is the signal of the informed in period \( t \), and \( P_t \) is the price at date \( t \), which allows him to derive the equilibrium in a straightforward manner.

Vives further shows that when agents receive information signals only at the first trading round, prices do not move and there is no trade in any of the subsequent trading rounds (see his Remark 4.3). This case is directly comparable to our special case in Proposition 1, in which there is no short-term private information, so that the only private signal long-term agents receive is the realization of \( \theta \) prior to trade at date 1. Comparing Equations (29) and (30), it can be seen that our date 1 demands do not have the same form as those in Vives, in the sense that the second term in Equation (29) represents an additional component of trade to exploit the expected price appreciation across dates 1 and 2. The two demand equations are equivalent only if the expected price appreciation across dates 1 and 2 is a constant times \((\theta + \delta - P_1)\), which is not true in general. Thus, Vives’ analysis does not carry over to our case. Indeed, in our case there are some parameter values that generate multiple equilibria.

The key lies in the differing nature of uncertainty faced by informed agents in the two models. Specifically, in Vives, the imprecision regarding the final asset value is uncorrelated across agents, whose positions therefore are not subject to any public information shocks. In our model, agents observe the same signal and are subject to a common public informational shock \( \epsilon \) at date 3.\(^{13}\) This gives the agents in our model an incentive to reduce the risk exposure arising from a position taken at date 1, by making a partially reversing trade at date 2. Thus our model can support trade in both rounds even when

\(^{13}\) In technical terms, we are stating that if we were to modify the Vives model so that the final liquidation value was \( v + \epsilon \) (instead of \( v \)), where \( \epsilon \) was an independent public informational shock, and agents were to observe \( v + \delta_i \), where \( \delta_i \) was i.i.d. across agents, we would be unable to obtain existence and uniqueness results using his methodology, primarily because the sufficient statistic for final value is more complex in this case. A formal proof of this assertion is available from the authors upon request.
agents receive a private signal only at date 1, whereas Vives’ cannot. With regard to related literature, Grundy and McNichols (1989, p. 504) also argue that common error terms in signals cause the demands of informed agents to become complicated in a dynamic setting. Further, Brown and Jennings allow for a general variance-covariance matrix of error terms and also find that the intertemporal demands of informed agents become complicated. Note that it is also not possible to adapt Vives’ “truth plus iid noise” structure to our case, as we are interested in how short- and long-term information gets incorporated into stock prices by different informed agents, a situation that must necessarily allow for some determinant of liquidation value that is unknown by any given informed agent.

While the linear, noisy rational expectations equilibria can be characterized in terms of the roots of a quadratic equation in the special case described in Proposition 2.2, no such simple characterization is possible in the general case. For all the parameter spaces for which we report results below, however, we numerically identified at least one linear equilibrium, which converged to that solution obtained by placing a positive sign on the discriminant of Equation (26), as we fixed \( \sigma^2_{z1} = \sigma^2_{z2} = \sigma^2_{z} \) and let the variances of \( \sigma^2_{\delta} \) and \( \sigma^2_{\eta} \) go to zero. The simulations below use this linear equilibrium to illustrate our comparative statics.

The base parameter values we use throughout this article are the following: \( \sigma^2_{\epsilon} = 10, \sigma^2_{\delta} = \sigma^2_{\delta} = \sigma^2_{\eta} = \sigma^2_{z1} = \sigma^2_{z2} = 1, \) and \( R = 1. \) The large value of \( \sigma^2_{\epsilon} \) relative to \( \sigma^2_{\eta} \) reflects the fact that long-term values are forecasted with far more uncertainty than value in the near term. Though we choose the above parameter values to illustrate our results, the qualitative features of all the results reported below continued to obtain when all the parameters (except the one on the x-axis of the relevant figure) were simultaneously varied between 1 and 10, with a grid size of unity.

### 2.3 Long-term informed trading around public announcements

Section 2.1 shows that the short-term informed behave as one-period agents by taking a position at date 1 and reversing it completely at date 2. We now describe some salient features of the equilibrium trading patterns of long-term informed agents within our model. Our goal is to establish a link between trading behavior in our model and the empirical literature on trading volume around public announcements.

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14 The analysis of Bhattacharya and Spiegel (1991) also suggests the possibility of the existence of nonlinear equilibria.
In this subsection, we interpret the date 2 shock \( \eta \) as an earnings announcement on date 2, so that \( \sigma^2_\eta \) measures the risk from the price reaction to the announcement. The simulations below characterize the trading behavior of the long-term informed agents as a function of \( \sigma^2_\eta \).

We begin by defining a measure of the volume of trade in each period. Note that according to our convention, \( x_{t2} \) denotes the holdings, rather than the trades, of the long-term informed agents. The volume of trade by the long-term informed agents can therefore be measured as \( M_{\text{std}}(x_{t1}) \) in period 1 and \( M_{\text{std}}(x_{t2} - x_{t1}) \) in period 2, where \( \text{std}(\cdot) \) denotes the standard deviation operator. Analogous expressions apply for the short-term informed agents. The total volume at each date can be measured as

\[
TV_{t1} = M_{\text{std}}(x_{t1}) + N_{\text{std}}(x_{s1}) + \text{std}(\omega_1) + \text{std}(z_1)
\]

\[
TV_{t2} = M_{\text{std}}(x_{t2} - x_{t1}) + N_{\text{std}}(x_{s2} - x_{s1}) + \text{std}(\omega_2) + \text{std}(z_2)
\]

where \( \omega_1 = Mx_{t1} + Nx_{s1} + z_1 \) and \( \omega_2 = M(x_{t2} - x_{t1}) + N(x_{s2} - x_{s1}) + z_2 \). The measure of total volume thus has four components: volume from the (i) long-term informed, (ii) short-term informed, (iii) market makers, and (iv) liquidity traders, and thus takes into account the trades crossed between traders as well as trading done with the market maker. [See Admati and Pfleiderer (1988b) for a similar definition of trading volume.]

Figure 2 plots the ratio of date 2 and date 1 long-term informed volume, fixing \( M = 1, N = 0 \), and varying \( \sigma^2_\eta \) from 0.1 to 10. The figure demonstrates that the ratio of the period 2 to period 1 trading volume by the long-term informed is increasing in \( \sigma^2_\eta \). The ratio of total date 2 volume to total date 1 volume is also plotted in Figure 2; as can be seen, this ratio is also increasing in \( \sigma^2_\eta \).

Figure 2 thus indicates that as the announcement risk across dates 1 and 2 increases, trading volume becomes more and more concentrated in round 2 as opposed to round 1. The reason is that long-term informed agents trade more heavily on their information after a large part of the uncertainty surrounding the security’s value is resolved, which causes trading volume to be heavier in round 2 as opposed to round 1. Thus, our model predicts that volume will be relatively high following an earnings announcement, which is consistent with the evidence of Morse (1981) and, more recently, Lee, Mucklow, and Ready (1993) (see their Figure 3). It is evident that our intuition applies more generally to any public information shock whose timing can partially be predicted in advance, not just earnings announcements.15

15 A significant literature also reports that market depth decreases following earnings announcements [see Lee, Mucklow, and Ready (1993), Skinner (1991), and Venkatesh and Chiang (1986)]. Market
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Figure 2
Long-term volume ratio and total volume ratio by variance of the date 2 information shock ($\sigma^2_\eta$)

The ratio of date 2 long-term volume to date 1 long-term volume becomes very large as $\sigma^2_\eta$ increases. The ratio of total date 2 volume to total date 1 volume is also increasing in $\sigma^2_\eta$. Long-term informed traders avoid the price risk generated by the uncertainty of the date 2 information shock $\eta$ by concentrating their trading in round 2 (when $\eta$ is known) as opposed to round 1. It is assumed that investor risk aversion $R = 1$, the mass of long-term agents $M = 1$, the mass of short-term agents $N = 0$, the variance of the date 3 information shock $\sigma^2_\varepsilon = 10$, the variance of the short-term signal $\sigma^2_\delta = 1$, the variance of the long-term signal $\sigma^2_\theta = 1$, the variance of date 1 uninformed trading $\sigma^2_{z_1} = 1$, and the variance of date 2 uninformed trading $\sigma^2_{z_2} = 1$.

It is worth noting that the results in Figure 2 are in contrast to the risk-neutral, strategic Kyle (1985)-type models such as Holden and Subrahmanyam (HS) (1992) and Foster and Viswanathan (FV) (1993), in which most of the trading and information revelation takes place in the early trading rounds. The intuition is simply that in our model, informed agents are competitive and their positions are constrained by their degree of risk aversion. In contrast, in the HS-FV models, the informed traders' positions are determined solely by competition between strategic agents.

We have ascertained that the result on total volume in Figure 2 depends somewhat on the mass of short-term informed agents. As $\sigma^2_\eta$ increases, the announcement risk for the short-term agents increases, so that they trade less at both dates 1 and 2. If the mass of short-term agents is large, the decrease in short-term volume at both dates can be sufficiently large to cause the ratio of the date 2 total volume to the date 1 total volume to decrease in $\sigma^2_\eta$.

depth in our model can be measured as the inverse of the coefficients of $\tau_1$ and $\tau_2$ in $P_1$ and $P_2$. We have verified that the period 2 depth does indeed increase in $\sigma^2_\eta$ under the parameter values of Figure 2; the intuition being that the long-term informed trade more and aggressively at date 2 as the date 2 announcement risk increases. Our model is therefore consistent with this stylized fact as well.
3. The Ex Ante Benefit From Trading on the Information

In this section, we endogenize $M$ and $N$ and thus explore comparative statics associated with the equilibrium proportions of investors who collect short-term and long-term information.

3.1 Calculation of the ex ante utility

The simplest way to calculate the ex ante benefit from trading on either the short- or long-term signal is to operate on matrices. Define a vector of random variables $x \equiv [\theta \, \delta \, \eta \, z_1 \, z_2]$. Thus $x$ is multivariate normally distributed with a mean vector of zeroes, and covariance matrix $\Sigma \equiv \text{Diag}(\sigma_\theta^2, \sigma_\delta^2, \sigma_\eta^2, \sigma_{z_1}^2, \sigma_{z_2}^2)$.

The long-term trader’s demands can then be written as $x_{l_2} = x\beta'_{l_2}$ and $x_{l_1} = x\beta'_{l_1}$, where

$$
\beta_{l_2} \equiv \begin{bmatrix}
-\frac{e}{\rho \sigma_r^2} & 0 & 0 & -\frac{f}{\rho \sigma_r^2} & -\frac{g}{\rho \sigma_r^2}
\end{bmatrix}
$$

and

$$
\beta_{l_1} \equiv \begin{bmatrix}
-\frac{a \sigma_\varepsilon^2 + m_2(S_1 - (S_2 + \sigma_\varepsilon^2)) + S_2 - S_1}{RS_1 \sigma_\varepsilon^2} & 0 \\
-\frac{b \sigma_\varepsilon^2 - (l_1(S_1 - S_2) + m_1(S_2 + \sigma_\varepsilon^2 - S_1))}{RS_1 \sigma_\varepsilon^2} & 0 \\
-\frac{c \sigma_\varepsilon^2 - (l_2(S_1 - S_2) + m_3(S_2 + \sigma_\varepsilon^2 - S_1))}{RS_1 \sigma_\varepsilon^2} & 0
\end{bmatrix}.
$$

Similarly, the short-term trader’s demand can be written as $x_{s_1} = x\beta'_{s_1}$ where

$$
\beta_{s_1} \equiv \begin{bmatrix}
\frac{k_1 - a}{\text{Var}(P_2|P_1)} & 0 & \frac{1 - b}{\text{Var}(P_2|P_1)} & 0 & \frac{k_1 - c}{\text{Var}(P_2|P_1)} & 0
\end{bmatrix}.
$$

The prices $P_1$ and $P_2$ and the final value $F$ can be written in vector form as $P_i = \lambda_i x'$, $i = 1, 2$, and $F = \lambda_3 x'$ where $\lambda_1 \equiv [a \, b \, 0 \, 0 \, c \, 0]$, $\lambda_2 \equiv [e \, 0 \, 1 \, 1 \, f \, g]$, and $\lambda_3 \equiv [1 \, 1 \, 1 \, 1 \, 0 \, 0]$.

The the ex ante benefit from trading on the long-term signal is given by

$$
E[-\exp(-RW_{l_2}^L)],
$$

where $W_{l_2}^L = x_{l_2}(F - P_2) - x_{l_1}(P_2 - P_1) + B_0 = x\beta'_{l_2}(\lambda_3 - \lambda_2)x' + x\beta'_{l_1}(\lambda_2 - \lambda_1)x'$. Similarly, the ex ante benefit from trading on the short-term signal is

$$
E[-\exp(-RW_{s_2}^S)],
$$

where $W_{s_2}^S = x_{s_1}(P_2 - P_1) + B_0 = x\beta'_{s_1}(\lambda_2 - \lambda_1)x' + B_0$. 708
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Figure 3

Difference in ex ante utility (short-term versus long-term) by the mass of long-term agents ($M$)

When risk aversion is high enough, the ex ante utility of the short-term informed is greater than ex ante utility of the long-term informed (i.e., the difference is positive) for any value of $M \in [0, 1]$. Thus, in equilibrium 100 percent of the agents choose the short-term signal. It is assumed that investor risk aversion $R = 1$, the variance of the date 3 information shock $\sigma_\varepsilon^2 = 10$, the variance of the date 2 information shock $\sigma_\eta^2 = 1$, the variance of the short-term signal $\sigma_\delta^2 = 1$, the variance of the long-term signal $\sigma_\theta^2 = 1$, the variance of date 1 uninformed trading $\sigma_z^2 = 1$, and the variance of date 2 uninformed trading $\sigma_z^2 = 1$.

Since the wealth levels $W^L_2$ and $W^S_2$ are quadratic forms of multivariate normal random variables, the ex ante utilities of each short-term and long-term informed agent, in terms of the $\beta$ and $\lambda$ vectors, can be evaluated by using standard results on the moment-generating functions of such quadratic forms; details are provided in the Appendix. The expressions for the ex ante utilities are complicated and do not permit the derivation of formal comparative statics. However, the expressions can be numerically calculated by first solving for the price coefficients $a$, $b$, $c$, $e$, $f$, and $g$ and then using them to calculate the $\beta$ and $\lambda$ vectors.

3.2 The role of risk aversion

In Figure 3 we explore how the difference between the utilities from collecting short-term and long-term information changes as the proportion of long-term (short-term) informed agents increases (decreases). As can be seen, in the entire range $M \in [0, 1]$, the utility from collecting short-term information is higher than that from collecting long-term information, which implies that the only equilibrium, fixing the (numerically) identified linear equilibrium in the trading stage, is one in which 100 percent of the agents collect the short-term information. We have found that this result obtains under a wide range of parameter values, so long as $R$ is sufficiently large.
Figure 4
Equilibrium proportion of short-term agents ($N^*$) by risk aversion
When risk aversion is not too high ($R < 0.4$), then an interior equilibrium is obtained with some proportion of the agents acquiring the short-term signal ($N^* < 1$) and the remaining proportion acquiring the long-term signal. When risk aversion is high enough ($R \geq 0.4$), then a corner solution is obtained with all agents choosing the short-term signal ($N^* = 1$). It is assumed that the variance of the date 3 information shock $\sigma^2_\epsilon = 10$, the variance of the date 2 information shock $\sigma^2_\eta = 1$, the variance of the short-term signal $\sigma^2_\delta = 1$, the variance of the long-term signal $\sigma^2_\theta = 1$, the variance of date 1 uninformed trading $\sigma^2_1 = 1$, and the variance of date 2 uninformed trading $\sigma^2_2 = 1$.

Figure 4 identifies interior equilibria for values of $R$ smaller than that considered in Figure 3. The figure plots the equilibrium proportion of agents who collect short-term information $N^*$ versus the risk aversion coefficient $R$. The simulations in Figure 4 use the same base parameter values as Figure 3, except that $R$ varies from 0.01 to 1.0. As can be seen, $N^*$ increases in $R$ until it reaches the maximum possible value of 1, suggesting that large values of the risk aversion coefficient encourage the acquisition of short-term information.

3.3 The role of uninformed liquidity trading
This subsection measures the effect of increasing the variances of both the period 1 and period 2 liquidity trading on the equilibrium proportion of agents collecting short-term information and, consequently, on the long-term and short-term informational efficiency of market prices. We set $\sigma^2_{z1} = \sigma^2_{z2} = \sigma^2_z$. The simulations use the same parameter values as Figure 4, except that $\sigma^2_z$ varies and $R$ is fixed at unity.

Figure 5 plots the difference between short-term and long-term utility for different values of $\sigma^2_z$. It can be seen that the gains from collecting short-term information relative to those from collecting long-term information increase monotonically in $\sigma^2_z$. This result suggests that large values of the variance of liquidity trading increase the ten-
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Figure 5
Difference in ex ante utility by the variance of uninformed trades ($\sigma^2_2 = \sigma^2_1 = \sigma^2_z$)

An increase in the variance of uninformed trades causes the ex ante utility of the short-term informed to increase by more than the ex ante utility of the long-term informed. Since short-term agents benefit by more, this increases the tendency to acquire short-term information. It is assumed that investor risk aversion $R = 1$, the mass of long-term agents $M = 0.5$, the mass of short-term agents $N = 0.5$, the variance of the date 3 information shock $\sigma^2 = 10$, the variance of the date 2 information shock $\sigma^2 = 1$, the variance of the short-term signal $\sigma^2 = 1$, and the variance of the long-term signal $\sigma^2 = 1$.

tendency to acquire short-term information. The intuition is as follows. The short-term informed agents trade more heavily on their information than the long-term agents because the holding-period risk of the short-term informed (measured by $\sigma^2$) is smaller than that of the long-term agents (measured by $\sigma^2_\theta$ and $\sigma^2_\eta$). Thus, an increase in $\sigma^2_z$, which tends to raise the ex ante utility of both types of informed agents, benefits the short-term informed agents more than it does the long-term agents.

We now explore the effect of varying $\sigma^2$ on the efficiency of prices about the short-term and long-term private information. As is standard in the literature, we measure informativeness (or efficiency) by the conditional precision of the security value innovation. Thus, we measure period 1 and period 2 long-term efficiency by $E_{11} \equiv [\operatorname{Var}(\theta|P_1)]^{-1}$ and $E_{12} \equiv [\operatorname{Var}(\theta|P_1, P_2, \delta, \eta)]^{-1}$. Explicit expressions for these measures are given by

$$E_{11} = \frac{a^2 \sigma^2_\delta + b^2 \sigma^2_\delta + c^2 \sigma^2_2}{\sigma^2_\theta (b^2 \sigma^2_\delta + c^2 \sigma^2_\delta)},$$

$$E_{12} = \frac{\sigma^2_\theta \sigma^2_2 (af - ce)^2 + a^2 g^2 \sigma^2_\theta \sigma^2_2 + c^2 g^2 \sigma^2_2 \sigma^2_\delta}{c^2 g^2 \sigma^2_2 \sigma^2_\delta}. \tag{32}$$

A corresponding expression for short-term efficiency (conditional on
An increase in the variance of uninformed trades causes a monotonic increase in both date 1 liquidity and date 2 liquidity. Liquidity on each date is measured by the inverse of the slope of each date’s price function relative to each date’s total order flow. It is assumed that investor risk aversion $R = 0.1$, the variance of the date 3 information shock $\sigma^2_\varepsilon = 10$, the variance of the date 2 information shock $\sigma^2_\eta = 1$, the variance of the short-term signal $\sigma^2_\delta = 1$, and the variance of the long-term signal $\sigma^2_\theta = 1$. The proportion of short-term and long-term agents is endogenous.

Public information at date 1) can be obtained from Equation (9) and is given by

$$ E_d = \left[ \text{var}(\delta|P_1) \right]^{-1} = \frac{a^2\sigma^2_\theta + b^2\sigma^2_\delta + c^2\sigma^2_{\sigma_1}}{\sigma^2_\delta(a^2\sigma^2_\theta + c^2\sigma^2_{\sigma_1})}. \quad (33) $$

Figure 6 plots date 2 long-term and short-term informational efficiency [from Equations (32) and (33)] versus $\sigma^2_z$, using the equilibrium proportion of investors $N^*$ that collect short-term information at each level of $\sigma^2_z$. As can be seen, an increase in the variance of liquidity trading increases the equilibrium proportion of agents collecting short-term information and thereby leads to an increase in short-term efficiency, while causing a corresponding decrease in long-term efficiency.16

Figure 7 plots the equilibrium values of the period 1 and period 2 depth parameters (the inverse of the coefficients of $\tau_1$ and $\tau_2$ in $P_1$ and $P_2$) for the same range of $\sigma^2_z$ as in Figure 6. Consistent with intuition, liquidity or depth increases monotonically as $\sigma^2_z$ increases. Thus, Figures 6 and 7 demonstrate that as $\sigma^2_z$ increases, the market

16 Though not presented for brevity, the effects of increasing the risk aversion coefficient on short-term and long-term price efficiency are qualitatively identical to that of increasing the variance of liquidity trading.
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Figure 7
Short-term and long-term informational efficiency by the variance of uninformed trades ($\sigma^2 = \sigma^2_1 = \sigma^2_2$)

An increase in the variance of uninformed trades causes a monotonic increase in short-term informational efficiency and a monotonic decrease in long-term informational efficiency. This example, together with Figure 6, demonstrates how it is possible for greater liquidity to result in a market that impounds more short-term information and less long-term information. It is assumed that investor risk aversion $R = 0.1$, the variance of the date 3 information shock $\sigma^2_3 = 10$, the variance of the date 2 information shock $\sigma^2_2 = 1$, the variance of the short-term signal $\sigma^2_3 = 1$, and the variance of the long-term signal $\sigma^2_4 = 1$. The proportion of short-term and long-term agents is endogenous.

becomes more and more liquid, the informativeness of prices about the long-term declines, while short-term efficiency increases.\(^\text{17}\)

The comparative statics in Figures 6 and 7 lend credence to the argument by some quarters that high market liquidity (in our case, via a high variance of noise trading) can be accompanied by an intensified focus on the short-term and decreased information production about the long-term. It is worth noting here that we formalize this notion by varying the exogenous parameter $\sigma^2_z$, not by varying liquidity itself, which is endogenous to our model. In models of informed trading, there is no necessary positive relationship between market liquidity and the variance of liquidity trading under endogenous information acquisition. This is because increased liquidity trading tends to promote information collection; the net effect on equilibrium liquidity can be positive or negative. In Figures 6 and 7, however, the beneficial effect of an increased variance of liquidity trading on market liquidity dominates.

\(^{17}\) If one were to vary $\sigma^2_3$ alone, as opposed to both $\sigma^2_3$ and $\sigma^2_2$ simultaneously, the results would be even stronger. This is because simultaneously increasing $\sigma^2_3$ and $\sigma^2_2$ increases the variability of the period 2 price; this effect tends to reduce the utility of both the short-term and the long-term informed traders. Of course the potential benefit is that the equilibrium can permit more liquidity in both periods. In the simulation in Figures 6 and 7, the former effect dominates the latter.
3.4 Discussion

Note that we do not perform a welfare analysis. Doing so would require an explicit modeling of the preferences of the liquidity traders, and/or introducing investment by the corporate sector into the model. These extensions are outside the scope of our article. Our perspective on the welfare issue is as follows. The current stock price is an important indicator to corporate managers in their investment decisions: more efficient prices lead to better allocation of resources. In making investment decisions, a corporate manager will care more about how efficient prices are about the long-term (e.g., about cash flows over the next 2 years), than about (say) next month’s earnings announcement. There is thus a correspondence, at least in this intuitive sense, between the incentives of investors to be short-term oriented and the efficiency with which corporate resources are invested.

Our model makes a number of special assumptions: for example, we assume that market makers are risk neutral and that all potentially informed investors have identical preferences. A model with risk averse market makers becomes too complicated to solve even numerically. If market makers are risk averse, the short-term informed agents would not completely reverse their positions at date 2, as the date 2 price would offer them a conditional risk premium. This would reduce the aggressiveness of short-term informed agents and thus reduce the parameter space under which our result on the effect of an increase in the variance of liquidity trading obtains (but would not eliminate the result). If informed investors have diverse preferences, one can envision an equilibrium in which the least risk averse investors pursue long-run analysis, while the most risk averse specialize in short-run information. Our goal, however, is to explore the comparative statics of the equilibrium proportion of short-term investors when all individuals are ex ante identical. Our general intuition, namely, that the risk of holding long-term positions can cause potentially informed agents to be short-run oriented, and that an increase in the variance of liquidity trading confers a greater marginal benefit to the short-term agents than to the long-term agents (and thus encourages short-term information acquisition), appears to be robust to the relaxation of the model’s assumptions.

For explicit models of the link between price efficiency and the efficiency of investment decisions, see Bradley, Khanna, and Slezk (1994) and Fishman and Hagerty (1989).

Jacklin and Bhattacharya (1988) develop a model in which agents have differing preferences for early versus late consumption, and show that it is optimal to finance short-term assets using deposit contracts and long-lived, illiquid assets using equity. We endogenize investor horizons, but do not focus on contract design in our article.
4. Shortening Horizons by Publicly Disclosing Long-Term Information

A central result from the previous analysis is that the disutility of holding long-term risky positions can give rise to an equilibrium in which all investors endogenously shorten their horizons by specializing in short-term information collection. The result, however, is obtained in a setting where informed agents cannot publicly reveal their private information before its exogenous incorporation into value. This assumption is consistent with much of the dynamic rational expectations literature and is reasonable because an informed agent may find it difficult to convey credibly to the market that he indeed has relevant information. However, Allen (1990) discusses ways to ensure credible disclosure of private information by agents by appropriate choices of signal-contingent portfolios and payment functions. With the possibility of such disclosure, another means by which agents can shorten their horizons is by voluntarily revealing their (long-term) private information after an initial round of trade.20 Motivated by this intuition, we now explore the implications of allowing informed agents to credibly disclose their information. Many empirical studies, for example, Barber and Loeffler (1995), Bjerring, Lakonishok, and Vermaelen (1983), Davies and Canes (1978), and Greene and Smart (1994) have found that recommendations by reputed analysts through the popular press lead to significant abnormal returns following the announcement date, which are only partially reversed up to 3 weeks following the announcement. Thus, there is reliable evidence that such announcements do have information content.

We show below that under certain parameter values, disclosure at date 2 is optimal for the long-term informed agents. The reason is that holding a position for long periods of time results in the bearing of risk which is unrelated to the information. If the utility loss from holding a position for the long-term dominates the benefit from not disclosing at date 2 and realizing gains in the second round of trade, disclosure at date 2 yields higher expected utility.

The securities market equilibrium with disclosure of long-term information at date 2 is easily calculated as follows. If $\theta$ is credibly disclosed by a long-term agent at date 2, then the net order flows $\omega_1$ and $\omega_2$ cannot convey any incremental information at this date, so

20 Note that there is a clear possibility of market manipulation here in that an analyst could buy a stock, publicly provide a (false) buy recommendation, and sell out as the price moves in response to the recommendation. Presumably, however, reputational considerations would preclude an agent from manipulating the market consistently in this fashion. We abstract from such market manipulation and reputational issues in our analysis. See, however, Allen and Gale (1992) and Benabou and Laroque (1992) for models of market manipulation by informed traders.
that $P_2 = E(F|\theta, \delta, \eta) = \theta + \delta + \eta$. Given that both the long-term and the short-term traders face a one-period problem in this case, they maximize standard mean-variance objectives, and their demands are

$$x_{l1} = \frac{\theta + E(\delta|P_1, \theta) - P_1}{R[\sigma_\eta^2 + \text{var}(\delta|P_1, \theta)]},$$  \hspace{1cm} (34)

$$x_{s1} = \frac{\delta + E(\theta|P_1, \delta) - P_1}{R[\sigma_\eta^2 + \text{var}(\theta|P_1, \delta)]}.$$  \hspace{1cm} (35)

The market makers observe the variable $Mx_{l1} + Nx_{s1} + z_1$ at date 1, and this net demand is observationally equivalent to

$$\tau_1 = M \frac{\theta + E(\delta|P_1, \theta)}{R[\sigma_\eta^2 + \text{var}(\delta|P_1, \theta)]} + N \frac{\delta + E(\theta|P_1, \delta)}{R[\sigma_\eta^2 + \text{var}(\theta|P_1, \delta)]} + z_1.$$  \hspace{1cm} (36)

By the market efficiency condition, we have

$$P_1 = E(\theta + \delta|\tau_1).$$  \hspace{1cm} (37)

The numerical solution to the disclosure case can be obtained by postulating a linear price function $P_1 = a\theta + b\delta + cz_1$, calculating the expectations in Equation (36), calculating the expectation in Equation (37), and equating coefficients to obtain implicit expressions for the constants $a$, $b$, and $c$. The ex ante utility of the long-term agents with disclosure can be calculated using a simple application of the techniques used to calculate the ex ante utility of agents without disclosure [see Lemma 1 and Equation (48) in the Appendix].

The masses of long-term and short-term agents, $M$ and $N$, are taken to be exogenous in this section, as endogenizing them does not lend any additional insight into the issues addressed here. An overall equilibrium, allowing informed agents to credibly disclose their information, can then be defined in a straightforward fashion. Define $EU_{ld}$ and $EU_{ln}$ as the utility of a long-term agent with and without disclosure of $\theta$ at date 2, respectively. An equilibrium is one in which the long-term agent chooses a strategy of either disclosing or not disclosing $\theta$ at date 2, based on whichever strategy yields higher ex ante expected utility.\(^{21}\) Thus, if $EU_{ld} > EU_{ln}$, the equilibrium involves disclosure at date 2, and vice versa. To see that this claim is correct, suppose $EU_{ld} > EU_{ln}$, and no long-term agent discloses at date 2. This cannot be an equilibrium, because a long-term informed agent

\(^{21}\) It is evident that neither the short-term agents nor the long-term agents would find it optimal to reveal their information at date 1, that is, before they have realized any gains from the private information. We have verified that this trivial point, while difficult to analytically prove, is true in all the simulations below.
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Figure 8
Difference in ex ante utility (disclosure versus no disclosure) by the variance of the date 3 information shock ($\sigma^2_\epsilon$) and by the variance of the long-term signal ($\sigma^2_\theta$).

For low values of $\sigma^2_\epsilon$, the ex ante is greater for disclosure than no disclosure. But this reverses for the high values of $\sigma^2_\epsilon$. Similarly, for high values of $\sigma^2_\theta$, the ex ante utility is greater for disclosure than no disclosure. But this reverses for low values of $\sigma^2_\theta$. It is assumed that investor risk aversion $R = 1$, the mass of long-term agents $M = 0.5$, the mass of short-term agents $N = 0.5$, the variance of the date 2 information shock $\sigma^2_{\eta_2} = 1$, the variance of the short-term signal $\sigma^2_\theta = 1$, the variance of date 1 uninformed trading $\sigma^2_{z_1} = 1$, and the variance of date 2 uninformed trading $\sigma^2_{z_2} = 1$.

has an incentive to deviate and disclose. Suppose, on the other hand, that one long-term agent does disclose. This is clearly an equilibrium, because the long-term agent does not have an incentive to change his strategy, and once the signal $\theta$ has been disclosed by one long-term informed agent and thus been incorporated into value, the other long-term agents are indifferent to whether they disclose or not. An analogous argument applies in the case of $EU_{ld} < EU_{ln}$.

Figure 8 illustrates how the incentives to disclose $\theta$ change with the model parameters. The figure plots the difference between the long-term utilities with and without disclosure as a function of the post-date 2 risk $\sigma^2_\epsilon$ and of the variance of private information $\theta$. In the figure, the values of $M$ and $N$ are each set to be 0.5. As can be seen, for high values of $\sigma^2_\epsilon$ and low values of $\sigma^2_\theta$, the long-term agent’s utility with public disclosure of his information is higher than that without. As the figure indicates, high values of $\sigma^2_\epsilon$ and low values of $\sigma^2_\theta$ encourage disclosure, and vice versa. While not presented here for brevity, the qualitative effect of increasing risk aversion on the incentive to disclose is similar to the effect of increasing $\sigma^2_\epsilon$. Thus if the long-term risk is large, or if the agent is very risk averse, or if the information is not very valuable, the agent is better off revealing the information at date 2 rather than holding on to a risky position until date 3, so that the equilibrium involves disclosure at date 2.
From an empirical standpoint, the above discussion suggests that (i) very long-term information is more likely to be announced publicly than shorter term information, and (ii) the less well-capitalized (more risk averse), smaller investors are more likely to disclose their information publicly than the relatively well-capitalized ones.

5. Summary

A primary goal of this article has been to investigate an unaddressed issue in the literature on information acquisition in financial markets; namely, what parameters govern the incentives of agents to acquire and produce information that will be reflected in security values in the short-term as opposed to the long-term?

In our model, competitive agents can choose ex ante to specialize either in short-term or in long-term information production. The information acquisition decision and all subsequent trading behavior of the informed agents are determined within our equilibrium. We show that risk aversion can cause all potentially informed investors in the economy to tilt away from the long-term and concentrate exclusively on the short-term. Also, in contrast to the existing literature on strategic trading models, we show that long-term informed agents delay trading heavily on their information till a later trading round due to risk aversion, so that prices reflect relatively more information in later trading rounds rather than in the earlier ones. We also show that if credible disclosure of private information is possible, risk aversion lends a natural incentive for long-term informed agents to shorten their horizons by voluntarily revealing their information after an initial round of trade.

There have been informal assertions that “highly liquid markets can breed too much short-term trading and overly myopic behavior” [Gammill and Perold (1989, p. 16)]. We formalize this notion by showing that a high variance of informationless trading (and thus, high liquidity) can encourage short-term trading and increase the tendency to specialize in collecting short-term information. This, in turn, makes prices reflect more short-term information and decreases their informativeness about the long-term.

Appendix

Derivation of Equation (8)

Note that in period 1, the long-term informed traders maximize the derived expected utility of their time 2 wealth which is given by

$$E[\exp\{-R[B_0 - x_{11}P_1 + x_{11}P_2 + (\theta + \delta + \eta - P_2)^2/(2R\sigma_x^2)]\}]\phi_{i1}. \tag{38}$$
Let $\hat{P}_2$ and $\hat{\delta}$ denote the expectations of $P_2$ and $\delta$, and $\Pi$ denote the variance-covariance matrix of $P_2$ and $\theta + \delta + \eta$, conditional on $\phi_1$. Then, the expression within the exponential above (including terms from the normal density) can be written as

$$-\left[\frac{1}{2}x' S x + b' x + l\right],$$

where

$$x' = [P_2 - \hat{P}_2, \delta - \hat{\delta}]$$

$$b' = [R x_l + \frac{(P_2 - (\theta + \delta))}{\sigma^2}, \frac{(\theta + \hat{\delta} - \hat{P}_2)}{\sigma^2}]$$

$$S = \begin{bmatrix} \Pi^{-1} + \begin{bmatrix} \sigma^{-2} - \sigma^{-2} \\ -\sigma^{-2} & \sigma^{-2} \end{bmatrix} \end{bmatrix}$$

$$l = R x_l (\hat{P}_2 - P_1) + g$$

where $g$ is an expression that does not involve $x_l$. From Lemma 1 in the text and Bray (1981, Appendix), Equation (38) is given by

$$-\frac{1}{(\text{Det}(\Pi))^{\frac{1}{2}} |\text{Det}(A)|^{\frac{1}{2}}} \exp\left(\frac{1}{2} b' S^{-1} b - l\right). \quad (39)$$

Thus, the optimal $x_l$ solves

$$\left[\frac{d b}{d x_l}\right]' S^{-1} b - \frac{d l}{d x_l} = 0.$$

Substituting, we have

$$x_l = \frac{\hat{P}_2 - P_1}{RS_1} + \frac{\theta + \hat{\delta} - \hat{P}_2}{R \sigma^2} \frac{S_1 - S_2}{S_1}, \quad (40)$$

where $S_1$ and $S_2$ are the elements in the first row of the matrix $S^{-1}$. Defining $\rho \equiv (S_1 - S_2)/S_1$, we obtain Equation (8). \(\square\)

**The constants in Equations (11) through (13)**

Using standard formulas for projections of normal random variables, the constants in Equations (11) through (13) are given by the following:

$$k_1 = \frac{a(a c \sigma^2 + c f \sigma^2)}{a^2 \sigma^2 + c^2 \sigma^2_1} \quad \text{(41)}$$

$$k_2 = \frac{c(a c \sigma^2 + c f \sigma^2)}{a^2 \sigma^2 + c^2 \sigma^2_1} \quad \text{(42)}$$
Derivation of the ex ante expected utilities of each short- and long-term informed agent

We begin by using the following lemma, which is a standard result on multivariate normal random variables [see, for example, Brown and Jennings (1989)].

**Lemma 1.** Let \( Q(\nu) \) be a quadratic function of the random vector \( \nu \): \( Q(\nu) = C + B'\nu - \nu'A\nu \), where \( \nu \sim N(\mu, \Sigma) \). We then have

\[
E[\exp(Q(\nu))] = |\Sigma|^{-\frac{1}{2}}|2A + \Sigma^{-1}|^{-\frac{1}{2}} \\
\times \exp \left( C + B'\mu + \mu'A\mu + \frac{1}{2}(B' - 2\mu'A')(2A + \Sigma^{-1})^{-1}(B - 2A\mu) \right).
\]

The calculation in our case is somewhat simplified because \( \mu = 0, C = -RB_0, \) and \( B = 0 \). The long-term trader’s ex ante utility becomes

\[
EU_l(N) = E((-\exp(-xA^l:\lambda'))|\phi_0) = -|\Sigma|^{-\frac{1}{2}}|2A^l + \Sigma^{-1}|^{-\frac{1}{2}}
\]

where

\[
A^l \equiv R[\beta_{12}'(\lambda_3 - \lambda_2) + \beta_{11}'(\lambda_2 - \lambda_1)]
\]

Similarly, the short-term trader’s ex ante utility can be written as

\[
EU_s(N) = E((-\exp(-xA^s:\lambda'))|\phi_0) = -|\Sigma|^{-\frac{1}{2}}|2A^s + \Sigma^{-1}|^{-\frac{1}{2}}
\]

where

\[
A^s \equiv R[\beta_{11}'(\lambda_2 - \lambda_1)]
\]

References

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The Review of Financial Studies / v 9 n 2 1996


