

Recovery of Face Value at Default: Empirical Evidence and Implications for Credit Risk Pricing

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Abstract

We examine defaulted bonds of predominantly US-based issuers who at the time of the initial default event had several publicly traded bonds outstanding. We find that in the vast majority of cases bonds of the same issuer and seniority are valued equally or within one dollar by the market, irrespective of their time to maturity. This strongly suggests that the amount recovered in default is best modelled as a fraction of face value at the time of default (RFV) as other assumptions, in general, cannot generate such a payoff pattern in bonds. We argue that this assumption is generated by the institutional framework which affects corporate bond pricing and thus is the only assumption with an underpinning theory. Within the context of structural credit risk models we find that the recovery value form does affect spreads and potentially to a degree as important as other institutional features which have been incorporated into structural credit risk models. This has important implications for empirical studies of credit risk which use structural models. While it has been remarked in the literature that RFV is often used in practice, up to now there has been little detailed discussion on why this should be the case nor any empirical support for such a hypothesis. In fact, some recent papers show support for alternative recovery assumptions. The results within strengthen the argument that the market incorporates (or should) RFV when valuing corporate debt securities.

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1 Introduction

In this paper we examine, at the time of the initial default event, the value of bonds for a sample of defaulted issuers having multiple publicly traded bonds outstanding. Considering the prices at the default date as a proxy for the recovery value of a given bond, we find that the data supports the *Recovery of Face Value at Default* (RFV) assumption as described in Duffie (1998)¹. This recovery assumption states that bondholders receive a fractional recovery of par *at* the default date. A primary implication of such an assumption is that bonds of the same issuer and seniority will converge to the same value regardless of the remaining maturity. Our focus on the *relative* recovery rates for bonds of the same issuer and seniority allows us to determine whether such an assumption better describes the data compared with alternative recovery forms. We find that in 83% of the cases in our sample the difference between the maximum and minimum bond price for a given issuer is less than \$1, a result which is hard to generate under most other recovery forms. To the best of our knowledge this is the first study which shows empirical evidence regarding relative recovery values for bonds with the same issuer and seniority. While the presentation and description of our dataset is the main contribution of this paper, we discuss two important and related topics: 1) the effect of various institutional factors on relative recovery rates, and 2) the implications such a recovery assumption has for credit risk pricing within the context of structural credit risk models.

There are a number of recovery value assumptions seen in the literature for defaultable debt modelling. While each embeds different empirical implications, it is unusual to see a theory associated with the recovery form assumed. We argue that a close study of the institutional framework affecting the corporate bond should generate this theory. Specifically, the bond indenture and relevant bankruptcy code should be the critical determinants of the relationship between recovery values across bonds of the same issuer and seniority. The debt acceleration clause found in the typical bond indenture generates the hypothesis that an RFV assumption should hold. Our examination of the US bankruptcy

¹This reference cites Brennan and Schwartz (1980) as an early example that used such a recovery assumption in a (convertible) debt pricing model. Further references are listed below.

code does not lead us to generate an additional or a modified hypothesis on what the relative recovery values should be. Specifically, we can interpret from the code that recovery values could easily deviate from RFV as a result of a specific bankruptcy plan, but not in a systematic or predetermined way. Our empirical results lend support to this assertion.

We examine the effect that this assumption has on theoretical credit risk pricing. Huang and Huang (2002) has demonstrated that almost all structural credit risk models significantly underestimate credit spreads for reasonable parameters after having been calibrated to empirical default frequencies. This leads the authors to conclude that much of the yield spread between corporate and treasury bonds has little to do with credit risk. This conclusion is surely one potential solution to the puzzle, however, in their study one parameter that does seem to affect spreads is the recovery value parameter. If the recovery value *level* can move spreads significantly, it might be that the recovery *form* may play a part in affecting the performance of these models. We do in fact find that the recovery form can affect model generated spreads rather significantly. Incorporating RFV can either increase or decrease spreads versus a recovery form traditionally used in this framework depending on the particular interest rate environment. While RFV cannot solve the traditional pricing puzzles exhibited by these models, its significance in affecting spreads seems at least as important as other institutional features which have been incorporated into corporate debt pricing models.

1.1 Related Literature

A major factor affecting the valuation of defaultable debt is the amount recovered by the debtholder when default occurs. Estimating this recovery value is a difficult task for a variety of reasons. First, the absolute priority rule is often violated in practice – due in part to the complex nature of the bargaining game that takes place between the corporate claimants at default. This feature seems to be especially important in the US bankruptcy code (see Franks and Torous (1989)). Second, empirical evidence suggest that recovery values vary across both firms and time. Characteristics of the company’s assets affect

the firm's liquidation value at default², while macroeconomic conditions defined by the current state of the economy also seem to play a major role.³ In addition, the recoveries can depend on whether default is triggered by financial rather than economic difficulties. These combined facts have led researchers to model recovery as an exogenous variable within pricing models which one can then estimate by using historical information.

In models with exogenous recovery a number of different assumptions have been made regarding its form. In structural defaultable debt models, such as Longstaff and Schwartz (1995) and Saa-Requejo and Santa-Clara (1999), it is usually assumed that the recovery amount is a fixed fraction of an equivalent coupon and maturity risk-free bond. Alternatively in this class of model, it could be easily assumed that the recovery amount of default is non-zero only for the principal. In this case, the recovery amount would be a fixed fraction of a zero-coupon risk-free bond with the same maturity as the defaultable bond (Collin-Dufresne and Goldstein (2001)). In intensity-based defaultable bond models the recovery assumption is the main feature that distinguishes one model from another. In Jarrow and Turnbull (1995) and Jarrow, Lando, and Turnbull (1997) debtholders, as mentioned above, receive a fixed fraction of the face amount at the maturity of the defaultable bond. In Duffie and Singleton (1999) debtholders receive a fraction of the market value of the defaultable bond just prior to default. This assumption is made mainly for technical reasons as it leads to much simplified expressions for the price of risky debt. Following the terminology used in Duffie and Singleton (1999) and Bakshi, Madan, and Zhang (2002), we label the first two recovery assumptions described the *Recovery of Treasury* (RT and RT-F)⁴ models; and the last assumption as the *Recovery of Market Value* (RMV) model.

Despite the large and expanding literature on defaultable bonds, the RFV assumption has not always been addressed, especially in the empirical literature. Duffie (1998)

²Altman and Kishore (1996) find that certain industries consistently have higher recovery rates. One explanation for this is the nature of the assets.

³Keenan (1999) documents that the yearly average lows of recovery rates correspond to recession periods as well as times where corporate default rates have peaked. Franks and Torous (1994) show that recovery is significantly and related to the past performance of the aggregate stock market.

⁴To distinguish between the case where coupons have non-zero recovery in addition to the face amount and that where only the face value has non-zero recovery, respectively.

discusses this recovery form in the context of intensity-based models. He provides a few tractable cases where prices of defaultable debt can be computed. This is important for practical applications as such models are used to extract implied risk-neutral default probabilities from traded credit instruments which can be used for valuation and risk management of credit derivative products. Finkelstein (1999 and 2000) carries this point forward and gives an example how this change in recovery assumption can substantially affect the relative valuation of bonds of the same issuer and seniority as well as implied risk-neutral default probabilities from market credit spreads. He also asserts that, in addition to recovery of par at default, the investor is owed the accrued interest on the last coupon and incorporates this feature into the model.⁵ A similar point is made in Jarrow and Turnbull (2000) when discussing which types of recovery assumptions can be used within intensity-based models. They label the assumption the “legal claim approach”, acknowledging that this form of recovery is often used by practitioners. However, in neither of these papers is strong evidence given that RFV is a more correct assumption. This paper attempts to provide this reasoning as well as empirical support. Hull and White (2000) compare the RFV assumption with the RT assumption for pricing credit default swaps.

Other papers have done more extensive work comparing the different recovery assumptions. While the intensity-based model presented in Duffie and Singleton (1999) assumes a RMV, the authors also recognize the fact that for the case of corporate bonds RFV may be more realistic assuming either liquidation at default or that absolute priority holds. The main arguments used to support their RMV model in such cases are: 1) computational advantages versus the RFV model; and 2) absolute priority is not always maintained in bankruptcy courts and liquidation at default is often avoided. Nevertheless, they investigate the pricing implications of models under both recovery assumptions and show under chosen parameters that the implied credit spreads under the two assumptions are similar. However, they also remark that it is possible under certain scenarios (e.g. bonds with significant premium or discount, steeply upward or downward sloping yield curve)

⁵As we will notice later, this is a slight overspecification of the general point that the language in bond indentures has a direct effect on the recovery value of bonds at default.

that the RMV and RFV assumptions can have markedly differing spread implications for a given exogenous recovery rate. Delianedis and Lagnado (2002) similar to Finkelstein (2000) compare three recovery assumptions, RT-F⁶, RMV, and RFV, for the pricing of credit default swaps and for inferring risk-neutral default probabilities from hypothetical zero-coupon risk-free and risky term structures. They demonstrate the sensitivity of implied conditional default probabilities to the recovery assumption and conclude that the choice of recovery model is an important consideration for credit risk pricing. They find that for most cases the RMV and RT-F models generate similar term structures of risk-neutral default probabilities assuming some initial credit risky term structure while the RFV assumption results in distinctly different probabilities. Interestingly, they argue that the RFV model fails to generate admissible long-dated risk-neutral default probabilities. The RFV assumption also leads to significantly higher default swap spreads than the other two assumptions.

Bakshi, Madan, and Zhang (2002) analyze the effect of recovery on the price of defaultable debt in the context of intensity-based defaultable debt models using individual corporate bond data. It is the first paper to provide an empirical test that compares different recovery concepts within such models. They compare three recovery assumptions: RT-F, RMV, and RFV in a robust setting where recovery rates of each firm are stochastic and a function of the underlying hazard rate which is in itself related to the stochastic riskfree rate process, all consistent with empirical data. The two main empirical findings are: 1) the bond data better supports the notion of the RT-F assumption in that pricing errors are significantly smaller; and 2) the RFV model provides the more stable implied recovery rates.⁷ The authors explain their first finding by the fact that the RT-F model recognizes time-value of money considerations and claim that such an assumption may be more closer to truth in market valuation, compared to either RFV or RMV. However, our findings on *actual* recovery values contradict this conjecture. Our empirical results suggest that assuming RT-F to price bonds of the same issuer and seniority while using the same recovery rate may lead to significant mispricing as the expected cashflow pattern of the

⁶No coupon-paying bonds were considered in their comparative analysis.

⁷Compared to the RT-F model. In the RMV model, the recovery rate is unidentified.

bond would be far different than from reality. A potential explanation for their results is our finding in the context of structural credit risk models that RT-F does lead to higher theoretical spreads versus the competing assumptions. This is due to the comparatively smaller recovery value in the RT-F assumption for a given recovery rate.

The paper is organized as follows. Section 2 shows an example of RFV for the publicized cases of Enron Corporation and Worldcom Incorporated. In Section 3 we discuss the institutional factors that provide the framework which underlies this assumption. Section 4 discusses the data sources and selection procedure used. The core of the paper is found in Section 5 where we describe the recovery values in the selected sample and compare our data to hypothetical values generated by different recovery models. Section 6 shows how this assumption can affect spreads in the context of structural credit risk models. We offer some concluding remarks in Section 7.

2 Recovery Convergence: Enron Corp. and Worldcom Inc.

We discuss in detail two “case-study” examples of RFV for corporate bonds. Specifically, we examine prices of bonds issued by Enron Corporation and Worldcom Incorporated over the period leading up to their initial default events. These companies are of particular interest as they represent the two largest Chapter 11 bankruptcy filings in U.S. history. While just examples, the two cases produce strong evidence of the phenomenon we wish to document and show later is a pervasive feature of the value of corporate debt in default.

Table I shows the prices and contractual details of nine bonds of the same seniority (senior unsecured) within Enron’s capital structure. The shortest bond, #1, matures on 3 April 2001; while the longest maturity bond, #9, matures on 15 July 2028. The coupon rate varies across bonds, ranging from 6.40% to 9.125%. Table II shows the dealer bid prices and respective yields to maturity for the bonds in the months and days leading up to the official initial default event, a Chapter 11 bankruptcy filing on Sunday, 2 December 2001. On the following day the bonds that have a face value of par are quoted at the same price of \$21. However, the table shows that the bonds converged to within one dollar of the same price a few days before, on 28 November 2001. On this date a previously

announced merger bid by a rival company, Dynergy Inc., was called off.

Figures 1 and 2 show the dramatic effect that convergence in recovery values has on the corporate yield curve. The most striking feature of RFV is that yields on short maturity bonds increase dramatically compared to long-maturity bonds. In price terms short bonds suffer more on a relative basis compared to long-maturity bonds. Bond 1 declines 64% in one day when the failed merger is announced while Bond 9 decreases 54%. At this same date compared with 28 September 2001, when the bonds were trading at pre-distress level, Bond 1 declines 80% versus 76% for Bond 9.

Table III and Figures 3 and 4 show a similar pattern for the case of Worldcom Inc. We observe the prices of 13 senior unsecured bonds as seen in Table I⁸. Bond #1, the shortest dated bond, matures on 15 May 2003. The longest dated bond, #13, matures on 15 May 2031. The official initial default event occurs on 15 July 2002 when the company misses interest payments on some of their outstanding bonds. At this stage bonds 1-5 are quoted at just a little over \$14, the quoted price for bonds 6-13. When Worldcom eventually files for Chapter 11 on 21 July, all 13 bonds are quoted at \$13.25. Though these cited dates may be the official dates of record for default and bankruptcy the bond start trading near this level on 26 June 2002; when Worldcom announced that it had hidden \$3.8 billion in losses for over a year. Immediately, the bonds were quoted at close to \$11.50, apart from bond 1 which was quoted at \$14. As in the Enron case the short maturity bonds suffer comparatively more than the longer maturity bonds. On the day of the news announcement the bond 1 declines by 80% while bond 13 decreases in value by 70%. These two examples display at least some evidence of RFV, a recovery assumption generally not considered in most empirical studies of credit risk pricing.

⁸For purposes of saving space in the tables we only show a table of the prices and yields for a subset of the 13 bonds. The 4 bonds omitted, however, show the same pattern as those bonds described in the table.

3 Institutional Factors and Relative Recovery Values

In this section we discuss the effect institutional features may have on the relative recovery value of bonds in the same seniority class. We restrict our discussion to the US market. Understanding which institutional factors may affect defaulted bond prices and to what extent is an important point. Any particular pattern that we may see in relative recovery values should have an underpinning theory. We believe the foundation of this theory can be found in the particular institutional details. We focus on: 1) the *bond indenture*, the legal instrument that describes a borrower's responsibilities and constrains their actions; and 2) the *bankruptcy code*, which will affect the value of debt claims if the company files for bankruptcy, and thus any ex-ante valuation of such claims.

3.1 Bond Indentures

The bond indenture sets out in detail the legally binding agreement between an issuer of a bond and the bondholder. They cover considerations such as, but not limited to: 1) contractual form of the bond; 2) amount of the issue; 3) collateral pledged (if a secured issue); 4) protective covenants; 5) credit enhancements; 6) redemption rights and call privileges; and 7) defaults and remedies. The indenture also provides for the appointment of a trustee to act on behalf of the bondholders, in accordance with the Trust Indenture Act of 1939 (TIA), to which, in general, all publicly sold corporate bonds must conform.

Indentures are necessarily bond specific documents and thus to some extent idiosyncratic. However, most indentures are based on a *model indenture form* produced by organizations such as the American Bar Association⁹ and these model forms provide a useful

⁹The most cited documents are the *American Bar Foundation Commentaries on the Model Debenture Indenture Provisions* (1971), the *American Bar Foundation Mortgage Bond Indenture Form* (1981), and the *Model Simplified Indenture of the Section of Business Law* (1983). Recent attempts to modernize these model indentures to recent changes in the capital markets include the *Form Indenture and Commentary* (2000), produced by the National Association of Bond Lawyers (NABL); and the *Revised Model Simplified Indenture* (1999), prepared by the American Bar Association. (ABA). We primarily refer to these latter two documents, both which can be downloaded from the internet (<http://www.nabl.org/library/topicalmats/modelindent/2000/indenture2000.PDF> and <http://www.abanet.org/buslaw/trust/rmsi0500.html>), in our paper.

guide to the contents of a typical-asked about indenture. The critical part of the indenture that influences recovery value is found in the article concerning “Events of Default and Remedies”; this is also outlined in the bond prospectus. In this article the term “Event of Default” is defined in detail and may include the following: 1) failure to make an interest or principal payment on a due date; 2) failure to perform any other covenants over a specified time period; 3) any event of bankruptcy or receivership; and 4) trigger of cross-default provisions. Next, the remedies available to the bondholder after default has occurred are listed. The key point here is that a *debt acceleration* clause is usually specified. That is, the trustee or the holders of a large percentage (usually at least 25%) of the aggregate principal amount of the bonds outstanding can declare the principal amount immediately payable. In many cases the accrued interest, if any, and the redemption premium, if any, is also declared immediately payable. Furthermore, the indenture usually gives the trustee, upon the event of default, the power to exercise a broad range of additional remedies on behalf of the bondholders, including the right to sue for past due principal and interest. The trustee or the majority of bondholders can also rescind any declared acceleration and restore the status quo if the events which led to default are cured or waived, although such an action is usually rare. As an example of the type of language that is commonly used for this acceleration of debt payments, we reproduce below a paragraph from the prospectus supplement of the 6.875% Enron Corp bond, maturing on 15 October 2007. Notice that in this particular case neither a cross-default clause nor an acceleration of any accrued interest/premium is specified:

“If an Event of Default described in clause (a) or (b) in the foregoing paragraph has occurred and is continuing with respect to Indenture Securities of any series, the Indenture provides that the Trustee or the holders of not less than 25% in principal amount of the outstanding Indenture Securities of that series may declare the principal amount of all of the Indenture Securities of that series to be due and payable immediately, and upon any such declaration such principal amount shall become immediately due and payable. If an Event of Default described in clause (c) or (d) of the foregoing paragraph occurs and is continuing, the Trustee or the holders of not less than 25% in principal amount of all of the Indenture Securities then outstanding may declare the principal amount of all of the Indenture Securities to be due and payable immediately, and upon any such declaration such principal amount shall become immediately

due and payable. (Section 502.)

A default under other indebtedness of Enron is not an Event of Default under the Indenture, and an Event of Default under one series of Indenture Securities will not necessarily be an Event of Default under another series.”

While additional information detailing the rights and remedies available to bondholders is typically included in this section of the indenture, the main conclusions to be drawn from bond indentures for our purposes are that: 1) the bondholders have an absolute and unconditional right to receive payment of principal and accrued interest (if any and specified) on the bonds and can institute suit for the enforcement of such payment; and 2) the debt becomes due immediately. Thus, the terms of the indenture mean that the stated maturity of the bonds should have no impact on the price of the *principal* portion of bonds within the same seniority class at default. This conclusion is independent of the specific bankruptcy code that is applicable if the company makes such a filing and provides the initial description of the recovery claim for defaulted issues within the same seniority class.

3.2 Bankruptcy Code

The consequences of the indenture may put pressure on the obligor to cure the defaults if they are able to do so and thus rescind the debt acceleration. Alternatively, the acceleration may force the debtor to seek the protection of the bankruptcy courts, if such a filing were not the initial default event. The Bankruptcy Act of 1978 is the main bankruptcy code law of the United States. It defines the procedures for a liquidation, through Chapter 7 of its code, and for a reorganization through Chapter 11. While the debtor may attempt to workout a reorganization outside Chapter 11, for most large companies with multiple bonds outstanding (the typical firm in our sample) Chapter 11 is the usual first option.

Chapter 11 protects the debtor company from its creditors while it attempts to work through its financial difficulties. The objective of Chapter 11 is to preserve the company while a plan for reorganization is worked out among creditors. The immediate effect following a filing is that the bankruptcy court automatically suspends all default notices

from lenders. The next step is that the debtor company proposes a plan of reorganization (which can be presented at the time of filing i.e. a prepackaged bankruptcy) describing how creditors and shareholders are to be treated under the new plan. The claimants in each class of creditors are entitled to vote on the plan. If the plan is approved, within each class, by a majority of creditors by number and two-thirds by value, then the debtor can seek confirmation of the plan from the bankruptcy court. If the court approves the plan, a newly reorganized company emerges from bankruptcy.

The stage of the Chapter 11 process that most directly affects the relative recovery values of bonds within the same class is the classification of claims as proposed in the reorganization plan. The rules for allocation are based on Section 1122 of the 1978 Act. We reproduce this section below:

“Section 1122. Classification of claims or interests

- a) Except as provided in subsection (b) of this section, a plan may place a claim or an interest in a particular class only if such claim or interest is substantially similar to the other claims or interests of such class.
- b) A plan may designate a separate class of claims consisting only of every unsecured claim that is less than or reduced to an amount that the court approves as reasonable and necessary for administrative convenience.”

The main point is that only substantially similar claims can be put into the same class. However, because the guidance provided by the code is relatively broad, it may be easier to determine what is *not* “substantially similar” than to say what is. Of course, we would not expect secured and unsecured debt to be grouped in the same class. Yet within the same seniority class no specific provisions are made for dealing with the bond-specific contractual details such as the maturity or coupon. As long as the court views a group of securities as reasonably similar, the claims can be placed in the same class. Once the allocation has been made, Section 1123 of the code, which describes the contents of the plan, states that a plan:

“(a)(4)provide the same treatment for each claim or interest of a particular class, unless the holder of a particular claim or interest agrees to a less favorable treatment of such particular claim or interest”

This allows for a wide range of allocations within each class as a result of bargaining. For example, the relative recovery values might be strongly affected by the concentration of claim ownership. One single creditor can block a plan if it holds one-third of the value of any class of claimants. However, creditors are typically dispersed and it is difficult to assess how the negotiating game may play out. Bondholder committees are usually formed with the help of the trustee to speak on behalf of each class of creditors, but their role for determining *intra*-class allocation is less clear. In general, it may be the case that to appease certain bondholders versus others within the same class concessions could be made in a proposed plan, leading to a recovery form inconsistent with RFV.

The role of the bankruptcy code on the ex-ante relative valuation of same seniority defaulted debt is complex. While the bond indenture provides strict language on the nature of the claim at default, the bankruptcy code only outlines a few general principles regarding the allocation of the new claims. Therefore, the US bankruptcy code neither implies that a RFV will hold, nor precludes it.

4 Data Description

4.1 Data Sources

This study uses data on issuers who have defaulted on their debt and on the details of individual corporate bonds once the issuers have been chosen. We identify defaulted issuers using the Moody’s corporate bond default database and we thus restrict the universe of debt issuers to those having outstanding debt rated by Moody’s. In addition, we employ Moody’s particular definition of a default event. The main features of the definition of a Moody’s default event is the occurrence of one of three events: 1) a missed or delayed interest and/or principal payment, 2) a bankruptcy filing or legal receivership occurs, or 3) a distressed exchange where either the exchange had the purpose of avoiding default or the new securities offered to the bondholders amount to a diminished financial obligation. A full description of Moody’s definition can be found in Hamilton, Cantor, and Ou (2002).

Moody's default research is also used for identifying the date and type of the initial default event. For cases where this information is not available in the Moody's reports we use Bloomberg. Bankruptcy filing dates, if such an event occurred and was not the initial default event, are also retrieved from Bloomberg.

We obtain individual corporate bond price data both before and after the default event from the Dow Jones Interactive website (<http://djinteractive.com>) owned by the Dow Jones Company, hereafter referred to as the "bond data source". According to the website the historical prices come from Tradeline, a product of SunGard Market Data Services (<http://www.tradeline.com>). The underlying source of data is specified by either the exchange where the bond is traded (e.g. NYSE) or labelled "Dealer-priced", which are dealer bid prices. We use only dealer bid prices due to the well known problems inherent with exchange-traded corporate bond prices (Warga (1991)). Apart from the coupon rate and final maturity date of each bond, very little static information for the bonds is available on this website. We use Bloomberg, Moody's Industrial Manuals, and prospectuses for each individual bond to obtain more detailed static information.

4.2 Data Selection

The full sample of defaulted issuers used for this study is determined as follows. First, with the help of an investment bank, we perform a search of the Moody's default database to find all bond issuers which defaulted from January 1997 to May 2002 in the US and Europe and filter those that had at least 5 (non-convertible) bonds outstanding at the time of the initial default event. This generates a list of 32 issuers. Since the information on the number of bonds outstanding for this search is taken from a different data source from the one used to obtain prices, we further filter this list by checking, for each issuer, whether price data are available for more than one bond. This step reduces the list to 30 issuers. We then check whether we have data for more than one bond of the same seniority type at the initial default event date. This further reduces the list to 26 issuers. We augment this sample by searching through all the defaulted bond issuers as listed in the Moody's default reports for 2000, 2001, and the first quarter of 2002 and which are not in the previous sample. This secondary search adds a further 16 issuers, a total of 42

issuers that are analyzed below.

We select bonds for each issuer as follows. For 33 of the 42 issuers all the bonds available on the bond data source are considered, while for the remaining nine issuers a subsample of the available bonds are used¹⁰. We then select bonds if a traded price exists at the initial default date. We are interested only in bonds from the same issuer and with the same seniority. Most of the bonds in our sample were senior unsecured debt. For five of the issuers, they were subordinated unsecured debt. Table IV shows the selected sample of issuers along with the number of bonds selected for each issuer.

4.3 Data Summary

We see in table IV that 38 of the 42 selected issuers are domiciled in either the US (34) or Canada (4). Thus, for the most part, focusing on just one type of bankruptcy regime (US) should be sufficient. Table V summarizes the issuer data by the year of default and by the initial default event type. 37 of the 42 issuers (88%) were defaults since the beginning of 2000. The table also shows the mean and median default volume of the selected sample of firms over time. Although the sample period is relatively short we find that the size of the mean and median defaulted issuer, if proxied by default volume, has increased. This is the case even though the largest default in history, Worldcom, is not included in the sample.

The vast majority of initial default events were either a Chapter 11 bankruptcy filing (48% of sample) or a missed interest or principal payment (38% of sample). However, even if the initial event is not a bankruptcy filing, bankruptcy may still be imminent. Table VI shows that in total 36 of the 42 issuers (86%) eventually file for Chapter 11 protection. Twenty-three cases were conventional filings while the balance were prepackaged. The majority (21 of 36) of the bankruptcies filed in the sample are still pending at the time of maturity while in 15 of the cases the company has either emerged from bankruptcy or completed through a different manner (i.e. sale of assets, merger). The table also describes the length of time between the initial default event and the bankruptcy filing. The mean

¹⁰In these case the number of available bonds were numerous enough that we saw it sufficient to download a subset of bonds which covered as wide a range of maturities as possible.

and median times to file are 141 and 76 days respectively. The maximum number of days between the two events is 409 days, while the minimum time is 7 days.

Table VII gives the distribution of the number of bonds in the sample issued by a given company and the distribution of the range of maturities of bonds, issued by a given company. The maturity range refers, for each issuer, to the difference between the final maturity dates of the longest and shortest dated bonds. The table also shows that we are able to examine only a small number of bonds for each issuer. For 26 of the firms we have data for only 2 or 3 bonds while we have data for at least 6 outstanding bonds from six issuers. For 25 of the issuers the maturity range is 5 years or less. The mean and median maturity ranges are 7.34 years and 3.78 years respectively with a standard deviation of 8.49 years. The maximum range is 27.85 years and the minimum range is zero.

5 Relative Analysis of Recovery Values

This section summarizes the relative recovery values at default in our sample. An important issue is the metric used for determining how close the recovery values are. We look at simple measures of dispersion in the data and these suggest that the RFV pattern in recovery is pervasive. We then compare our data to hypothetical values generated by alternative recovery assumptions.

5.1 Simple Dispersion Measures

The simplest measure of dispersion is the range, defined here as RV_{Range} , i.e. the difference between the maximum and minimum values in the data. Under strict RFV where no accrued interest is included as part of the claim at default RV_{Range} would be equal to zero. When accrued interest is part of the claim, as specified in some indentures, we would expect the measure to be within the difference between accrued interest due on respective bonds. Panel A of table VIII shows a distribution of this measure at two dates: 1) the initial default date and 2) the bankruptcy filing date. In 17 of the 42 cases (40%) RV_{Range} is zero, i.e. the recovery value is identical across issues. Furthermore, 29 of the 42 issuers (69%) have a RV_{Range} less than or equal to \$2. On the other hand, 6 of the

issuers show a RV_{Range} greater than \$5, the largest being \$52.16 for Owens Corning. The mean and median of the RV_{Range} at the initial default date are 3.30 and 0.88 respectively. The RV_{Range} at the Chapter 11 filing date display a broadly similar pattern with a slightly higher percentage (77%) of issuers having a value less than or equal to \$2. In 15 of 36 issuers (42%) the RV_{Range} is equal to zero while 3 of the issuers have a RV_{Range} greater than \$5.

Panel B of table VIII shows the average deviation, RV_{AD} , of each issuer's bonds at the initial default and bankruptcy filing dates. This measures the average absolute deviation from the mean of an issuer's bond prices at the specified date. This metric may be more reliable as it averages across all bonds of a particular issuer, thus measurement errors due to stale data from one particular bond are less problematic. We see that at the initial default date 34 of the 42 issuers (81%) RV_{AD} is at or below \$2 and only two cases have an RV_{AD} greater than \$5. Results at the bankruptcy filing date are similar where 31 of the 36 issuers (86%) have a RV_{AD} less than or equal to \$2. The mean and median of the average deviation of recovery values in sample are 1.20 and 0.31 respectively.

We also determine if for each issuer whether a mode in the set of bond prices exists. A mode exists if the same price is quoted for more than one bond at a particular date. Although the information is binary it may lend more support to RFV compared to other recovery assumptions. For example, in the case of an RT or RT-F assumption we would not expect, in general, to see any mode for bonds of differing maturities due to the discounting effect. For the RMV assumption we would only expect to see a mode if the pre-default prices of the different bonds were trading at the same level. Panel C of table VIII shows that a mode exists for 27 of the 42 (64%) firms at the initial default date and 26 of the 36 firms at the bankruptcy filing date (72%).

5.2 Examination of Outliers

The results of the previous section suggest that for a great majority of the default events in our sample RFV is a good approximation to relative default values. Nevertheless, there are cases where the pattern of default values is different and where such differences cannot be accounted for by accrued interest. A closer look at these cases may help determine if

there are systematic reasons why RFV does not hold in all cases. We focus on the 4 cases with the highest RV_{Range} at the initial default date. These cases also had the 4 highest RV_{AD} at the initial default date and in two of these cases no mode in the recovery value of defaulted bonds existed. These 4 selected issuers are: 1) Owens Corning; 2) Fruit of the Loom, Inc.; 3) Southern California Edison Company; and 4) Polaroid Corporation. We summarize these cases below. More detailed accounts can be found in Appendix A.

For the case of Owens Corning we find that 4 of the 6 bonds we obtain data for do in fact display RFV at the initial default date. On further examination of the two remaining bonds we find that the majority of amount outstanding for these bonds were tendered by the company nearly two years before the default date. We believe these bonds suffered from thin trading problem, and therefore the prices in our database did not reflect the true economic value of the bonds. This conclusion is supported by the fact that eventually the prices do trade at the same prices as the other bonds.

We analyze 3 bonds issued by Fruit of the Loom, Inc. At the initial default date (a bankruptcy filing) two of the bonds, with a maturity differential of 20 years, show a RFV pattern. However, the third bond of the same seniority, trades at a much lower price at the default date and continues to do so throughout the sample. It is difficult to say precisely why this bond is an outlier. It was issued in 1981, a little over a decade before the other two bonds. Perhaps related to this, is the fact that this bond is managed by a different trustee from that of the other bonds. We believe the best explanation for this departure from RFV for this bond is that it is being considered to be in a different class than the other bonds.

Southern California Edison Company is a unique case in our sample, since, although it does experience a default event, no bankruptcy is filed and eventually the company is able to pay back the defaulted interest payments within the time period considered. Perhaps, this resolution of the company's difficulties was expected by the market and therefore their issued bonds did not experience RFV at the time of default.

For the case of Polaroid Corporation, we find that only one day after the initial default date the RV_{Range} is equal to \$2 as opposed to the \$9 seen at the initial default date. By the time Chapter 11 is filed, the three bonds we analyze display an RFV pattern. We can

conclude for this case that at the initial date there was some measurement error in the data and that the prices use to measure the dispersion were not an accurate reflection of their true value.

From these outlying cases we find no systematic large deviation away from the RFV assumption that would either cause us to believe RFV is an inferior description for the general case of defaulted debt or lead us to favor an alternative recovery assumption. In only one case (FTL) does there exist data that we believe that we cannot fully explain if RFV did indeed always hold.

5.3 Comparison of Recovery Assumptions

The basic dispersion measures seem to provide empirical support for RFV. Nevertheless, it is useful to compare our results to what the data would look like if alternative recovery forms were to hold. We first introduce some notation to describe the recovery assumptions we will compare¹¹.

The recovery value at default date u for a bond maturing at time T and paying semi-annual coupons at an annual rate c , $RV_c^T(u)$, can be written as the product between the *expected* recovery rate, ω , $0 \leq \omega \leq 1$, which we assume constant for the purpose of this paper; and the expected claim amount. This latter term defines the particular recovery assumption we wish to consider.

Recovery of Face Value at Default (RFV):

$$RV_c^T(u) = \omega F \tag{1}$$

This assumption takes the claim amount to be the promised face value of the bond F . We ignore here for simplicity the accrued interest component that may be due to the bond investor as a result of the language in the bond indenture.

¹¹This classification closely follows that described in Bakshi, Madan, and Zhang (2002) and Delianedis and Lagnado (2002).

Recovery of Treasury - Face Value (RT-F):

$$RV_c^T(u) = \omega D(u, T)F \quad (2)$$

where $D(u, T) = E_u^Q \left[\exp \left(- \int_u^T r(s) ds \right) \right]$, the value of the default-free zero-coupon bond at time u maturing at time T . This assumption takes the claim amount to be the promised face value F discounted at the default-free interest rate r .

Recovery of Treasury (RT):

$$RV_c^T(u) = \omega \left[D(u, T)F + \sum_{i=1}^{2T} D(u, T_{i/2}) \frac{c}{2} \right] \quad (3)$$

RT is similar to RT-F but also discounts any promised coupon payments at the default-free interest rate and assumes those to be part of the claim amount.

Recovery of Market Value (RMV):

$$RV_c^T(u) = \omega P_c^T(u_-) \quad (4)$$

where $P_c^T(u_-)$ is the price of the risky coupon bond with coupon rate c and maturity T just before the default event.

We construct the hypothetical recovery values as follows. First we impute the expected recovery rate ω from the data by assuming that RFV holds exactly at the initial default date. For 27 of the 42 cases this means we use the mode of the issuer's defaulted bond values even if the bonds have not converged to a unique value. This requires us to assume that measurement error exists for those bonds observed not converging to the mode. In the remaining 15 cases we assume the mean of the bond values to be the expected recovery rate. We discount future promised coupon and principal payments at the default date using zero-coupon bonds estimated from the US dollar interest rate swap curve downloaded from Datastream.

This procedure is sufficient for computing recovery value for the RFV, RT-F, and RT cases. Constructing recovery values for the RMV requires some judgement. The

straightforward approach would be to assume the claim amount to be the bond price a day before the Moody's initial default date. However, there are cases where the default event is perhaps not a surprise and bond prices have declined to close to their recovery value leading up to their default date. In such cases, using the imputed recovery rate ω would lead to markedly lower recovery values in default. Another possible interpretation is that there is a lag between the Moody's default date and the actual default event¹². In an attempt to correct for this we consider traded prices 30 days before the initial default date to be the claim amount for the RMV assumption. Unfortunately, this proxy for pre-default market prices does not succeed too well. Figure 5 shows that there is a close correspondence between these values and the defaulted bond values. This shows the need for further research on the actual timing of default events compared to those events which may designate the official occurrence of default. In addition, it may also call to question what value of the expected recovery rate, δ , should be inputted in models where an RMV assumption holds.

Table IX shows the comparative dispersion statistics for the different recovery assumptions along with the empirical distribution. The RFV assumption, assuming no accrued interest is due to bond investors, predicts that the RV_{Range} and RV_{AD} be equal to zero in all cases. However, in a version where accrued interest is due we would expect that many of the cases would fall between 0 and 1. As mentioned above this matches the empirical data rather well. The RT-F assumption results in a range and average deviation of recovery values noticeably exceeding those found in the empirical distribution. There are 13 cases where RV_{Range} is greater than 10. The mean of RV_{Range} and RV_{AD} for the RT-F assumption is 9.34 and 3.36 respectively, versus 3.30 and 1.20 found in the empirical distribution. This results from the cases where there exists a large maturity differential between the shortest and longest dated bonds. Such an assumption could better fit the data if the recovery rate, ω , were assumed to be time-dependent. The RT assumption seems to fit the data better than RT-F with means for RV_{Range} and RV_{AD} of 3.75 and 1.45 respectively. The medians of RV_{Range} and RV_{AD} for the RT assumption are noticeably higher than those found in the actual data (1.98 and 0.92 versus 0.88 and 0.31). This

¹²The 19-day lag seen for the Worldcom Bonds in section 2 is a typical example of this.

shift in the distribution is due to the fact that under the RT assumption bonds can never converge to exactly the same recovery value unless they have the same coupon and maturity. The RMV assumption, as mentioned, produces recovery values that are much lower than those found in the data due to the pre-default prices being close to the defaulted bond prices. Thus, we view any results regarding this assumption with caution. However, to be complete we show that this same feature results in dispersion statistics for the RMV assumption which are broadly similar to those found in the data. For example, in 8 of the 42 cases the bonds had converged to the same level, $RV_{Range} = 0$, and a further 16 cases show an RV_{Range} less than one.

The RFV assumption seems better at matching the empirical data than the RT-F and RT assumptions. Moreover, it is the only assumption described which has a theory underlying its implications. These two reasons argue strongly in favor for use of the RFV assumption for most, if not all, applications of defaultable debt models. The RT seems to result in a decent approximation to RFV, especially versus RT-F. We conclude that if one were to use a Recovery of Treasury model in pricing defaultable debt, it would be better to include any promised coupon payments as part of the claim in recovery. It is hard for us to judge for or against the RMV assumption given our data. More work is needed to understand the timing of default events and to discover accurately what represents a pre-default price.

6 Implications for Credit Risk Pricing

As mentioned above several papers have examined the effects of alternative recovery assumptions, including RFV, within the context of intensity-based defaultable debt models. This is important for many practical purposes such as imputing risk-neutral default probabilities from market prices. However, this class of models, as it takes prices as inputs, is ill-suited to deal with questions dealing with how credit risk should be priced. The contingent claims, or structural approach, initiated by Black and Scholes (1973) and Merton (1974) and extended by many others is the traditional framework for answering this question. These models consider a firm's equity and bonds as contingent claims of the

firm's assets. In many of the models the risk-neutral default probability is modelled as the first-passage time probability that the firm value hits some default boundary. We incorporate our RFV assumption into this framework to study its effect on theoretical credit risk pricing. To the best of our knowledge it is the first work to measure the effect of different recovery assumptions in this context.

In a recent paper Huang and Huang (2002) examine a large number of structural credit risk models and find that they significantly underestimate credit spreads even after they are calibrated to historical default frequency. These findings imply, rather than a general failure of such models, that only a limited portion of the observed yield spreads is related to credit risk. Their results hold along the spectrum of models, from the simplest first-passage time model to those which embed more realistic features such as stochastic interest rates, strategic debt service, and jumps in asset value. However, one parameter the authors stress test and which does seem to affect spreads significantly is the recovery rate (Table 15 of their paper). This is perhaps not surprising given the central role recovery plays in corporate debt pricing. In turn we think it will be a valuable exercise to assess the importance of varying not just the recovery value *level* but its *form*.

6.1 Structural Model for Default

We first conduct our analysis in the simplest of structural models, a first-passage time model with constant interest rates and a constant default boundary. This is essentially the model presented in Black and Cox (1976) and extended by Longstaff and Schwartz (1995) to include stochastic interest rates. This simple model is also considered the *base case* model in Huang and Huang (2002) against which more complicated models are judged. Apart from ease in computation, one motivation for using such a simple model is that the previous paper found that even when more features are introduced, the main qualitative results related to spread underestimation remain unchanged. The underlying stochastic variable is the firm value process whose dynamics under the risk-neutral measure are modelled as follows:

$$dV_t = (r - \delta) V_t dt + \sigma_V V_t dz_V^Q \quad (5)$$

where r is the riskfree interest rate, δ is the firm payout rate, and σ_v is the assumed constant volatility of firm's asset value. W_t^Q is a standard Brownian motion under the risk-neutral probability. Default occurs at the first-passage time τ of V hitting the constant default boundary, K . Defining x_t as the log of the inverse leverage ratio, $\ln \frac{V_t}{K}$, and employing Ito's lemma, we can write the risk-neutral cumulative default probability Q_t ($\tau < T$) as the probability of the following process for x_t hitting zero between times t and T starting from an initial value x_0 :

$$dx_t = \left(r - \delta - \frac{\sigma_V^2}{2}\right)dt + \sigma_V dz_V^Q \quad (6)$$

Using the first-passage time density of x this is solved as (see Musiela and Rutkowski (1997)) :

$$Q_t(\tau < T) = N\left(\frac{-x_0 - \mu_*(T-t)}{\sigma_V \sqrt{T-t}}\right) + e^{-\frac{2\mu_* x_0}{\sigma_V^2}} N\left(\frac{-x_0 + \mu_*(T-t)}{\sigma_V \sqrt{T-t}}\right) \quad (7)$$

where N is the standard normal cumulative distribution function and μ_* is the risk-neutral drift for x_t : $\mu_* \equiv r - \delta - \frac{\sigma_V^2}{2}$.

6.2 Coupon Bond Pricing

Expressions for defaultable coupon bond prices are straightforward in this simple setting. Our main objective is to compare the implications of the different recovery assumptions. To make this clear we split the value of a defaultable bond with a face value of F maturing at time T paying semi-annual coupons at an annual rate of c into two components: 1) valuation of payments conditional on no default occurring, and 2) valuation of payments conditional on default occurring.

$$P_{0,c,T} = P_{0,c,T}^{ND} + P_{0,c,T}^D \quad (8)$$

The latter portion will vary with the recovery form we choose while the former part is independent of the recovery value. In other words, the first expression is equal to the

value of the bond assuming a zero recovery value which can be written in closed-form in our setting:

$$P_{0,c,T}^{ND} = D(0,T)F [1 - Q_0(\tau < T)] + \frac{c}{2} \sum_{i=1}^{2T} D(0, T_{i/2}) [1 - Q_0(\tau < T_{i/2})] \quad (9)$$

where $D(0, T_{i/2})$ is as in (2) the value of the risk-free discount bond maturing at $T_{i/2}$, or in this specific case: $\exp(-rT_{i/2})$. The value conditional on default will depend on the recovery form. We consider three forms: RT-F, RT, and RFV assuming an expected recovery rate of ω , all of which have closed-form expressions under our constant interest rate setting:

Recovery of Treasury-Face Value (RT-F):

$$\begin{aligned} P_{0,c,T}^{D,RT-F} &= E_0^Q [\mathbf{1}_{\{\tau < T\}} \exp(-rT)\omega F] \\ &= \omega D(0, T)F Q_0(\tau < T) \end{aligned} \quad (10)$$

Recovery of Treasury (RT):

$$P_{0,c,T}^{D,RT} = \omega \left[D(0, T)F Q_0(\tau < T) + \frac{c}{2} \sum_{i=1}^{2T} D(0, T_{i/2}) Q_0(\tau < T_{i/2}) \right] \quad (11)$$

Recovery of Face Value at Default (RFV):

$$\begin{aligned} P_{0,c,T}^{D,RFV} &= E_0^Q [\mathbf{1}_{\{\tau < T\}} \exp(-r\tau)\omega F] \\ &= \omega F \Phi(0, T) \end{aligned} \quad (12)$$

where $\Phi(0, T)$ is the closed-form solution of $E_0^Q [\mathbf{1}_{\{\tau < T\}} \exp(-r\tau)]$. The solution is given

in the Appendix. This term pays a unit amount at the time of the default versus at the time of maturity in RT-F case. This expression is a particular type of binary option seen the barrier option literature, known by some as the *down-and-in cash-at-hit option*¹³(see Haug (1998)). As said before we can also include any accrued interest due in this recovery form if this is mentioned in the specific bond’s indenture.

6.3 Comparing Spreads across different Recovery Assumptions

We wish to compare hypothetical spreads generated by the different recovery forms using the basic structural model we have assumed. It is useful to do this exercise across different credit rating classes and various maturities. Implementing the above model with different recovery forms only requires a reasonable set of parameters. For those parameters we assume invariant across credit rating classes, we look to Huang and Huang (2002). They include: the constant interest rate, $r(8.00\%)$ the payout rate $\delta(6.00\%)$, the default boundary K , assumed to be 60% of the firm’s total liabilities; and an expected recovery rate ω of 51.31% We consider a hypothetical semiannual coupon-paying bond under three different annual coupon rate scenarios: 1) 8.162%, 2) 12.00%, and 3) 4.50%. The first corresponds to a par bond under our interest rate assumption while the latter two assumptions consider a premium and discount bond, respectively.

Consistent with empirical evidence we assume that the asset volatility and leverage ratio vary across credit rating classes. We use empirical results found in Davydenko and Strebulaev (2002) which are shown in Table X¹⁴. These parameters allow us to compute spread for a typical company belonging to a particular credit rating class. While the credit spreads in absolute terms will depend highly on our choice of parameters it should be noted that we are concerned here with the *relative* difference in spreads across the different recovery forms. We compute prices for 3 recovery assumptions: RT, RT-F, and RFV for 6 different credit rating classes and three maturities (2, 10, and 30-year). Results

¹³Also referred to as American binary/digital/bet options (Taleb (1997) and Zhang(1998)) or the rebate price for a down knock-out barrier option (Sbuelz (1999)).

¹⁴In the HH paper the asset volatility parameter is calibrated to match the data, and thus vary for different maturities. While we could use their data on leverage ratios for the different credit rating classes, we decide to take these two parameters from a consistent source.

are found in Table XI.

In the top three credit rating classes (Aaa, Aa, A) the recovery form generally does not make much difference. This follows from default being an extremely unlikely event. This is especially true in the 2 and 10-year bonds. In the case of the A-rated 30-year maturity bond there is a 20 basis point difference between RT and RT-F for the par bond case and similar numbers in the premium and discount cases. A particular noticeable feature is how much higher the spreads in RT-F versus RT and RFV are as we increase maturity and as we move further down the credit rating class. For the B-rated 10-year maturity par bond the RT-F assumption results in a yield spread of 501 bps versus 335 and 343 bps for the RT and RFV forms respectively. This is not too surprising given that we have assumed fixed the expected recovery rate. As we increase the maturity the discounting effect leads to a smaller amount being recovered at default for RT-F and therefore significantly larger spreads. This result may also lead us to understand why some recent studies have found the RT-F assumption to be a superior recovery form for pricing bonds. Bakshi, Madan, and Zhang (2002) find that RT-F does significantly better at fitting individual corporate bond prices than RFV or RMV both in-sample and out-of-sample. Their framework is much different than ours and incorporate richer features in their recovery process. They also estimate the recovery parameters separately across the different models and test for robustness in their results, so it should be the case that the estimated parameters should take into account the discounting effect that features in RT-F. Yet the authors themselves hypothesize that the market may incorporate RT-F as an assumption due to the uncertainty in receiving full recovery at default. However, once we consider the valuation of same issuer and seniority bonds within the institutional framework we have discussed, it becomes clear that RT-F cannot generate cash flow patterns across the different bonds that the market would realistically expect.

Spreads in the RFV case, as expected, are always smaller than the RT-F case as the bondholder will receive the recovery amount at least as soon as in the RT-F case. Relative to the RT case the RFV spreads can be smaller or larger depending on whether the bonds are at a premium or discount. In the par bond case RFV and RT are virtually

the same¹⁵ with the largest difference between RFV and RT spreads being 8 bps in the 10-year maturity B-rated bond. For high coupon bonds we see that the RFV assumption can produce noticeably larger spreads versus the RT case especially as the credit rating worsens and the maturity increases. For the 10-year maturity B-rated premium bond RFV produces a spread of 406 bps versus 335 bps for the RT case. These results come directly from the fact that the assumed claim in default is higher than par in RT while the recovery claim in RFV is fixed at par. The opposite results are seen in the discount bond case. RFV spreads can become significantly lower than RT spreads. Again in the 10-year maturity B-rated bond case, the RT spread is 334 bps while the RFV spread is 263 bps.

While comparing spreads across recovery forms under such a simple environment may only be of moderate use, a few conclusions can be made. Firstly, RFV does not help much in addressing the problem of credit spread underestimation. This feature is most puzzling for high-grade credits where we find little difference between the spreads generated by the various recovery forms and indeed there are environments where RFV produces comparatively the lowest spreads. Secondly, RT-F produces significantly larger spreads than the other assumptions. However, as we have seen above, this assumption provides a bad model for the actual recovery value data assuming a constant expected recovery rate for bonds of the same issuer. Thirdly, unless bonds have very high or low coupons the RT assumption provides a close approximation to RFV. Yet there seems to be little justification for using RT versus RFV especially since the latter embeds an theory generated by the bond indenture and bankruptcy code. From a computational standpoint both forms can be valued with closed-form equations in the constant interest rate case. In more realistic models where interest rates are driven by multiple factors, possibly correlated with the default process, advanced numerical methods would likely be needed in both recovery forms.

¹⁵In the Huang-Huang paper the authors assume the par coupon (8.162%) in their calibration exercises. Therefore, they effectively approximate the RFV assumption, although they price bonds in their base case using a RT assumption.

6.4 Incorporating Stochastic Interest Rates

6.4.1 Choosing a Term Structure Model

It is useful to see how the different recovery forms affect pricing within the setting of stochastic default-free interest rates. This requires us to choose a sensible term structure model. While most of literature on structural credit risk pricing with stochastic interest rates assume simple one-factor models for the riskfree rate (e.g. Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Cathcart and El-Jahal (1998), Collin-Dufresne and Goldstein (2001)) we believe a model with multifactor dynamics is more sensible. The empirical literature shows vast evidence (e.g. Litterman and Scheinkman (1991)) across all major interest rate markets that two or three factors are necessary to adequately describe the empirical behavior of yield curve movements. In addition, to jointly examine how different initial riskfree yield curve shapes can affect credit risk pricing across the different recovery forms, we would like that a model can be calibrated to fit the initial yield curve. Since the initial yield curve would be an input to the model we can determine whether recovery forms matter more in a certain interest rate environment, say an upward-sloping yield curve, versus another, say a humped-shaped yield curve. Our last requirement for choosing a term structure model is that it allows us to consider non-zero correlation between the stochastic terms driving the risk-free rate and those driving the default risk of the corporate bonds, in this case the firm-value process. Such covariation would be consistent with empirical studies (e.g. Collin-Dufresne, Goldstein, and Martin (2001)) showing significant negative relation between interest rate levels and credit spreads.

6.4.2 Model and Implementation

We incorporate the above elements in a relatively simple two-factor *affine* term structure model as described by Brown and Schaefer (1994) and Duffie and Kan (1996), and which is *admissible*(see Dai and Singleton (2000)). The short rate path $r(t)$ is assumed as the sum of two stochastic factors, $l(t)$ and $s(t)$, and a deterministic factor $g(t)$. The dynamic

system of equations, under the risk-neutral measure, is

$$\begin{aligned}
dl(t) &= -\kappa_l l(t)dt + \sigma_l dz_l^Q \\
ds(t) &= -\kappa_s s(t)dt + \sigma_s dz_s^Q \\
r(t) &= g(t) + l(t) + s(t)
\end{aligned} \tag{13}$$

where $E(dz_l^Q dz_s^Q) = \rho_{ls}dt$, $E(dz_l^Q dz_V^Q) = \rho_{lV}dt$, $E(dz_s^Q dz_V^Q) = \rho_{sV}dt$ and the initial values of both l and s are equal to zero. We can refer to κ_l and κ_s as *reversion* parameters of the two factors; while σ_l and σ_s can be termed *volatility* parameters. By making the deterministic term, $g(t)$, time dependent we can calibrate the model to the initial yield curve. A full description of this particular specification within the general affine case as well its solution and calibration procedure can be found in the Appendix.

Implementing the model requires values for the parameters which are assumed constant. In two-factor gaussian models as the one described, typically one factor can be thought of as the interest rate *level* factor while the other factor can be interpreted as a *slope* factor. The level factor is likely the one with a considerably smaller reversion parameter value, indeed usually close to zero. Using the estimates of Brown and Schaefer (2000) from US Treasury STRIP data we set $\kappa_l = 0.0393$ and $\kappa_s = 0.2060$ thus we consider $l(t)$ as the level factor and $s(t)$ the slope factor. For the volatility parameters we set $\sigma_l = 0.010$ and $\sigma_s = 0.015$, values found in both Brown and Schaefer (1994) and He (2000). We set the term structure correlation parameter, ρ_{ls} , equal to -0.336 as estimated by Brown and Schaefer (2000). Using the author's own estimates from a subset of companies and relevant interest rate data we set $\rho_{lV} = -0.15$ and $\rho_{sV} = 0.00$. Ideally we would like these latter correlation parameters to vary by credit rating, but given the size of data which the estimates would have been taken from we consider only one value. Given these parameters and the initial yield curve the term structure model has been fully specified.

6.4.3 Implementation and Results

We price defaultable bonds by jointly simulating the default variable process found in (6)¹⁶ and the interest factor processes. The three different recovery forms (RT, RT-F,

¹⁶Now with the short rate a stochastic variable rather than a deterministic constant.

and RFV) are taken into account when determining cash flows path by path. Details of the simulation procedure and the appropriate variance reduction techniques used for reducing pricing bias are discussed in the Appendix. We examine 4 different initial yield curve shapes in terms of the instantaneous forward rate across different maturities: 1) flat curve; 2) upward-sloping curve; 3) humped-shape curve; and 4) downward sloping curve. The curves are chosen such that the average forward rate over the first ten years is equal to 0.08, thus somewhat comparable to our base results found in the previous section which assumed a constant interest rate across maturities of 0.08. Figure 6 plots the 4 different term structure shapes. Details on how the yield curves are produced using the Nelson and Siegel (1987) methodology is found in the Appendix.

Table XII shows the results in terms of basis point spreads over the default-free promised yield corresponding to the particular interest rate environment. The first noticeable result is that when we assume an initial flat curve the spreads are little unchanged from the our base case result with constant interest rates, despite the fact that the firm value process is correlated with the interest rate process. This result is especially valid when we are concerned with the relative effects of the different recovery forms.

The results from the other interest rate environments assumed support our argument that the form of the recovery assumption can affect pricing significantly. This is true even if the coupon rate of the bond is not markedly high or low. Consistent with our base case result is that spreads produced by the RT-F assumption are consistently and in some cases dramatically higher than those produced by RT and RFV. Going forward we will concentrate on comparing the RT and RFV assumptions. As we saw in our base case analysis as the maturity of the bond increases and the credit quality decreases the model spreads for the various recovery forms can differ significantly. Here, in addition, we can examine the effect of different yield curve shapes.

In the upward sloping yield curve, the respective default-free promised yields for the three different maturities are: 4.45%(2-year), 7.56%(10-year), and 8.94% (30-year). For the short-dated bond the RFV spread is higher than RT in the B-rated bond due to the coupon of the bond considered being at a significant premium. In the medium term bond, while the bond coupon rate is still at a premium relative to the default-free promised yield

the RT spread is higher than RFV, by up to 30 basis points for the B-rated bond. This is a result of the fact that since the payments upon default will be paid at their promised maturity in the RT case, the higher long-dated interest rates decreases the value of the RT bond relative to the RFV bond

In the humped-shape yield curve, the respective default-free promised yields for the three different maturities are: 9.67%(2-year), 8.36%(10-year), and 7.86% (30-year). The results in the case are similar to the flat curve case since the forward rates assumed flatten out to a constant 7.00% after the humped-shape rise in the early maturities. The positive difference between the RFV and RT assumptions in the 2 and 10-year case even though the hypothetical bond would be a discount coupon bond can be attributed to the higher interest rates assumed in the shorter maturities making any immediate recovery attained in an RFV setting worth relatively less. The difference between the two forms increases as the maturity increases to 30 years as the bond considered becomes a premium coupon bond.

In the downward sloping yield curve, the respective default-free promised yields for the three different maturities are: 9.66%(2-year), 8.36%(10-year), and 6.08% (30-year). Contrasting with the upward sloping case we find that for the medium term bonds RFV produces a slightly higher spread compared with the RT assumption despite the fact that the bond is at slight discount to the promised default-free yield. This is the opposite effect seen in the upward sloping case. Lower interest rates increase the recovery value under an RT assumption on a relative basis versus RFV. In the case of the 30-year maturity bond the significant premium of the coupon rate versus the default-free yield (8.16% versus 6.08%) exacerbates the difference between RFV and RT spreads.

Several conclusions can be made from our analysis under a stochastic interest rate setting. Firstly, our base case results, though assuming constant interest rates, are in general robust as long as the yield curve is not too extensively upward or downward sloping. This implies that using the closed-form solutions presented earlier can be used with confidence in defaultable bond analysis in many interest rate environments. Secondly, while the coupon premium/discount effect is strong the yield curve shape can also affect the how the recovery form affects model credit spreads. *Ceteris paribus*, in an upward

sloping curve spreads under spreads under RT will be higher than RFV spreads, while the opposite holds in a downward sloping yield curve. Indeed the two recovery forms imply different spread sensitivities to interest rate movements. Guha and Sbuely (2002) examine this point in detail and show that the practical hedging implications can be significant for lower-rated bonds. A more general point is that the results from past empirical studies on corporate bonds which implement structural credit risk models (e.g. Eom, Helwege, and Huang (2002)) may be biased. To the best of our knowledge none of these studies have incorporated RFV into their estimated pricing models but rather either incorporate RT or RT-F. If RFV is indeed the recovery form incorporated into market prices, properly analyzing the empirical performance of structural credit risk models still remains to be done.

7 Concluding Remarks

Corporate debt securities typically embed a long list of complicated institutional features that lead to difficulty in applying standard asset pricing techniques. Indeed the first meaningful, and seminal in its own right, extension of the contingent claims approach outlined by Black and Scholes (1973) and Merton (1974) for valuing corporate debt was Black and Cox (1976), titled “Valuing Corporate Securities: Some Effects of Bond Indenture Provisions”. It was immediately realized by the research profession that corporate bond models needed to take into account features within the institutional framework they existed under, as they significantly affect valuation.

A number of the recent extensions to the classical approach to credit risk pricing have also focused on the asset pricing implications of institutional details. The strategic debt service models¹⁷, for example, focus on a particular aspect of the bankruptcy procedure in an attempt to explain why credit spreads are higher than those implied by traditional firm-value models. We view this paper as a continuation of this literature stream . The debt acceleration clause found in bond indentures and the features of the US bankruptcy

¹⁷Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) are the key theoretical papers here. See Davydenko and Strebulaev (2002) for empirical work on this topic.

code generate a strong hypothesis that the RFV assumption should hold in default. Our data shows that such an assumption does explain the data well, especially compared to other standard recovery assumptions. These results may be particularly interesting given that some recent research in this area has found RFV to be a suboptimal recovery form under certain measures. A joint understanding of the results presented here and those found in these other papers still remains to be done. One hypothesis explaining this result comes from Tables XI and XII. They shows that some recovery assumptions which do not provide consistent descriptions of actual relative recovery values can nonetheless generate high spreads versus more consistent recovery assumptions. In general we believe that incorporating RFV in future work concerning corporate debt models is necessary to properly model such debt or to accurately assess credit risk in empirical studies. Our preliminary work on calculating spreads using RFV shows that the assumption can affect spreads significantly within the context of structural credit risk models. While it does not solve any of the well known puzzles of credit spread underestimation, its effect on pricing may be as great as some other institutional features which have been incorporated into debt pricing and can significantly affect empirical analyses of corporate bonds which implement structural credit risk models.

One obvious source of concern here is the quality of the data for these distressed bonds. It is well known that liquidity in such markets can often be scarce and that the pricing of individual bonds can be greatly affected by the dispersion in ownership of the particular bonds. These are all legitimate concerns that may affect some of the points we have made. However, given that we witness a certain empirical regularity across a sizeable number of issuers, along with conversations we have had with various market participants we feel confident in our main arguments. However, it still may be useful to study more in depth the microstructure of the distressed debt market. Another area of further research within this topic is the effect that the credit derivatives market may have on recovery values. The most widely traded credit derivative is the plain vanilla credit default swap¹⁸. In a typical credit swap the counterparty who has bought protection against a default event for a particular issuer must physically deliver a bond to the protection seller in exchange

¹⁸See Duffie(1999) for a description.

for par if the issuer has indeed defaulted. In general, the protection buyer can also choose among a range of *pari passu* bonds of a particular issuer in deciding which one to deliver. It seems possible that this recent development in the credit markets will have an effect on both the absolute and relative levels of defaulted bond values. Determining exactly how may be useful for corporate bond valuation.

A Description of Outlier Cases

In this appendix we give a more detailed description of the four cases outlined in Section 5.2. The four selected cases are: 1) Owens Corning; 2) Fruit of the Loom, Inc.; 3) Southern California Edison Company; and 4) Polaroid Corporation.

Owens Corning

On 5 October 2000, Owens Corning filed for Chapter 11 protection, this was also the initial default event. We obtained data for 6 of their bonds with the same seniority (senior unsecured), with maturity dates ranging from 1 June 2002 to 1 August 2018 and coupon rates ranging from 7% to 8.875%. On this announced date 4 of the 6 bonds converge to the same price of \$28¹⁹. However, the 8.875% bond due 1 June 2002 has a quoted price of 80.16, while the 9.375% bond due 1 June 2012 is quoted with a price of \$53.23. Examination of the prospectuses show no covenants or other features of their indentures that should cause these two bonds to diverge from the other bonds. We study the news reports related to these two bonds and find that on 20 July 1998 Owens Corning announced a debt tender offer for this pair of bonds. Of the original \$150 million of the 2002 bonds, \$110 million was purchased by the company leaving \$40 million outstanding. Of the original \$150 million of the 2012 bonds, \$143 million was purchased leaving \$7 million outstanding. This is in contrast to the other 4 bonds which had an amount outstanding equal to the debt amount originally issued, ranging from \$250 million to \$400 million. One reasonable conclusion to make is that these two bonds suffered, relatively speaking, from thin trading problems which affected their price quotes. Such a conclusion seems especially plausible for the 2012 bond as in just a few weeks time (15 November 2000) it is trading at the same level (28) as the 4 bonds which had converged at the default date. The 2002 bond, although it decreases in price substantially during these weeks after default, stays at a quote on average 30 dollars higher than the other 5 bonds until the last week of July 2001 when it too also converges to the same price of 28 on 30 July 2001. We believe that this anomaly is due to the relative lack of marketability of the two bond which deviate from RFV. This is a result of: 1) the small amount outstanding; and 2) the relative short remaining maturity left on the bond. If this interpretation is correct then we can regard Owens Corning, the most deviating case under the simple dispersion measures in our sample ($RV_{Range} = 52.16$ and $RV_{AD} = 17.20$), as another example of a company which experiences RFV.

Fruit of the Loom, Inc.

Fruit of the Loom, Inc. filed for Chapter 11 protection, its initial default event, on 29 December 1999. The company was eventually bought by Berkshire Hathaway, Inc., a deal announced in November 2001 and approved by the court in April 2002. We obtained data for 3 senior unsecured bonds: 1) 6.500% Note due 15 November 2003; 2) 7.000% Debenture due 15 March 2011; and 3) 7.375% Debenture due 15 November 2023. At the default date bonds 1 and 3 converge to a price of 36 and stay converged throughout the bankruptcy period. However, bond 2 at the default date is quoted at 23 and throughout the bankruptcy period is quoted at, on average, two-thirds of the price of the other two bonds. From examining the details of the bond we find that bond 2 is callable unlike the other bonds, which are all straight. Yet this feature is unlikely to cause such a price discrepancy, given the unlikelihood of the company calling the debt back. We look at the amount of the issue outstanding to see if marketability may be the cause of the discrepancy. While only \$75 million of the original \$125 million issued is outstanding for bond 2, we believe that this size is adequate for the bond to be traded as frequently as the other bonds. Bond 2 was issued in 1981 as opposed to 1992 and 1993 for bonds 1 and 3. In fact, it was an original Northwest Industries (which included Fruit of the Loom) bond before Fruit of the Loom emerged as its own entity. However, we cannot find any information on why this would affect the relative valuation of the bonds. Bond 2 is managed by a different trustee than bonds 1 and 3, which are managed by the same trustee. Thus, one conclusion could be that the \$300 million of debt overlooked by the trustee for bonds 1 and 3 has a better recovery due to these two bonds being put into a different class than bond 2, although all 3 bonds are senior unsecured debt. With the information²⁰ available this seems to be the most plausible conclusion. Not considering bond 2 and its various idiosyncrasies, the behaviour of bonds 1 and 3 does support the RFV hypothesis, and is particularly significant given the maturity differential of 20 years.

¹⁹All bond prices are quoted as dollar amounts per \$100 face value.

²⁰We are unable to find the bond indenture for bond 2.

Southern California Edison Company

On 15 January 2001 Southern California Edison defaulted on interest payments on \$200 million of notes. We collected data on 6 bonds, with maturity dates ranging from 1 June 2001 to 1 April 2029. Immediately following the default event the RV_{Range} decreased from 22.58 to 9.26. However, in only a few weeks time this metric had drifted back up to 15 and we see little evidence going forward of the bonds converging. This case is unique in our sample in the sense that, although the firm defaults, no bankruptcy is filed and eventually the company is able to pay back the defaulted interest payments within the time period we consider. The default occurred due to the power crisis California was experiencing at that time. Over the following months it defaulted on due interest and principal payments while struggling to convince state lawmakers to approve a rescue plan to avoid bankruptcy. In October 2001 the company reached an agreement with the California Public Utilities Commission to use a portion of customer bills to pay its debt, and in March 2002 it was able to pay its creditors defaulted payments. We can conclude that in this case the bond prices did not represent true defaulted prices in the sense that the market had incorporated a strong probability of a rescue in its valuation. If the company would have had to file for Chapter 11 protection, a RFV could have been a definite possibility

Polaroid Corporation

On 11 July 2001 Polaroid Corporation announced it would not be making its scheduled bond interest payment due later that month, signifying its initial default event. A few months later on 12 October 2001 the company filed for Chapter 11. We have data for 3 unsecured bonds: 1) 6.75% Note due 15 January 2002; 2) 11.50% Note due 15 February 2006; and 3) 7.25% Note due 15 January 2007. Prior to the default announcement the quoted prices for the 3 bonds in the above order are: 44.5, 28.5, and 24.5. After the default event the prices are 24.5, 17.5, and 15.5, an asymmetric decrease of value percentage-wise, but still not exemplary of RFV ($RV_{Range} = 9.00$). However, the following day the prices are 27.5, 26.5, and 25.5. Indeed, the RV_{Range} does not go above 4 over the following weeks and usually is within the \$3 range. As the filing date approaches the bonds converge and at filing date all bonds are quoted at price of 7. We can conclude for this case that at the initial date there was some measurement error in the data and that the prices used to measure the dispersion were not an accurate reflection of their true value.

B Closed-Form Pricing of RFV Assumption

We wish to value the expression for $P_{0,c,T}^{D,RFV}$:

$$\begin{aligned} & E_0^Q [\mathbf{1}_{\{\tau < T\}} \exp(-r\tau) \omega F] \\ &= \omega F E_0^Q [\mathbf{1}_{\{\tau < T\}} \exp(-r\tau)] \end{aligned} \quad (B1)$$

The expression in brackets of (B1) pays a unit amount at the stopping time τ when the process for V_t given in (5) hits the constant default boundary K or alternatively when the process for $x_t \equiv \ln\left(\frac{V_t}{K}\right)$ given in (6) hits zero

$$\tau \equiv \inf \left(u > 0, x_u \equiv \ln \left(\frac{V_u}{K} \right) = 0 \right)$$

The price of this security at time 0 for a final maturity of T with an initial value of x_0 and defining the risk-neutral drift for x_t as $\mu_* \equiv r - \delta - \frac{\sigma_v^2}{2}$ is given as

$$\Phi(0, T) = \left[e^{\frac{-x_0(\mu_* + \lambda)}{\sigma_v^2}} N \left(\frac{-x_0 + \lambda T}{\sigma_v \sqrt{T}} \right) + e^{\frac{-x_0(\mu_* - \lambda)}{\sigma_v^2}} N \left(\frac{-x_0 - \lambda T}{\sigma_v \sqrt{T}} \right) \right] \quad (B2)$$

where

$$\lambda \equiv \sqrt{\mu_*^2 + 2\sigma_v^2 r}$$

and N is the standard normal cumulative distribution function.

Proof. See Nelken (1996) □

C Default-Free Term Structure Model Specification

General Affine Model Under the standard assumption of no-arbitrage we can write the value of a default-free riskless zero coupon bond at time t with maturity date T as

$$P(t, T) = E_t^Q \left[\exp \left(- \int_t^T r(s) ds \right) \right] \quad (C1)$$

where the expectation is taken under the risk-neutral probability measure Q , and $r(t)$ is the process for instantaneous riskless nominal short rate. We consider a term structure model in the multivariate affine class as described by Brown and Schaefer (1994), Duffie and Kan (1996) and Dai and Singleton (2000). In the general case the short rate is an affine function of N unobserved state variables or factors $X(t) = (X_1(t), X_2(t), \dots, X_N(t))$,

$$r(t) = g + h'X(t) \quad (C2)$$

where g is scalar and h is a N -vector. The factors follow the diffusion process

$$dX(t) = K(\Theta - X(t))dt + \Sigma S(t)dW(t) \quad (C3)$$

where $W(t)$ is an N -dimensional independent Wiener process under Q , K and Σ are $N \times N$ matrices, and Θ is an N -vector. $S(t)$ is a diagonal matrix with elements

$$S(t)_{ii} = \sqrt{\alpha_i + \beta_i'X(t)} \quad (C4)$$

where α_i is scalar and β_i is an N -vector. We can stack the scalars in the N -vector α , and as well stack the β_i vectors into an $N \times N$ matrix β where β_i' is the row i of β .

Given such a model, and with parameters chosen so that the dynamics are *admissible* [see Dai and Singleton (2000)], Duffie and Kan (1996) show that the zero coupon bond price is exponential affine with respect to the factors

$$P(X(t), t, T) = \exp[A(t, T) - B(t, T)X(t)] \quad (C5)$$

where $A(t, T)$ is a scalar function and $B(t, T)$ is an N -valued function.

Model Specification We choose and implement a particular case of this general model. Specifically we first set $N = 2$, $\beta = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and allow the intercept in (C2) to be time-dependent, i.e. $g(t)$. The first two settings result in a two-factor Gaussian model where the level of the factors do not affect the volatility of the factors. While in this case there is a non-zero probability that the short rate can fall below zero, the enhanced tractability and the good fit relative to other popular models seen in empirical studies make it a sensible specification to use. Setting the intercept g to be time-dependent allows us to calibrate the model to exactly fit the initial yield curve. This is analagous to making a component in the drift time-dependent to fit the yield curve which is often seen in the implementation of one-factor affine term structure models. With regard to the other parameters in (C3) if we substitute $K = \begin{bmatrix} \kappa_{11} & 0 \\ \kappa_{12} & \kappa_{22} \end{bmatrix}$, $\Theta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ we achieve the *canonical* form (Dai and Singleton (2000)) of this particular sub-class of Affine models:

$$\begin{aligned} d \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & 0 \\ \kappa_{12} & \kappa_{22} \end{pmatrix} \left[- \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \right] dt + d \begin{pmatrix} W_1(t) \\ W_2(t) \end{pmatrix} \\ r(t) &= g(t) + h_1 X_1(t) + h_2 X_2(t) \end{aligned} \quad (C6)$$

The existence of a non-zero off-diagonal element in K , κ_{12} , implies that we allow for interdependencies between the two factors.

We choose to transform (C6) to an equivalent form (Dai and Singleton (2000)) which generates identical prices for all interest rate instruments but which may be more intuitive in representing the factors underlying the term structure. Such practice is the usual one seen in the literature where in many cases one of the factors is taken to be the short rate itself. Examples of reparametrised models in the two-factor time-homogenous Gaussian case include additive models such as Langetieg (1980) where the short rate is equated to the sum of the state variables and stochastic mean models where the two factors are the short rate and the mean reversion level of the short rate. We introduce as the two factors: $l(t)$ and $s(t)$. The new dynamic system of equations, again under Q , is

$$\begin{aligned} dl(t) &= -\kappa_l l(t)dt + \sigma_l dz_l^Q \\ ds(t) &= -\kappa_s s(t)dt + \sigma_s dz_s^Q \\ r(t) &= g(t) + l(t) + s(t) \end{aligned} \quad (C7)$$

where $E(dz_l^Q dz_s^Q) = \rho_{ls} dt$ and the initial values of both l and s are equal to zero. We can refer to κ_l and κ_s as *reversion* parameters of the two factors; while σ_l and σ_s can be termed *volatility* parameters.

Solution and Calibration of Model We substitute our affine representation of the short rate into the basic bond valuation equation (C1)

$$\begin{aligned} P(t, T) &= E_t^Q \left[\exp \left(- \int_t^T g(u) + l(u) + s(u) du \right) \right] = \\ &\exp \left(- \int_t^T g(u) du \right) \cdot E_t^Q \left[\exp \left(- \int_t^T x(u) + y(u) du \right) \right] \end{aligned} \quad (C8)$$

We see from the right hand side of the equation that the bond value can be viewed as the product of a deterministic factor and a ‘‘bond price’’ with zero mean factors. The expectation on the right hand side has a solution form as seen in (C5) so we can write the bond value as

$$P(t, T) = \exp \left(- \int_t^T g(u) du \right) \cdot \exp [A(T-t) - B_l(T-t)l(t) - B_s(T-t)s(t)] \quad (C9)$$

where after solving the PDE and denoting $\tau = T - t$, we find

$$\begin{aligned} B_l(\tau) &= \frac{1 - \exp(-\kappa_l \tau)}{\kappa_l} & B_s(\tau) &= \frac{1 - \exp(-\kappa_s \tau)}{\kappa_s} \\ A(\tau) &= \int_0^\tau A'(u) du \text{ where } A'(\tau) = \frac{1}{2} \sigma_l^2 B_l^2(\tau) + \frac{1}{2} \sigma_s^2 B_s^2(\tau) + \rho_{ls} \sigma_l \sigma_s B_l(\tau) B_s(\tau) \end{aligned} \quad (C10)$$

We would like to calibrate the model to be able to fit the initial yield curve. First we write the equation for the short rate path which is the solution to the term structure model (13)

$$r(t) = g(t) + \sigma_l e^{-\kappa_l t} \int_0^t e^{\kappa_l u} dz_l(u) + \sigma_s e^{-\kappa_s t} \int_0^t e^{\kappa_s u} dz_s(u) \quad (C11)$$

It is evident that the time dependent intercept $g(t)$ will embed any information regarding the initial yield curve. From (C9) we can express the initial yield curve as the set of zero coupon bond prices at time 0

$$P(0, T) = \exp \left[- \int_0^T g(u) du + A(T) \right] \quad (C12)$$

since $l_0 = 0$ and $s_0 = 0$ by construction. Taking logs and differentiating with respect to T on both sides of the equation and then substituting from (C10) we arrive at

$$g(T) = -\frac{d \log P(0, T)}{dT} + \frac{dA(T)}{dT} = f(0, T) + \frac{1}{2} \sigma_l^2 B_l^2(T) \quad (\text{C13})$$

$$+ \frac{1}{2} \sigma_s^2 B_s^2(T) + \rho_{ls} \sigma_l \sigma_s B_l(T) B_s(T) \quad (\text{C14})$$

where $f(0, T)$ is the initial instantaneous forward rate curve. We can substitute this term into [C11] to arrive at our new expression for the short rate path

$$r(t) = f(0, t) + \frac{1}{2} \sigma_l^2 B_l^2(t) + \frac{1}{2} \sigma_s^2 B_s^2(t) + \rho_{ls} \sigma_l \sigma_s B_l(t) B_s(t) \quad (\text{C15})$$

$$+ \sigma_l e^{-\kappa_l t} \int_0^t e^{\kappa_l u} dz_l(u) + \sigma_s e^{-\kappa_s t} \int_0^t e^{\kappa_s u} dz_s(u)$$

From this expression it is clear how the short rate path used to value all interest rate sensitive claims can be calibrated to initial market data – via the forward rate curve. All that is needed for implementation are the time-invariant parameters: $\kappa_l, \kappa_s, \sigma_l, \sigma_s$, and ρ_{ls} .

D Simulation Pricing Procedure

This section describes the Monte Carlo simulation procedure used for pricing the defaultable bonds under a stochastic interest rate environment with the term structure model specified above in (13). The dynamic system of equations with the correlated processes are:

$$\begin{aligned} dl(t) &= -\kappa_l l(t) dt + \sigma_l dz_l^Q \\ ds(t) &= -\kappa_s s(t) dt + \sigma_s dz_s^Q \\ dx_t &= (r(t) - \delta - \frac{\sigma_V^2}{2}) dt + \sigma_V dz_V^Q \\ r(t) &= g(t) + l(t) + s(t) \end{aligned} \quad (\text{D1})$$

where $E(dz_l^Q dz_s^Q) = \rho_{ls} dt$, $E(dz_l^Q dz_V^Q) = \rho_{lV} dt$, $E(dz_s^Q dz_V^Q) = \rho_{sV} dt$. The initial values of $l(t)$ and $s(t)$ are equal to zero, while the initial value of $x(t)$ is x_0 . Default occurs the first time $x(t)$ hits zero. It is useful to do a Cholesky factorisation of the correlation matrix Σ of the random processes $z_i, i = \{V, l, s\}$ so we can write the system in terms of independent processes $w_i, i = \{V, l, s\}$. That is we find the lower triangular matrix \mathbf{M} such that

$$\mathbf{z} = \mathbf{M} \mathbf{w}$$

and

$$\mathbf{M} \mathbf{M}' = \Sigma$$

In our case:

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ \rho_{ls} & \sqrt{1 - \rho_{ls}^2} & 0 \\ \rho_{lV} & \frac{\rho_{sV} - \rho_{ls} \rho_{lV}}{\sqrt{1 - \rho_{ls}^2}} & \sqrt{1 - \rho_{lV}^2 - \frac{(\rho_{sV} - \rho_{ls} \rho_{lV})^2}{1 - \rho_{ls}^2}} \end{pmatrix} \quad (\text{D2})$$

After we have rewritten the dynamics in terms of the independent processes $w_i, i = \{V, l, s\}$ we discretise our continuous time equations at equal time steps Δt . For $x(t)$, we do a simple Euler discretisation. For $l(t)$ and $s(t)$ we discretise their continuous closed-form solutions²¹ for $0 \leq j \leq t$

$$\begin{aligned} l(t) &= e^{-\kappa_l(t-j)} l(j) + \sigma_l e^{-\kappa_l t} \int_j^t e^{\kappa_l u} dz_l(u) \\ s(t) &= e^{-\kappa_s(t-j)} s(j) + \sigma_s e^{-\kappa_s t} \int_j^t e^{\kappa_s u} dz_s(u) \end{aligned} \quad (\text{D3})$$

²¹The increments of the correlated processes would be replaced by independent processes multiplied by the Cholesky factorisation matrix.

The variance terms on the righthand side have a zero mean and variance equal to

$$\frac{\sigma_i^2}{2\kappa_i}(1 - e^{-2\kappa_i(t-j)}), i = \{l, s\}$$

Once we have discretised our dynamic processes and chosen an appropriate time step we produce N random paths as follows. First we generate independent standard normal variates $\epsilon_i, i = \{l, s, V\}$ each vectors of length $\frac{N}{2}$ at each time step for the three independent stochastic terms. We then use the antithetic variable technique which means that for the remaining $\frac{N}{2}$ paths we use $-\epsilon_i$. Thus, for each time step we generate $\begin{bmatrix} \epsilon_i \\ -\epsilon_i \end{bmatrix}, i = \{l, s, V\}$. With this methodology we can generate N paths of x and the short rate r . Bond valuation is attained as follows. First, the promised cash flows would be generated for each path. Second, if default has occurred for a specific path the promised cash flows would be altered in a way specific to the recovery form assumed. Once the default-adjusted cash flow paths have been generated we discount each cash flow path at the *relevant* riskfree short rate path. That is the rate path which produced the default path associated with the default-adjusted cash flow path. The mean of these N values can be considered the *crude* Monte Carlo estimated bond price.

A problem with using the crude Monte Carlo price as the estimate is that in almost all practical cases there will be an overpricing bias. This is due to the fact that default, i.e. when $x(t)$ reaches the zero barrier, is in general a highly improbable event, and once we sample our continuous time processes at discrete time intervals we do not get as many ‘‘hits’’ in our simulation as we would expect under our risk-neutral pricing measure. This is due to the fact that for any nonzero time interval discretisation induces a bias since many time points at which low values of the variable could occur are ignored. Narrowing the time step shrinks the bias toward zero, but time steps small enough to produce accurate answers may be computationally quite burdensome. To correct this bias we implement the technique developed by Beaglehole, Dybvig, and Zhou (1997). Their technique draws on the theory of the Brownian bridge. After we have generated the paths for $x(t)$ in our crude simulation we treat each discrete time interval as two ends of a Brownian bridge. We then draw the minimum (as we are interested in $x(t)$ reaching zero) of the path process on the interval using the known theoretical distribution of a Brownian bridge on an interval. To explain further consider the euler discretisation of our default state variable x ²²

$$x(t_{i+1}) - x(t_i) = (r_t - \delta - \frac{\sigma_V^2}{2})\Delta t + \sigma_V [z_V(t_{i+1}) - z_V(t_i)] \quad (D4)$$

We can write the continuous time process as

$$x(t) = x(t_i) + \sigma_V B_t, t_i \leq t \leq t_{i+1} \quad (D5)$$

where B_t is the standard Brownian bridge connecting zero and $b = \frac{x(t_{i+1}) - x(t_i)}{\sigma_V}$. Drawing the minimum of the Brownian bridge over all (t_i, t_{i+1}) will clearly lead to more potential default barrier ‘‘hits’’. The draw of the minimum value over the interval (t_i, t_{i+1}) is given by

$$x_i^* = x(t_i) + \sigma_V B_{\min} \quad (D6)$$

where B_{\min} is a draw from the distribution of the Brownian bridge minimum. We compute this latter value as follows. The distribution of the Brownian bridge maximum is well known:

$$P(\max_{t_i \leq t \leq t_{i+1}} B_t \leq y \mid B_{t_{i+1}} = b) = 1 - e^{-2y(y-b)/\Delta t} \quad (D7)$$

and using the reflection principle we know that minus of the maximum of the Brownian bridge connecting 0 and $-b$ results in the minimum of the Brownian bridge connecting 0 and b . Assuming u is taken from a standard uniform distribution the following has the same distribution as the Brownian bridge minimum.

$$\frac{b - \sqrt{b^2 - 2\Delta t \ln(1 - u)}}{2} \quad (D8)$$

Thus at each interval we generate a N -length vector of standard uniform variables and apply this technique for generating the new Brownian bridged $x(t)$ paths which are used to identify default events in the simulation. We then calculate model bond prices as described above.

²²See previous footnote.

E Generating Hypothetical Yield Curves

We generate 4 yield curve shapes: 1) flat curve; 2) upward-sloping curve; 3) humped-shape curve; and 4) downward sloping curve. We produce curves in terms of the instantaneous forward rate curve. Generating the flat curve is straightforward. For the remaining types we used the Nelson and Siegel(1987) methodology which fits term structures using the following form for the initial instantaneous forward rate curve:

$$f(0, m) = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau}\right) + \beta_2 \left(\frac{m}{\tau} \exp\left(\frac{-m}{\tau}\right)\right) \quad (\text{E1})$$

where m is a particular maturity point on the yield curve while $\beta_0, \beta_1, \beta_2,$ and τ are parameters. We choose parameters such that the sought for shapes are generated and that the average instantaneous forward rate over the the first ten years is equal to 0.08. The curves are shown in Figure 6. The parameter values chosen are as follows:

Flat: $\beta_0 = 0.08, \beta_1 = 0, \beta_2 = 0$

Upward-Sloping: $\beta_0 = 0.08, \beta_1 = -0.05, \beta_2 = 0.14, \tau = 12.05$

Humped-Shape: $\beta_0 = 0.07, \beta_1 = -0.04, \beta_2 = 0.143, \tau = 1$

Downward-Sloping: $\beta_0 = -.01, \beta_1 = 0.1068, \beta_2 = 0.085, \tau = 10.00$

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Table I: Contractual Details of Selected Bonds Issued by Enron and Worldcom

Enron Corporation			Worldcom Incorporated		
ID	Coupon	Maturity	ID	Coupon	Maturity
1	9.125%	01-Apr-03	1	7.875%	15-May-03
2	7.875%	15-Jun-03	2	6.250%	15-Aug-03
3	6.625%	15-Nov-05	3	7.550%	01-Apr-04
4	6.400%	15-Jul-06	4	6.500%	15-May-04
5	7.125%	15-May-07	5	6.400%	15-Aug-05
6	6.875%	15-Oct-07	6	7.375%	15-Jan-06
7	6.750%	01-Aug-09	7	8.000%	15-May-06
8	7.375%	15-May-19	8	7.750%	01-Apr-07
9	6.950%	15-Jul-28	9	8.250%	15-May-10
			10	7.500%	15-May-11
			11	7.750%	01-Apr-27
			12	6.950%	15-Aug-28
			13	8.250%	15-May-31

Table II: Enron Bonds: Prices and Yields Leading up to Bankruptcy

Shown below are the dealer-bid prices and yields of 9 selected Enron bonds whose contractual details are found in Table I. All bonds are senior unsecured debt. The initial default event date defined by when Enron files for Chapter 11 bankruptcy is 2 December, 2001. On 28 November, 2001 it became apparent that a potential merger bid by a rival company would not take place.

Panel A: Prices

<u>ID</u>	31-Jul-01	31-Aug-01	28-Sep-01	31-Oct-01	21-Nov-01	23-Nov-01
1	106.23	106.28	106.89	87.09	69.89	64.95
2	104.87	104.91	105.47	82.19	62.88	63.92
3	102.61	102.74	103.75	78.19	57.76	61.84
4	101.13	101.78	103.13	77.7	57.74	60.82
5	104.3	104.59	105.74	77.9	56.72	59.82
6	102.53	103.25	104.33	76.89	65.4	59.81
7	100.49	100.81	100.68	77.73	61.66	58.76
8	101.04	101.93	97.69	74.66	56.63	57.81
9	94.5	96.15	91.37	73.93	54.86	56.81
<u>ID</u>	26-Nov-01	27-Nov-01	28-Nov-01	29-Nov-01	30-Nov-01	03-Dec-01
1	50.01	58.07	21	22.03	19	21
2	49.99	57.07	21	22.03	19	21
3	47.95	56.12	21	22.1	19.01	21
4	46.97	55.14	21	22.12	19.01	21
5	44.99	54.15	21	22.12	19.01	21
6	44.99	53.16	20.99	22.13	19.01	21
7	44.98	53.19	21.01	22.13	19	21
8	41.97	49.2	22	22.12	19	21
9	40.99	47.99	21.99	22.08	18.98	21

Panel B: Yields

<u>ID</u>	31-Jul-01	31-Aug-01	28-Sep-01	31-Oct-01	21-Nov-01	23-Nov-01
1	5.17%	4.95%	4.36%	20.01%	39.78%	46.76%
2	5.11%	4.97%	4.52%	21.41%	42.69%	41.48%
3	5.92%	5.88%	5.59%	13.85%	23.53%	21.33%
4	6.13%	5.97%	5.64%	12.86%	21.09%	19.63%
5	6.23%	6.16%	5.91%	12.82%	20.68%	19.34%
6	6.37%	6.23%	6.01%	12.49%	16.19%	18.29%
7	6.67%	6.62%	6.64%	11.11%	15.42%	16.36%
8	7.27%	7.18%	7.61%	10.58%	14.12%	13.83%
9	7.42%	7.28%	7.72%	9.69%	13.05%	12.61%
<u>ID</u>	26-Nov-01	27-Nov-01	28-Nov-01	29-Nov-01	30-Nov-01	03-Dec-01
1	73.38%	58.05%	185.17%	178.31%	201.70%	187.26%
2	62.71%	51.22%	155.33%	149.69%	168.90%	157.04%
3	29.86%	24.55%	62.34%	60.08%	66.98%	62.48%
4	27.18%	22.46%	54.76%	52.76%	58.82%	54.89%
5	26.88%	21.92%	51.24%	49.32%	55.10%	51.30%
6	25.34%	21.15%	48.15%	46.33%	51.76%	48.20%
7	21.82%	18.34%	41.23%	39.65%	44.47%	41.28%
8	18.75%	16.17%	34.00%	33.82%	39.09%	35.52%
9	17.25%	14.84%	31.58%	31.45%	36.53%	33.06%

Table III: Worldcom Bonds: Prices and Yields Leading up to Bankruptcy

Shown below are the prices and yields of 9 selected Worldcom bonds whose contractual details are found in Table I. All bonds are senior unsecured debt. The official initial default event date defined by when Worldcom misses an interest payment is 15 July, 2002. The Chapter 11 bankruptcy filing occurs on 21 July, 2002. On 26 June, 2002 it became public information that the firm had concealed \$3.9 billion of losses for more than a year.

Panel A: Prices

<u>ID</u>	31-Jan-02	28-Feb-02	28-Mar-02	30-Apr-02	31-May-02	18-Jun-02
1	103.27	99.69	95.62	75	84.5	81
3	103.06	98.50	91.47	60	73.5	70
5	99.22	96.45	86.51	50	57	54.5
6	100.00	97.66	87.40	49	56	53
8	101.96	97.80	86.65	45.5	54	49.5
9	103.09	101.06	88.83	48	50	47.5
10	97.68	96.35	83.88	47	49	46.5
12	85.28	82.31	69.66	42	38.5	39
13	99.05	96.12	80.66	44	43	42
<u>ID</u>	21-Jun-02	25-Jun-02	26-Jun-02	27-Jun-02	15-Jul-02	22-Jul-02
1	75	68.5	14	17	14.25	13.25
3	61.5	57.5	11.5	13.75	14.25	13.25
5	49	46.5	11.5	13.75	14.25	13.25
6	48.5	45	11.5	13	14	13.25
8	45.5	42	11.5	13	14	13.25
9	44	42	11.25	14	14	13.25
10	43.5	41	11.5	13.5	14	13.25
12	37.5	36.5	11	14	14	13.25
13	40	38	11.25	13	14	13.25

Panel B: Yields

<u>ID</u>	31-Jan-02	28-Feb-02	28-Mar-02	30-Apr-02	31-May-02	18-Jun-02
1	5.21%	8.13%	12.11%	39.36%	27.36%	33.82%
3	6.02%	8.34%	12.48%	39.01%	26.72%	30.51%
5	6.65%	7.58%	11.31%	32.04%	27.37%	29.49%
6	7.37%	8.09%	11.56%	31.68%	27.20%	29.51%
8	7.28%	8.29%	11.31%	29.35%	24.48%	27.22%
9	7.73%	8.07%	10.31%	22.52%	21.72%	22.90%
10	7.85%	8.07%	10.26%	20.68%	19.93%	21.02%
12	8.34%	8.66%	10.31%	16.87%	18.33%	18.11%
13	8.34%	8.61%	10.36%	18.87%	19.30%	19.75%
<u>ID</u>	21-Jun-02	25-Jun-02	26-Jun-02	27-Jun-02	15-Jul-02	22-Jul-02
1	44.43%	57.89%	438.35%	373.50%	471.50%	518.88%
3	39.67%	44.76%	203.29%	181.19%	182.57%	194.27%
5	33.97%	36.29%	111.19%	99.51%	98.80%	104.19%
6	32.95%	35.97%	105.48%	97.54%	94.26%	98.05%
8	29.84%	32.42%	87.78%	80.80%	77.34%	80.51%
9	24.67%	25.78%	76.27%	63.57%	63.59%	66.53%
10	22.40%	23.67%	67.62%	59.04%	57.30%	59.98%
12	18.80%	19.30%	62.64%	49.38%	49.45%	52.27%
13	20.71%	21.78%	72.63%	62.98%	58.45%	61.67%

Table IV: Sample of Defaulted Issuers

Listed are the defaulted issuers whose outstanding bonds at the time of default are examined in this paper. The source of the data is Moody's corporate bond default database. The filtering procedure which we used to arrive at this sample is described in Section 4. The column labelled "Data Selection" denotes whether the Issuer was selected with the help of the Investment Bank's database. The far right column shows the number of bonds for each company available for the study.

	Name of Company	Default Year	Issuer Domicile	Selected Bonds
Panel A: Issuers from Bank Database				
1	Armstrong World Industries, Inc.	2000	US	4
2	Call-Net Enterprises Inc.	2001	Canada	2
3	Chiquita Brands International, Inc.	2001	US	4
4	Comdisco, Inc.	2001	US	6
5	Enron Corp.	2001	US	9
6	Exodus Communications, Inc.	2001	US	3
7	Federal-Mogul Corporation	2001	US	3
8	FINOVA Capital Corporation	2001	US	6
9	Frontier Corporation	2002	US	2
10	Fruit of the Loom, Inc.	1999	US	3
11	Gaylord Container Corporation	2002	US	2
12	Integrated Health Services, Inc.	1999	US	3
13	Kmart Corporation	2002	US	11
14	Laidlaw Inc.	2000	Canada	6
15	Loewen Group International, Inc.	1999	Canada	4
16	Lomas Financial Corporation	1995	US	2
17	LTV Corporation (The)	2000	US	2
18	McLeodUSA Incorporated	2002	US	4
19	NTL Communications Corp.	2002	US	2
20	Owens Corning	2000	US	6
21	Pacific Gas and Electric Company	2001	US	5
22	Penn Traffic Company	1998	US	4
23	Southern California Edison Company	2001	US	5
24	USG Corporation	2001	US	2
25	Viatel, Inc.	2001	US	2
26	XO Communications, Inc.	2001	US	5
Panel B: Issuers Collected from Moody's Reports				
1	Amresco, Inc.	2001	US	2
2	Bethlehem Steel Corporation	2001	US	2
3	Burlington Industries Inc	2001	US	2
4	ContiFinancial Corporation	2000	US	3
5	Doman Industries Limited	2002	Canada	2
6	Global Crossing Holdings Limited	2002	Bermuda	4
7	Globalstar, L.P.	2001	US	3
8	Metromedia Fiber Network	2002	US	2
9	Netia Holdings II B.V.	2001	Poland	2
10	Paging Network, Inc.	2000	US	3
11	Polaroid Corporation	2001	US	3
12	PSINet Inc.	2001	US	4
13	Regal Cinemas, Inc.	2000	US	2
14	RSL Communications PLC	2001	Bermuda	3
15	Teligent Inc	2001	US	2
16	United Pan-Europe Communications N.V.	2002	Netherlands	3
Total:42				

Table V: Time-series Distribution of Initial Default Events by Year and Type

Initial default events by year and type of selected sample of issuers as listed in table IV. The data selection procedure is described in section 4. The “Other” column aggregates 3 types of default events (distressed exchange, payment moratorium, rating withdrawn) that occurred in the time sample. Mean and median (by issuer) default volumes are also shown on the right 2 columns.

Year	Chapter 11	Missed Payment	Suspension of Payments	Other	Total	Mean Default Volume(MM)	Median Default Volume(MM)
1995	1	0	0	0	1	\$480	\$480
1998	0	1	0	0	1	\$1,025	\$1,025
1999	2	1	0	0	3	\$976	\$910
2000	4	2	1	0	7	\$1,034	\$800
2001	10	7	2	2	21	\$2,556	\$1,950
1Q2002	3	5	0	1	9	\$3,042	\$2,596
Total	20	16	3	3	42	\$2208	\$1397

Table VI: Type and Timing of Chapter 11 Bankruptcy

The number of firms in the sample which eventually filed for Chapter 11 and if it was done on a prepackaged basis versus a traditional filing. A description of the timing between the initial default event and the Chapter 11 filing is shown as is the bankruptcy status as of 15 August 2002.

Type of Bankruptcy Filing	
Prepack	13
Normal	23
Total: 36 ²³	

Bankruptcy Status	
Emerged	9
Completed	6
Pending	21

Days between Default and Filing	
0	20
0-30	5
30-100	5
over 100	6

Table VII: Distribution of Issuers in Terms of Bond Data

The firms in the sample are distributed with respect to the number of bonds selected for each firm and the maturity range in years of the selected bonds. The maturity range is defined as the difference between final redemption maturities of the longest dated bond and the shortest dated bond for each firm. The right column shows the number of firms for each designated bin shown in the left column.

Number of Selected Bonds for Each Company	
2	17
3	9
4	7
5	3
6	4
>6	2

Bond Maturity Range in Years	
0-2	13
2-5	12
5-10	8
15-25	5
>25	4

Table VIII: Dispersion Statistics of Relative Recovery Values at Initial Default and Bankruptcy Filing Dates

Reports the distribution of three dispersion measures: the Range (Panel A), the Average Deviation (Panel B), and the Existence of a Mode (Panel C) of the relative recovery values in the sample. Two dates are examine: 1) the initial default date and 2)the bankruptcy filing date. The range, denoted by RV_{Range} is the difference between the maximum and minimum prices of an issuer's bonds at the specified date. The average deviation, RV_{AD} , is the average of absolute deviation from the mean of an issuer's bond prices at the specified date. Sample mean and median values are also shown. T-statistic for testing the null hypothesis that the mean is zero is shown below the sample mean. A mode exists for a particular issuer if the same price is quoted for more than one bond.

Panel A: Range of Recovery Values		
	<i>Default Date</i>	<i>Filing Date</i>
$RV_{Range} = 0$	17	15
$0 < RV_{Range} \leq 1$	8	7
$1 < RV_{Range} \leq 2$	4	6
$2 < RV_{Range} \leq 5$	7	5
$5 < RV_{Range} \leq 10$	4	1
$RV_{Range} > 10$	2	2
Total	42	36
Mean	3.30	2.97
Median	0.88	1.00

Panel B: Average Deviation of Recovery Values		
	<i>Default Date</i>	<i>Filing Date</i>
$RV_{AD} = 0$	17	15
$0 < RV_{AD} \leq 1$	13	15
$1 < RV_{AD} \leq 2$	4	1
$2 < RV_{AD} \leq 5$	6	3
$5 < RV_{AD} \leq 10$	1	1
$RV_{AD} > 10$	1	1
Total	42	36
Mean	1.20	1.08
Median	0.31	0.35

Panel C: Existence of Mode in Recovery Values		
	<i>Default Date</i>	<i>Filing Date</i>
Mode Exists	27	26
No Mode	15	10
Total	42	36

Table IX: Comparative Dispersion Statistics of Theoretical Relative Recovery Values at the Initial Default Date

Reports the distribution of two dispersion measures: the Range (Panel A) and the Average Deviation (Panel B) for the 4 different recovery assumptions described in Section 5.3. The assumptions are: 1) *RFV* (Recovery of Face Value), 2) *RT-F* (Recovery of Treasury - Face Value), 3) *RT* (Recovery of Treasury), and 4) *RMV* (Recovery of Market Value). The assumptions were used to create *hypothetical* recovery values at the initial default date for comparison with the empirical distribution from table VIII which is reproduced in the last column. Accrued interest which may be due to the bond investor under some interpretations of *RFV* is ignored. Sample mean and median values are also shown.

Panel A: Range of Recovery Values					
	<i>RFV</i>	<i>RT-F</i>	<i>RT</i>	<i>RMV</i>	<i>Empirical</i>
$RV_{Range} = 0$	42	1	0	8	17
$0 < RV_{Range} \leq 1$	0	10	15	16	8
$1 < RV_{Range} \leq 2$	0	7	6	7	4
$2 < RV_{Range} \leq 5$	0	7	8	5	7
$5 < RV_{Range} \leq 10$	0	4	10	2	4
$RV_{Range} > 10$	0	13	3	4	2
Total	42	42	42	42	42
Mean	0.00	9.34	3.75	2.08	3.30
Median	0.00	3.40	1.98	0.31	0.88

Panel B: Average Deviation of Recovery Values					
	<i>RFV</i>	<i>RT-F</i>	<i>RT</i>	<i>RMV</i>	<i>Empirical</i>
$RV_{AD} = 0$	42	1	0	8	17
$0 < RV_{AD} \leq 1$	0	18	21	25	13
$1 < RV_{AD} \leq 2$	0	6	11	3	4
$2 < RV_{AD} \leq 5$	0	8	8	5	6
$5 < RV_{AD} \leq 10$	0	7	2	1	1
$RV_{AD} > 10$	0	2	0	0	1
Total	42	42	42	42	42
Mean	0.00	3.36	1.45	0.75	1.20
Median	0.00	1.59	0.92	0.14	0.31

Table X: Parameter Values for Computing Hypothetical Spreads

The table shows the parameters used for computing model spreads which vary across credit rating class. The numbers are taken from Davydenko and Strebulaev(2002). The remaining parameters needed in the model, which are invariant across credit rating classes are taken from Huang and Huang (2002).

Credit Rating	Leverage Ratio	Asset Vol
Aaa	.12	.22
Aa	.15	.24
A	.29	.24
Baa	.36	.25
Ba	.45	.28
B	.64	.37

Table XI: Recovery and Credit Spreads: Constant Interest Rates

Calculated yield spreads are shown in basis points across different credit rating classes for 2, 10, and 30-year maturities using a first passage defaultable debt model e.g. Black and Cox (1976) and Longstaff and Schwartz (1995). Parameter assumptions are described in Table X and Section ???. The hypothetical bond pays a semi-annual coupon at an annual rate of either 8.162% (Par Bond), 12.00% (Premium Bond), or 4.50% (Discount Bond). The recovery assumptions considered are: 1) Recovery of Treasury (RT), 2) Recovery of Treasury - Face Value (RT-F), and 3) Recovery of Face Value (RFV).

Panel A: 2-Year Maturity									
	Par Bond			Premium Bond			Discount Bond		
	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>
Aaa	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Aa	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
A	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Baa	0.06	0.06	0.06	0.06	0.06	0.08	0.08	0.06	0.06
Ba	3.13	3.27	3.20	3.06	3.26	3.20	3.20	3.29	3.20
B	235.09	251.62	240.55	231.63	254.88	244.29	238.70	248.22	236.65

Panel B: 10-Year Maturity									
	Par Bond			Premium Bond			Discount Bond		
	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>
Aaa	0.09	0.09	0.09	0.08	0.09	0.08	0.10	0.10	0.09
Aa	0.82	0.93	0.84	0.75	0.88	0.81	0.92	1.00	0.89
A	10.98	13.09	11.22	10.19	12.83	11.24	12.05	13.43	11.20
Baa	28.74	35.33	29.36	27.03	35.29	30.19	31.04	35.38	28.26
Ba	86.98	112.83	88.89	83.53	115.91	95.43	91.62	108.69	80.19
B	335.21	501.05	343.40	335.91	545.15	406.35	334.27	442.56	262.51

Panel C: 30-Year Maturity									
	Par Bond			Premium Bond			Discount Bond		
	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>
Aaa	2.78	4.17	2.83	2.50	4.01	3.03	3.33	4.50	2.46
Aa	8.45	13.39	8.63	7.81	13.14	9.65	9.72	13.89	6.64
A	28.21	48.67	28.82	27.11	49.15	34.52	30.41	47.72	17.66
Baa	48.09	86.84	49.14	46.97	88.65	60.76	50.34	83.21	26.65
Ba	96.69	189.09	98.91	96.28	195.49	127.95	97.54	176.07	44.54
B	261.87	655.15	268.84	267.36	689.92	382.60	250.48	581.95	84.46

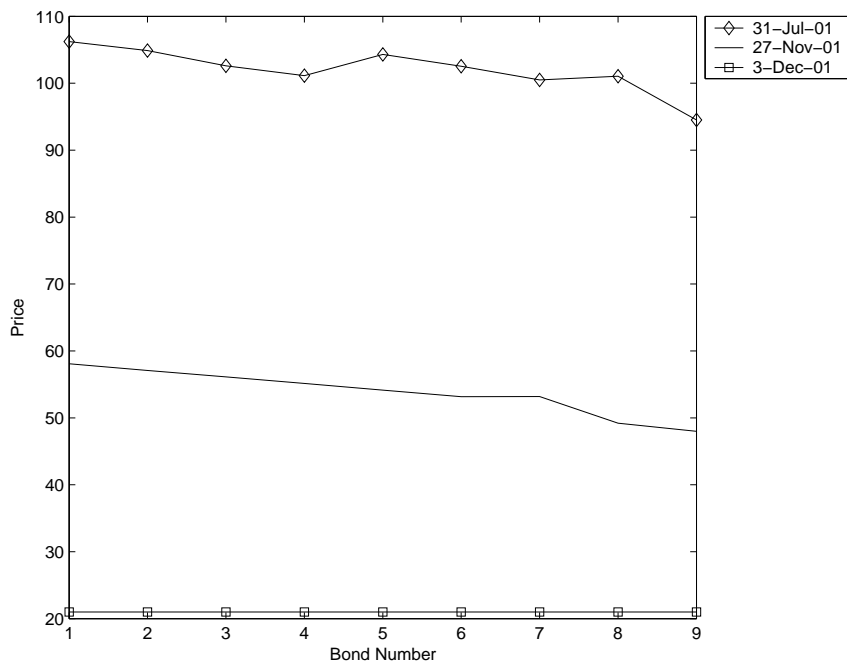
Table XII: Recovery and Credit Spreads: Stochastic Interest Rates

Calculated yield spreads are shown in basis points across different credit rating classes for 2, 10, and 30-year maturities using the first passage firm value model described in Section 6.1 combined with the default-free multifactor term structure model described in Section 6.4. The parameters for the firm value model are described in Table X and Section 6.3 while the parameters for the term structure model are described in Section 6.4.2. We consider a hypothetical bond paying a semi-annual coupon at an annual rate of 8.162%. We consider four different initial interest rate environments as described in the Appendix: 1) Flat Curve, 2)Upward Sloping Curve, 3)Humped Shape Curve, and 4)Downward Sloping Curve. The respective promised yields for the 2,10, and 30-year maturity bonds under the different environments are: 1)8.16%(2-year), 8.16%(10-year), 8.16%(30-year); 2)4.45%, 7.56%, 8.94%; 3)9.67%, 8.36%, 7.86%; and 4)9.66%, 8.36%, 6.08%. The recovery assumptions considered are: 1) Recovery of Treasury (RT), 2)Recovery of Treasury - Face Value (RT-F), and 3) Recovery of Face Value (RFV).

Panel A: 2-Year Maturity												
	Flat Curve			Upward Sloping			Humped Shape			Downward Sloping		
	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>
Aaa	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Aa	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
A	0.01	0.01	0.01	0.02	0.02	0.02	0.00	0.00	0.00	0.01	0.01	0.01
Baa	0.06	0.07	0.07	0.15	0.15	0.15	0.04	0.05	0.05	0.00	0.00	0.00
Ba	3.48	3.63	3.55	4.94	5.17	5.10	1.89	1.99	1.92	3.08	3.22	3.14
B	232.33	248.60	237.40	294.22	315.18	304.95	212.18	227.40	213.37	212.88	227.89	215.78

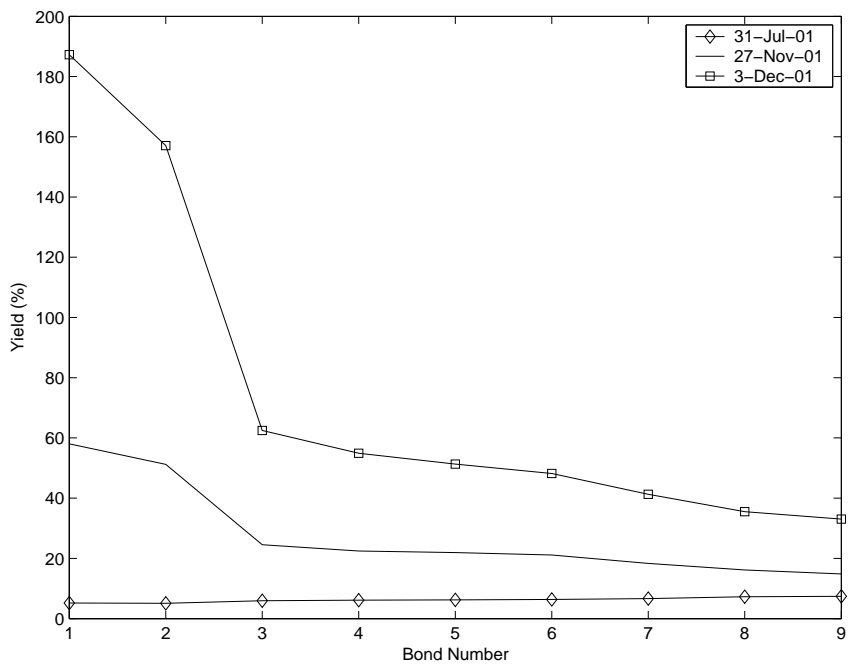
Panel A: 10-Year Maturity												
	Flat Curve			Upward Sloping			Humped Shape			Downward Sloping		
	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>
Aaa	0.08	0.08	0.08	0.01	0.01	0.01	0	0	0	0.04	0.05	0.04
Aa	0.79	0.89	0.80	0.16	0.23	0.14	1.15	1.26	1.18	0.73	0.81	0.75
A	10.63	12.58	10.88	10.4	12.78	9.88	10.81	12.78	11.29	10.25	12.07	10.68
Baa	28.62	35.08	29.32	27.27	34.55	25.72	26.35	31.97	27.52	25.92	31.28	27.19
Ba	86.23	111.6	87.83	86.38	115.53	80.45	81.26	103.53	85.15	80.09	102.01	83.82
B	332.89	497.19	338.41	343.3	532.67	316.85	322.86	474.86	336.72	319.2	468.86	331.54

Panel A: 30-Year Maturity												
	Flat Curve			Upward Sloping			Humped Shape			Downward Sloping		
	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>	<i>RT</i>	<i>RT-F</i>	<i>RFV</i>
Aaa	3.70	5.67	4.01	0.41	0.91	0.33	5.89	8.53	6.61	18.72	25.21	24.61
Aa	9.83	15.21	10.38	3.4	5.48	2.85	13.39	20.53	15.01	33.12	46.9	44.68
A	30.43	52.24	31.87	15.93	28.38	12.22	36.95	61.8	41.7	70.33	108.38	99.08
Baa	50.00	89.87	51.81	31.56	59.29	22.66	56.96	101.08	64.48	93.27	152.07	134.35
Ba	96.82	188.74	99.79	75.53	152.03	51.75	105.9	203.85	119.61	140.36	251.63	207.23
B	261.03	651.67	265.13	254.74	651.74	174.61	261.6	640.64	295.81	275.85	632.17	419.73



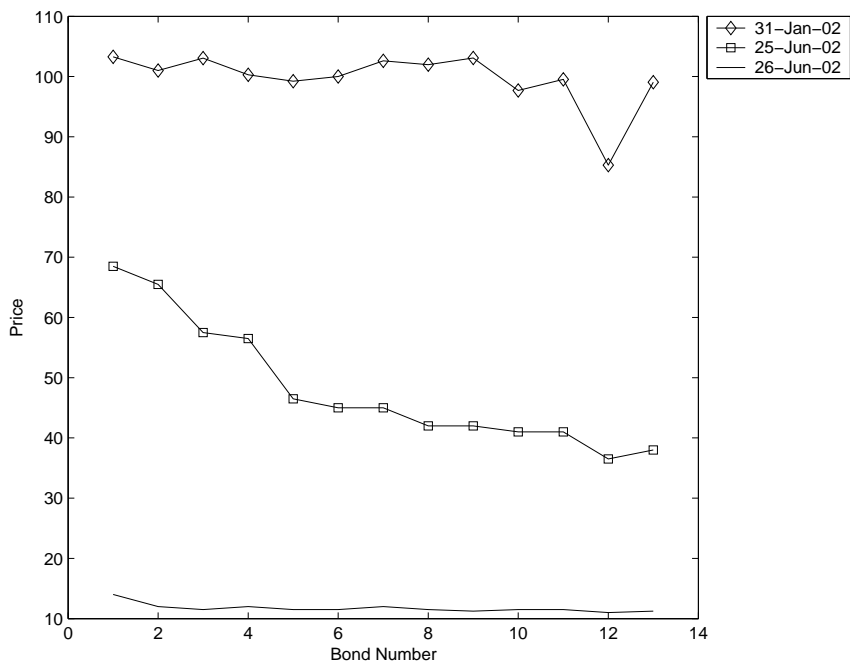
The plot shows prices for nine bonds issued by Enron Corporation at three different dates leading up to their official default date on 3 December 2001. The bonds are sorted by maturity, their contractual details are found in Table I

Figure 1: Enron Corporation Bond Prices



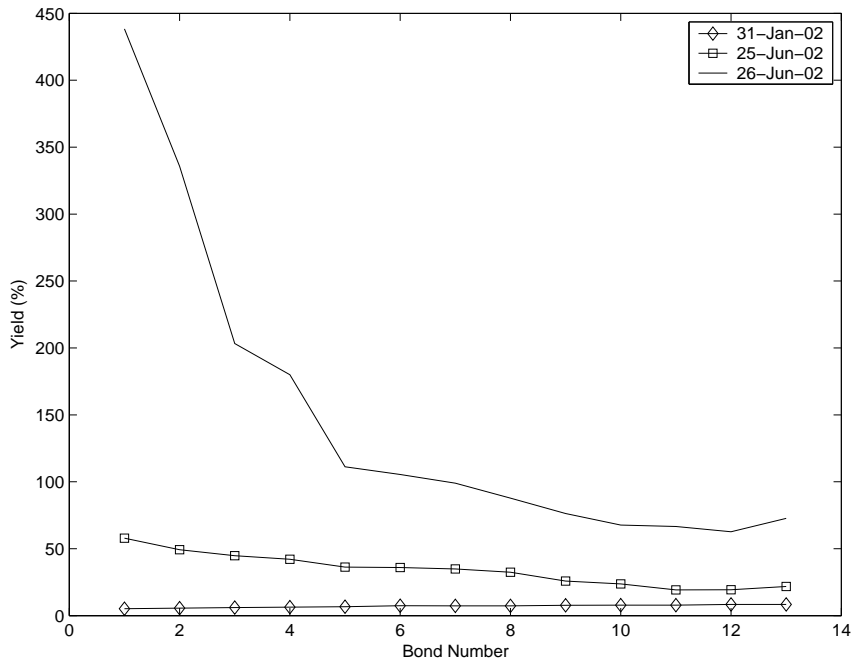
The plot shows redemption yields for nine bonds issued by Enron Corporation at three different dates leading up to their official default date on 3 December 2001. The bonds are sorted by maturity, their contractual details are found in Table I

Figure 2: Enron Corporation Bond Yields



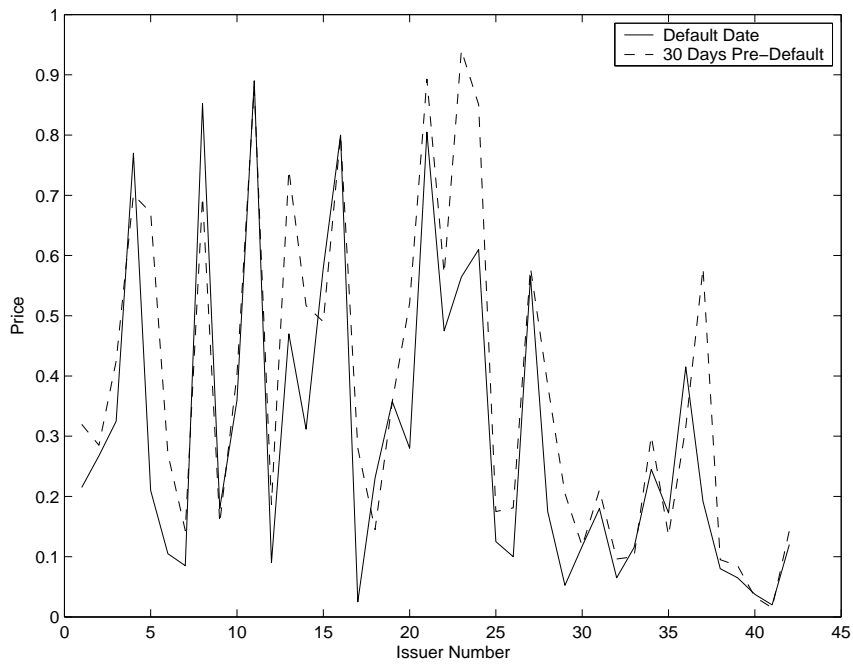
The plot shows prices for thirteen bonds issued by Worldcom Incorporated at three different dates leading up to their official default date on 15 July 2002. The bonds are sorted by maturity, their contractual details are found in Table I

Figure 3: Worldcom Inc. Bond Prices



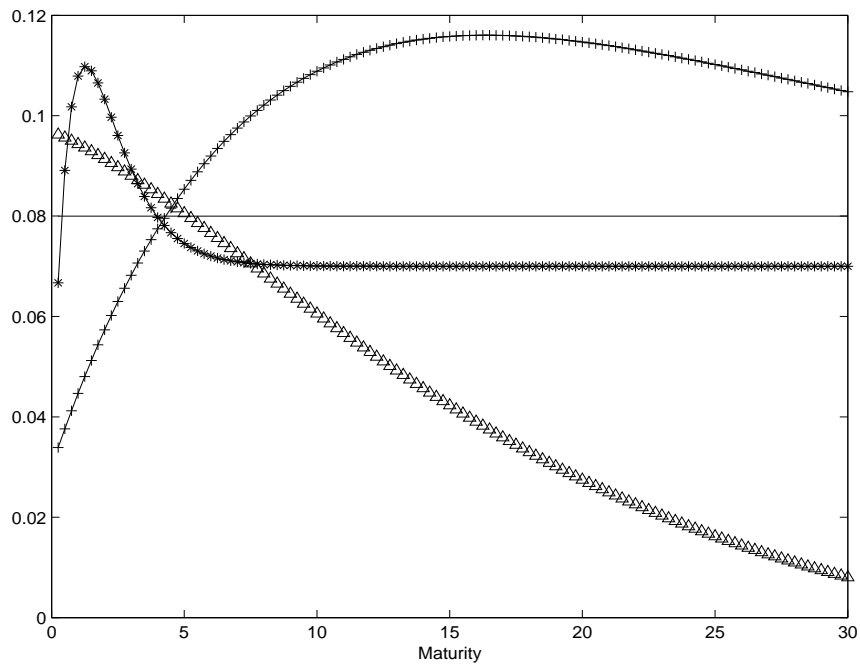
The plot shows redemption yields for thirteen bonds issued by Worldcom Incorporated at three different dates leading up to their official default date on 15 July 2002. The bonds are sorted by maturity, their contractual details are found in Table I

Figure 4: Worldcom Inc. Bond Yields



The plot shows the average bond price as a percentage of par across all issuers both at the official Moody's default date and 30 days prior to this date

Figure 5: Average Bond Prices at Default Date and 30 Days Prior



The plot shows the 4 different yield curve shapes we assume in comparing defaultable bond pricing in a stochastic interest rate environment. The yield curves are in terms of the instantaneous forward rates and are generated by Nelson-Siegel(1987) methodology using the parameters given in the Appendix.

Figure 6: Hypothetical Yield Curve Shapes used in Bond Pricing