OPTION-BASED BANKRUPTCY PREDICTION

by

Andreas Charitou and Lenos Trigeorgis*

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Address for correspondence: Dept of Business Administration, University of Cyprus, P.O.Box 537, Nicosia, CY 1678 Cyprus. Tel: 357-2-338763; Fax: 357-2-339063; e-mail: charitou@atlas.pba.ucy.ac.cy lenos.trigeorgis@gsb.uchicago.edu

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ABSTRACT

This study builds on option-pricing theory to explain business bankruptcy based on a sample of 139 matched pairs of bankrupt and control U.S. firms for the period 1983-94. Our results indicate that the primary option-motivated variables, such as firm volatility, play an important role in predicting default, one, two and three years prior to bankruptcy. When the model is extended to account for the probability of default on interest payments due at intermediate times prior to debt maturity (either due to voluntary equityholder default or due to cash flow problems), related profitability and cash-flow/liquidity variables are shown to have incremental predictive power, while the primary option variables remain statistically significant. Our theory-driven model has significant explanatory power and prediction ability in all years tested.

Key Words: option pricing, bankruptcy.

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I. INTRODUCTION

The economic and social costs of corporate failure are substantial.[1] Despite the significant costs of business failure and the empirical research efforts of many academics, there has been little attempt to develop or apply theoretical models to identify the financial variables that might explain business failure more rigorously. The lack of a theoretical framework concerning the primary variables that are relevant in distinguishing between failing and non-failing firms has been a serious impediment to the development of a truly scientific approach to bankruptcy prediction. Without a solid economic understanding of the determinants of bankruptcy it is difficult to ascertain whether a model developed based on data from one set of companies at a particular time period is appropriate for predicting business failure in a different economic or temporal setting.[2] If we must really understand bankruptcy, rather than just attempt to predict it, it is important to impute economic interpretation to bankruptcy models based on a sound theoretical foundation.[3]

This study builds on a conceptual model using option pricing theory or contingent claims analysis (CCA) to theoretically derive the factors associated with the probability of business default (see the seminal work of Black and Scholes (1973) and Merton (1973, 1974)).[4] The basic intuition

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[1] Evidence shows that the market value of distressed firms declines substantially. Hence, the suppliers of capital, investors and creditors, as well as management and employees are severely affected from business failures. Auditors also face the threat of potential lawsuit if they fail to provide early warning signals about failing firms through the issuance of qualified audit opinions (Boritz, 1991; Jones, 1987; Zavgren, 1983).

[2] Researchers have argued that this theoretical weakness is partly mitigated to some extent by validating the ad-hoc bankruptcy prediction model with a holdout sample from a different time period (Jones, 1987).

[3] The bankruptcy prediction literature is substantially based on the original work of Beaver (1966), Altman (1968), and the methodological improvements of Ohlson (1980). More recent work, linking market value of equity with book value of equity and earnings for financially distressed firms was conducted by Barth, Beaver and Landsman (1998). See also Boritz (1991) for a review of the related bankruptcy literature.

[4] Other related studies based on CCA include Jones, Mason and Rosenfeld (1985), and Fremault Vila and Schary (1995) who attempt to determine the default premium in bond valuation. The former incorporates several features of standard bond covenants into a complex CCA model , comparing the results to market bond values. The latter

behind the standard option-pricing or contingent claims model (e.g., Merton, 1974, 1977) is that the equity of a levered firm can be viewed as a European call option to acquire the value of the firm´s assets by paying off (i.e., having as exercise price) the face value of the debt (B) at the debt´s maturity (T).[5] From this perspective, a firm will be insolvent if the value of the firm´s assets falls below what the firm owes its creditors at debt maturity (i.e., when $V_T < B$). In that event, equityholders will default on the debt (file for bankruptcy) and simply hand over the firm´s assets to its creditors and walk away free (protected by their limited liability rights). The probability of default in this case, Prob($V_T < B$), is driven by the 5 primary option-pricing motivated variables: (the logarithm of) the book value of total liabilities (lnB) due at maturity representing the option´s exercise price, (the log of) the current market value of the firm´s assets (lnV), the standard deviation of % firm value changes (σ), the (average) time to the debt´s maturity (T) representing the option´s expiration, and the difference between the riskless return (r) and the firm´s payout yield (interest payments as proportion of asset value, D).[6] [7]

The standard option-based or CCA model described above is fairly parsimonious in that it uses only the aforementioned five primary option variables. Since this model focuses on default at maturity only, we extend it in a conceptual compound-option framework with the addition of other related financial variables that arise once we incorporate the probability of default on interest payments due at other intermediate times, before the debt´s maturity. Specifically, we motivate the

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paper discusses valuation of risky debt based on both involuntary as well as voluntary bankruptcy (equity´s put option to default).

[5] Essentially, from an economic perspective it is the creditors who are considered to be the owners of the firm (rather than the equityholders, who are the legal owners), with equityholders having the right to acquire the firm after paying off what they owe.

[6] Because of the ability to construct a riskless hedge that allows risk-neutral valuation of options (corporate liabilities), the mean drift in firm asset value (α) becomes irrelevant (and is replaced by the riskless rate, r).

[7] As shown in eq. 7 below, Prob($V_T < B$) = $N(-d'_2)$ where $d'_2 = \{\ln(V/B) + [(r - D) - ½\sigma^2]T\}/ \sigma \sqrt{T}$.

use of additional profitability, cash flow/liquidity and interest related variables as proxies for the equityholders´ option value of defaulting voluntarily or continuing (as a going concern) whenever an interest instalment comes due, as well as for capturing the probability of intermediate involuntary default on an upcoming interest payment if the firm has insufficient cash or liquid assets.[8]

A sample of 139 matched pairs of bankrupt and control US firms was selected for the period between 1983-1994. Consistent with the predictions of option theory, the results indicate that the book value of total liabilities (lnB), the market value of the firm´s assets (lnV), and the standard deviation of firm value changes (σ) play an important role in predicting bankruptcy. Moreover, when we extend the option framework to take into consideration the probability of default on intermediate interest payments due before the debt´s maturity, the three additional extended option variables involving profitability, cash-flow/liquidity, and interest related variables are found to be significant factors, beyond the above primary option variables, in predicting financial distress. Our theory-driven model has significant explanatory power and prediction ability in all years tested.

The rest of the paper is organized as follows. The conceptual framework used to motivate the selection of the underlying default determinants based on option pricing theory is presented next. Section III describes our data set and methodology. The empirical findings are discussed in section IV. The last section concludes.

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[8] Laitinen (1995) describes two different bankruptcy processes in Finland: (1) solidity bankruptcy, that happens when $V < B$ (as in the standard option model), and (2) liquidity bankruptcy, when available financing is less than the interest due to be paid. His approach was based on the gambler´s ruin model. For more details, see also Scott (1981).

II. A CONCEPTUAL OPTION-PRICING FRAMEWORK FOR BUSINESS DEFAULT

This section discusses: a) the standard option-pricing (or CCA) model of business default, and b) our extended option-pricing conceptual framework that includes additional financial variables to account for the probability of default at intermediate times.

A. The Standard Option-pricing (CCA) Model of Business Default

Since the seminal work of Black and Scholes (1973) and Merton (1973, 1974, 1977) option valuation or Contingent Claims Analysis (CCA) has been applied to the valuation of various corporate securities seen as packages of claims or options on the total value of the firm's assets, V; the various corporate liabilities, such as the stockholders' equity, risky debt, warrants, and convertible bonds, could now be valued as claims contingent on V as the underlying asset.

The total market value of the firm's assets at time t, V_t, is assumed to follow a standard diffusion process of the form:

$$dV_t/V_t = (\alpha - D)\,dt + \sigma\,dz, \tag{1}$$

where α denotes the (instantaneous) total expected rate of return on firm value, D is the payout by the firm (including coupon payments to debtholders) expressed as a % of V, σ is the (instantaneous) standard deviation of the firm's returns (% asset value changes), and dz is the increment of a standard Wiener process.

As shown by Merton (1977), any claim, F(V, t), whose value is contingent on a traded asset (portfolio) with value V, having a dividend payout D, and time to maturity τ ($\equiv$ T – t) must satisfy the fundamental partial differential equation (p.d.e.)

$$\tfrac{1}{2}\sigma^2 V^2 F_{VV} + (r - D)V F_V - F_\tau - r F + d = 0, \tag{2}$$

where d is the payout from the firm to the particular claim F. Each individual contingent claim (corporate liability) is uniquely represented by specifying its particular terminal and boundary conditions, along with the payout d it receives.

Consider a simple firm with only stockholders' equity of market value E and a single issue of zero-coupon (discount) debt of market value D. Suppose for now that there are no dividends of any form until after the promised face value of the bond, B, is paid off at maturity T, τ ($\equiv$ T - t) years from now. On the debt's maturity (t = T), $\tau = 0$, equity will be worth either (V - B) or zero, whichever is best for the equityholders, i.e., E(V, 0) = Max(V - B, 0).[9] The equity of such a levered firm is thus analogous to a European call option on the value of the firm's assets, V, with exercise price equal to the bond's promised payment, B, and time to expiration equal to the debt's maturity (τ).

The fundamental p.d.e. in (2) above, applied to the equity claim (F = E) when the firm pays no dividends to equity (d = 0) or coupon payments to debt (D = 0), becomes:

$$\tfrac{1}{2}\sigma^2 V^2 E_{VV} + r V E_V - E_\tau - r E = 0 \qquad (3)$$

s.t. $E(V, 0) = \text{Max}(V - B, 0)$

$E(0, \tau) = 0$

$E(V, \tau)/V \rightarrow 1$ as $V \rightarrow \infty$.

The solution to the above p.d.e. for the value of equity is given by the Black-Scholes solution for a European call option (after a transformation of variables):

[9] On the debt's maturity (T), if the value of the firm exceeds the face value of the debt, $V_T > B$, the bondholders will receive the full promised payment, B, and the equityholders will receive any residual claims, V - B. If on the other hand, $V_T < B$, the stockholders will find it preferable to exercise their limited liability rights, i.e., default on the promised payment and instead surrender the firm's assets V to its bondholders and receive nothing.

$$E(V, \tau) = V\, N(d_1) - B\, e^{-r\tau}\, N(d_2) \quad (4)$$

where $d_2 = \{\ln(V/B) + (r - \frac{1}{2}\sigma^2)\tau\} / \sigma\sqrt{\tau}$; $d_1 = d_2 + \sigma\sqrt{\tau}$

$N(d)$ = (univariate) cumulative standard normal distribution function (from $-\infty$ to d)

B = face value (principal) of the debt

V = value of firm's assets

σ = standard deviation of firm value changes (returns in V)

τ ($\equiv T - t$) = time to debt's maturity

r = risk-free interest rate

The first term in eq. (4) above is the discounted expected value of the firm if it is solvent. $N(d_2)$ in the second term of eq. (4) is the (risk-neutral) probability the firm will be solvent at maturity, i.e., $\text{Prob}(V_T > B)$, in which case it will pay off the debt principal B (with a present value cost of $B\, e^{-r\tau}$). The (risk-neutral) probability of default at the debt's maturity is given by:

$$\text{Prob. default (on principal B at maturity T)} = \text{Prob}(V_T < B) = 1 - N(d_2) = N(-d_2) \quad (5)$$

with $d_2 = \{\ln(V/B) + (r - \frac{1}{2}\sigma^2)\tau\} / \sigma\sqrt{\tau}$.[10]

If instead the debt promises regular (periodic) coupon interest payments (that are paid out or lost for equityholders while they maintain alive their option to acquire the firm but are (re)captured once they exercise their option, analogous to "dividends"), equity in the presence of the coupon-paying debt then becomes analogous to a European call option on a *dividend-paying* asset.[11] If

[10] Note that d_2 and hence $\text{Prob}(V_T < B)$ depend on V, B, σ, τ, r.

[11] Generally, if the firm makes any form of "dividend" payments (e.g., coupon interest payments on the debt), its

the firm makes a total constant payout yield D, the value of stockholders' equity can be approximated by the Black-Scholes call-option solution adjusted for a constant payout D (see Merton (1973)):[12]

$$E(V, \tau) = V e^{-D\tau} N(d'_1) - B e^{-r\tau} N(d'_2) \quad (6)$$

where $d'_2 = \{\ln(V/B) + [(r - D) - \frac{1}{2}\sigma^2]\tau\} / \sigma\sqrt{\tau}$, and $d'_1 = d'_2 + \sigma\sqrt{\tau}$.

In this case, the (risk-neutral) probability of default at the debt's maturity is given by

$$Prob(V_T < B) = 1 - N(d'_2) = N(-d'_2)$$

with $d'_2 = \{\ln(V/B) + [(r - D) - \frac{1}{2}\sigma^2]\tau\} / \sigma\sqrt{\tau}$. (7)

This standard option-based model has some interesting implications for the determinants of corporate bankruptcy. The probability of business default at the debt's maturity depends on the 5 primary option variables influencing d'_2 in eq. (7). Namely, the (risk-neutral) probability of default, $Prob(V_T < B) = N(-d'_2)$, is higher (d'_2 is lower) when:

(1) the (logarithm of) current firm value V (lnV) is low;

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value will be reduced after each "dividend" payment so that it may become optimal to exercise the equityholders' call option early in order to capture the "dividend". "Dividends" generally affect the drift of the underlying stochastic process of firm value as well as the probability of default. Shortcuts are often used in adjusting for "dividend" effects that basically attempt to sidestep the complication arising from the possibility of early exercise. If the firm pays a continuous constant "dividend payout", D (which is lost for the equity option holder), then the Black-Scholes solution for the value of a call option (eq. 6) can still be used, provided V is replaced by $V e^{-D\tau}$. This represents the current value of the asset minus the present value of the (stochastic) future "dividends" over the life of the option (debt maturity). That is, payment of a continuous "dividend yield" at the rate D reduces (or "drags down") the growth rate of firm value V at the constant rate D. Since the total return in a risk-neutral world must be r (including the "dividend yield" D), the expected growth rate in V must be (r - D).

[12] More precisely, if the firm pays dividends to equity, δ, and coupon interest payments to bondholders, d, the total payout yield would be $D = (\delta + d)/V$.

(2) the (logarithm of the) face value of the debt B due at maturity (lnB) is high –alternatively, when ln(V/B) is low (or the firm's leverage B/V is high);

(3) the volatility of the firm's return σ is high;

(4) the maturity of the debt τ is longer;

(5) the difference between the riskless interest rate, r, and the firm's payout D (i.e., r - D) is lower.[13]

B. An Option-pricing Extension for Intermediate Default

The above simplified option model (in eqs. 6-7) actually provides a lower-bound approximation for the true equity option value since it does not account for the option to default on intermediate coupon interest payments before maturity. More precisely, equity in the presence of coupon-paying debt is more like a compound option where each interest payment made by the stockholders represents the exercise price that must be paid to continue with the next stage (maintaining an option toward eventual ownership of the firm), i.e., it is the exercise price that must be incurred when interest payments come due to acquire an option on firm value V.

For example, with just one interest payment, I, due at intermediate time T' (< T) giving equityholders the option to continue (provided they pay I) with an option to acquire the firm by debt maturity T, the value of equity can be seen as a call on a call (or a compound call) option (e.g., see Geske (1979)) given by:

$$E(V, \tau) = V \; N(d^*_1 \;,\; d_1 \;;\; D) - B \; e^{-rJ} \; N(d^*_2 \;,\; d_2 \;;\; D) - I \; e^{-rJ} \; N(d^*_2) \qquad (8)$$

where $d^*_2 \equiv d^*_2\,(V^*,\, J') = \{\ln(V/V^*) + (r - ½\sigma^2)J'\}/\,\sigma\,\sqrt{J'}$; $\qquad d^*_1 = d^*_2 + \sigma\,\sqrt{J'}$

[13] Note again that the mean drift in firm value returns (α) is not a primary determinant of the probability of default.

$d_2 \equiv d_2\,(B, J) = \{\ln(V/B\,) + \,(r - ½\sigma^2)\tau\}/\ \sigma\ \sqrt{\tau}$; $\qquad d_1 \ = \ d_2 \ + \sigma\ \sqrt{\tau}$

$J \ \equiv T - t;\ \ J' \equiv T' - t$

N(d) = (univariate) cumulative standard normal distribution function (from $-\infty$ to d)

N(a, b; D) = bivariate cumulative standard normal distribution function with upper integral limits a and b and correlation coefficient D, where $D = \sqrt{J'/\tau}$

V* is the cut-off firm value, V, at the intermediate time T' when interest I comes due, above which equity's call option (to pay the interest installment in order to continue with its option toward acquiring the firm) should be exercised, obtained from solving E(V*, J') - I = 0, where E(V*, J') is obtained from the solution to earlier eq. (4). The Black-Scholes formula is actually a special case of above eq. (8), as can be seen by setting I = 0 (no intermediate interest payments) or $T = \infty$.

In this extended option-based formulation, equityholders may default on the debt not only at the debt's maturity T when $V_T < B$ (assuming that the firm has not previously defaulted on its interest payment I), related to the default probability $N(-d_2\ (B,\ J))$ given by eq. (5), but now they may also default at an intermediate time T', just before the coupon interest payment I comes due, if the value of the firm at that time falls below its cutoff option value as a going concern V* (where E(V*, J') = I, with E(V*, J') as given from the earlier option solution in eq. (4)). This latter (marginal) probability of equityholders *voluntarily* defaulting on the debt interest payment I at an intermediate time T' (with payout D) is given by

Prob. default (on interest I at T') = $\text{Prob}(V_{T'} < V^*)$ = Prob(E(V*, J') < I) = $1 - N(d^*_2) = N(-d^*_2)$ (9)

where $d^*_2 = d^*_2\,(V^*,\ J') = \{\ln(V/V^*) + [(r - D) - ½\sigma^2]J'\}/\ \sigma\ \sqrt{J'}$.

Of course, the higher the interest burden (I) from more leverage, the higher this probability of voluntary default at an intermediate time T'. The probability that equityholders exercise their call option to buy the firm by paying off the principal B at the maturity of the debt T, given that they previously decide to keep alive their option to continue (by not defaulting on the interest payment I at intermediate time T') is given by the bivariate cumulative normal distribution $N(-d^*_2, -d'_2; D)$.[14]

The above illustrates how the presence of intermediate debt payments opens up the possibility that equityholders may choose to default just before the next interest installment I comes due if their (option) value from continuing is not sufficient to cover the next payment (voluntary liquidation). However, default may additionally be triggered (this time initiated by creditors) if the firm (even when the firm is profitable and equity is valuable, i.e., ($E(V^*, J') > I$ or $V_T > B$) does not have sufficient cash flows or other liquid assets to make the next interest payment when due (involuntary liquidation).

If the firm maintains a constant proportion of its value in cash (or liquid assets), c, involuntary early default would be triggered at time T' (< T) if $c\,V_{T'} < I$ or $V_{T'} < I/c$. The (marginal) cumulative probability of default if the firm does not have sufficient cash to make the interest payment due at T' is (by analogy to eq. (9)) given by

Prob. default (on interest I due to insufficient cash) = $\text{Prob}(c\,V_{T'} < I) = N(d''_2)$

where $$d''_2 = \{\ln(cV/I) + [(r - D) - \tfrac{1}{2}\sigma^2]\, J'\} / \sigma \sqrt{J'}. \qquad (10)$$

The above probability of intermediate (involuntary) default on interest payments due to liquidity problems is higher the lower d''_2 and specifically (in addition to the other 5 primary option-pricing variables already discussed) the lower c V/I, i.e.,

(6) the higher the interest payments I (or the lower the interest coverage ratio);

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[14] The correlation among the two different events of default –on the principal B at maturity T and on interest I at T'-- is related to the timing of the intermediate coupon payment T' relative to the principal (face value) repayment at the end T, as captured by $D = \sqrt{J'/\tau}$. Note that when the intermediate payments occur rather early (J'/τ and D are small) the two different probabilities that equityholders will default are relatively independent; but otherwise (if J'/τ and D are large) they may interact so that the joint bivariate probability of default, $N(-d^*_2, -d'_2; D)$, may approximately just equal the highest of the two marginal default probabilities, $N(-d^*_2)$ and $N(-d'_2)$, given in eqs. (7) and (9).

(7) the lower the proportion of the firm’s cash flows (liquid assets) c, and

(8) the lower the firm’s profitability and value (V).

Note again that the early or delayed timing of interest payments (T') relative to maturity (T) also affects how the different events of default are correlated (D), although this is harder to operationalize in empirical testing compared to the other bankruptcy determinants. This latter effect makes the impact of the debt’s maturity (which in practice is a weighted average of the maturities of the various kinds of debt issued by the firm) less clear. Furthermore, firms in financial distress may have difficulty issuing long-term public debt and may opt for the greater flexibility of short-term bank financing, which may effectively reduce the average maturity (duration) of their debt.[15] Thus the relationship between average debt maturity (T) and the risk of default may be endogenous (circular). Also, the net effect of r - D may be ambiguous or insignificant since the two influences (of r and of D) partly offset each other.

The above extended compound-option framework motivates the use of interest coverage, cash flow/liquidity and profitability variables in assessing the probability of default at some intermediary stage before debt maturity, in addition to the 5 primary option variables identified in eq. (7) earlier.

Our option-motivated determinants of the probability of business default (with the predicted signs) can be summarized as follows:

$$\text{Prob. default} = f(\underset{-}{\ln V},\ \underset{+}{\ln B},\ \underset{+}{\sigma},\ \underset{?}{T},\ \underset{?}{r - D},\ \underset{-}{I},\ \underset{-}{\text{profitability}},\ \underset{-}{c}) \qquad (11)$$

[15] Laitinen (1995) suggests another reason why debt maturity does not have a strong effect on business failure in Finland, where the Companies Law provides that the inequality V < B is evaluated not only at the debt´s maturity (T) but in any year (effectively making the default option an American one).

III. DATA AND METHODOLOGY

This section discusses the dataset and the empirical models used in the study.

A. Dataset

The dataset consists of all U.S. firms meeting the following criteria: a) were identified in the *Wall Street Journal Index* as having filed a Chapter 7 or Chapter 11 bankruptcy petition in the period 1983-1994, b) were included in the Standard and Poor´s Research File of the Compustat database, c) have data available for at least 3 years prior to bankruptcy to enable calculating all option-motivated variables used in the model.[16] 139 firms met the above criteria and make up our bankrupt-firm sample.[17] These bankrupt firms were matched with 139 non-bankrupt firms from the same industry based on firm asset size and the year of bankruptcy (giving a total sample of 278 firms).[18]

B. Empirical Models

Logistic regression methodology is used to test the significance of the above option-motivated models, namely: a) the standard option-pricing model using only the five primary option variables, and b) the extended option model that includes additional profitability, cash-flow/liquidity and interest coverage variables to also capture the probability of intermediate default. The models used are the following:

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[16] The following Compustat data items were used to calculate the variables: Market value of the firm (V) = Compustat items (#199 * # 25) + item # 181; book value of total liabilities (B): item # 181; interest coverage (TIE): item IC; cash flow margin: ((item # 237 + total accruals) / item #12); net income: item # 172; operating income: item #178; dividend yield: item # DVYDF.

[17] The names of all bankrupt and matched non-bankrupt firms as well as the year of bankruptcy are available upon request. Three and four-digit SIC codes were used to match the bankrupt with the control firms.

[18] Jones (1987) points out that bankrupt firms are often disproportionately small and concentrated in certain failing industries. If non-bankrupt firms are drawn at random, there may be substantial differences between the two groups in terms of size and industry. As a result, a model attempting to discriminate between failing and healthy firms may actually be distinguishing between large and small firms or between different industries.

a) Standard option-pricing models (default only at debt maturity):

Model 1: Prob. default = f(lnV, lnB, σ, T, r – D)

Model 1´: Prob. default = (N(-d_2)), where $d_2 \equiv \{\ln(V) - \ln(B) + [(r - D) - \frac{1}{2}\sigma^2]T\} / \sigma\sqrt{T}$

b) Extended option-pricing models (also with intermediate default):

Model 2: Prob. default = f(lnV, lnB, σ, T, r – D, TIE, PROFIT2N, CFM)

Model 2´: Prob. default = (N(-d_2), TIE, PROFIT2N, CFM)

Models (1) and (1´) present results for the five primary option variables that account for default at debt maturity only, whereas models (2) and (2´) include the five primary and the three extended option-motivated variables (to also capture intermediate default), namely interest coverage (TIE), profitability (PROFIT2N), and cash-flow margin (CFM)). Models (1´) and (2´) combine the five primary option variables into a single predictor measure of the risk-neutral probability of default at maturity, N(-d_2), while models (1) and (2) treat them as five independent variables.

The 5 primary option variables and their measurement are as follows:[19]

(1) the log of the current market value of the firm´s assets, lnV, where V is measured as the market value of equity plus the book value of debt during the fiscal year.[20] The greater the current worth of the firm´s assets, the lower the probability of default at maturity.

(2) the logarithm of the book value of total liabilities, lnB. The higher the principal amount owed at maturity (the exercise price of the equityholders´ option), the greater the probability of default.

(3) the standard deviation (σ) of % monthly changes in firm value (monthly returns) estimated by going back over a 36-month period, starting with the first fiscal-year-end prior to bankruptcy.

[19] The mean growth rate of firm value (V) changes, estimated over a prior 36-month period, was also tested but found insignificant, as expected by option theory; it was therefore, not included in our models.

[20] We adopt the commonplace assumption in the literature of using the book value of debt as a proxy for its market value, e.g., see Gaver and Gaver (1993) and Smith and Watts (1992).

The greater the firm´s volatility, the greater the value of equityholders´ default option.

(4) the average time to debt´s maturity (T), measured as the average duration of all outstanding debt maturities. From a strict option-theory perspective, the longer the maturity, the greater the default option value, other things constant.[21]

(5) the difference between the riskless return and the firm´s payout yield, r – D, where r is obtained from the 3-month US Treasury-bill rate, and D is the coupon interest payment as a proportion of the market value of the firm (V) at fiscal year end.[22]

The 3 extended option-motivated variables are:[23]

(6) the interest coverage ratio, measured as the ratio of earnings to interest expense (Times Interest Earned or TIE). It shows the ability of the firm to generate earnings to cover its interest expense. In general, the higher the firm´s interest coverage, the lower the probability of default.

(7) profitability (successive losses) over the last two years, measured as a dummy (PROFIT2N) taking a value of 1 if the firm had losses in two successive years prior to the event year, and 0 otherwise. In general, the higher the profitability the lower the probability of default.

(8) cash-flow margin (CFM), measured as the ratio of operating cash flows to sales. The higher the cash-flow margin, the better the firm´s ability to meet its upcoming obligations and the lower the probability of default.[24]

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[21] In general and in the presence of a dividend yield the (European) option is not monotonic in time to maturity, although often for practical purposes the change in sign might occur after three years. Furthermore, in practice, firms facing financial difficulties are likely to have more difficulty in maintaining long-term debt, and so, by necessity, the sample of bankrupt firms may be associated with a lower duration of debt than healthy firms.

[22] The debt interest payment from equity´s perspective is analogous to a dividend on a stock option (e.g., see Merton 1974). The value of the equityholders´ call option to default would be higher the higher the interest rate r and the lower the payout D, so the net effect of r – D is indeterminate.

[23] There is no unique selection of proxies for the interest coverage, cash flow and profitability variables motivated by option theory. Alternative proxies examined produced qualitatively similar results. The presented proxies had slightly higher predictive performance.

[24] For more information about the usefulness of cash flow variables see Gilbert et al (1990), Charitou and Ketz (1991), Casey and Bartzak (1985), Gentry et al (1985).

We test the model´s significance and explanatory power using –2 log-likelihood statistics and pseudo-R^2. The significance of the difference across the bankrupt and non-bankrupt groups is examined via the parametric paired t-test (for means) and the non-parametric Wilcoxon signed-rank matched-pair test (for medians). We also test the predictive ability of the above option-motivated variables using a holdout sample. The training sample includes 82 pairs of firms for the period 1983-90. The forecasting (holdout) sample includes 57 pairs for the period 1991-94.[25] The results are presented next.

IV. EMPIRICAL RESULTS

Table 1 presents descriptive statistics on the 8 option-motivated variables for the total sample of 139 matched-pairs of bankrupt and non-bankrupt firms over the period 1983-1994, in each of the three years prior to bankruptcy. The significance of the mean and median differences across the groups is given in the third and fourth column for each year. The trends in the predictor variables over the three years prior to bankruptcy are illustrated graphically in Figure 1. The risk-neutral probability of default $N(-d_2)$ is significantly higher for bankrupt firms (at 1% for both the paired t-test and the Wilcoxon test) and increasing as the year of bankruptcy approaches. The median firm value (lnV) for bankrupt firms is only slightly lower 2-3 years prior to bankruptcy, but drops significantly in the year before bankruptcy. The amount of debt (lnB) owed by bankrupt firms appears higher in all 3 years, although it is not statistically significant. The ratio of the market value of the firm to the book value of debt (lnV/B) is lower for bankrupt than for non-bankrupt firms, and declines as the year of bankruptcy approaches. The difference among the two groups is statistically

[25] As Jones (1987) points out, if a holdout sample is obtained from a later period, one can test for both overfitting and a violation of the stationarity assumption. The stationarity assumption implies that the relationship between

significant (at 1%, using both tests). Mean firm volatility (σ) is higher for bankrupt than for non-bankrupt firms.[26] The median debt maturity is somewhat shorter for the sample of bankrupt firms. The times interest coverage (TIE) and the cash-flow margin (CFM) are significantly lower for bankrupt firms and decline as the year of bankruptcy approaches. These trends are generally consistent with our expectations from option theory and prior bankruptcy-prediction literature based on financial ratios.

Table 2 presents the univariate results for all option-motivated variables used in logistic regression one, two and three years prior to business failure. These results indicate that the risk-neutral probability of default $N(-d_2)$ is statistically significant (at 1%) in all years tested, with a pseudo-R^2 of 6%, 9% and 16% in the third, second, and first year prior to bankruptcy. The other primary option variables lnV, σ, T, and $r - D$ are statistically significant at least in the last year prior to bankruptcy. The ratio ln(V/B) is statistically significant (at 1%) in all 3 years (with pseudo-R^2 of about 2%, 5% and 15% in years 3, 2 and 1). The other three extended option variables, TIE, PROFIT2N and CFM, are statistically significant in all three years prior to bankruptcy.

Table 3 shows the multivariate logistic regression results for all 8 option-motivated variables, one, two and three years prior to bankruptcy.[27] All models tested are statistically significant at the 1% level based on the –2 log-likelihood test. In model (1) all individual primary option variables are statistically significant at 1%, with the exception of $r - D$. The coefficient of $r - D$ may be

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the independent variables and the dependent variable will hold over time (Mensah, 1984).

[26] The difference of means is statistically significant using the paired-t test in years 1 and 3. The difference of medians is not statistically significant.

[27] As noted, models (1) and (1´) present results for the primary option variables only, whereas models (2) and (2´) include the 5 primary and 3 extended option-motivated variables (namely, interest coverage (TIE), profitability (PROFIT2N), and cash-flow margin (CFM)). Models (3) and (3´) are similar to models (2) and (2´) but exclude the interest coverage ratio (TIE) for sensitivity purposes. Models (1´), (2´) and (3´) combine the five primary option variables into a single predictor measure for the risk-neutral probability of default at maturity, $N(-d_2)$, while models (1), (2) and (3) treat them as five independent variables.

insignificant because of the offsetting impact of the interest rate (r) and the firm payout (D). As expected, the probability of bankruptcy is higher the lower the value of the firm (lnV), the higher the amount of debt owed (lnB), and the higher the firm volatility (σ). The average debt maturity (T) has a negative sign, in partly because firms in financial distress have more difficulty in raising long-term debt and so they tend to hold more short-term debt.[28] The model´s pseudo-R^2 is 9.5%, 16.4% and 29.3% in the third, second, and first year before bankruptcy. The model´s testing results using a holdout sample over the period 1991-94 are 78%, 66.7% and 65.8% in years one, two, and three, respectively.[29] When the effect of all five primary option variables above is combined into the single (risk-neutral) default probability measure $N(-d_2)$, model (1´) gives significant (at 1%) positive coefficients in all three years tested, validating the prediction of option theory. This model (1) has a pseudo-R^2 of 6.3%, 9.9%, and 13.2% for the third, second, and first year. The predictive power of this single-variable model is 70.1%, 62.3% and 65.7%, one, two and three years prior to bankruptcy, respectively. These findings provide initial support for our option-based model as being useful in explaining financial distress.

When we expand the model to include the three additional extended-option variables to also account for intermediate default on interest payments (see model (2)), the primary option variables maintain their sign and significance (except for maturity T), while the explanatory power and predictive performance of the model improves. Results for the first year prior to bankruptcy (panel A) show that all the primary as well as the extended option variables are statistically significant, except for the r - D variable. The signs of the slope coefficients of the three extended option

[28] As stated earlier, generally in the presence of dividends the relationship between (European) option value and maturity is not monotonic.
[29] The type I error rates are relatively lower than the type II error rates in all years tested. The model´s training results are 78.7%, 72% and 64% one, two and three years prior to bankruptcy.

variables are as expected. Specifically, the probability of bankruptcy is higher the lower the interest coverage (TIE) and the cash-flow margin (CFM), and the greater the firm´s losses during the previous two years (PROFIT2N). The model´s classification (testing) accuracy is 85.1%. Results for the second and third years prior to bankruptcy (in panels B and C) show that all other variables remain statistically significant, except for average time to maturity (T) and interest coverage (TIE). The pseudo-R^2 of model (2) is 24.9%, 30.5% and 45.5% in the third, second, and first year. The model´s predictive ability is about 70%. The extended option variables maintain their sign and significance (except for TIE), even when the effect of the five primary option variables is combined into the single measure $N(-d_2)$ in model (2´).

Since the interest coverage ratio (TIE) is insignificant in years two and three prior to bankruptcy, models (3) and (3´) show sensitivity of the results from repeating the analysis with all variables except TIE. In model (3) all option-motivated variables are again significant and of the expected sign in all years, except for $r - D$ and T. The predictive ability of the model is 82.5% in the first year, and about 70% in the second and third years prior to bankruptcy. When the five primary variables are combined in model (3´), $N(-d_2)$ as well as the extended option variables remain significant and of the expected sign in all three years tested. The predictive ability of this 3-variable model is 78% in year one, and about 70% in years 2 and 3 prior to bankruptcy. [30] Overall, the above results confirm the usefulness of extended option theory in the selection of financial variables to explain firm bankruptcy.

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[30] The pseudo-R^2 for model (3) is 24.4%, 29.2%, and 42.8%, and for model (3´) is 19.8%, 23.6%, and 29.9%, in the third, second, and first year before bankruptcy.

V. CONCLUSIONS

This study uses an extended option framework to select the explanatory variables for understanding business bankruptcy. Using a sample of matched U.S. firms during the 1983-94 period, our results indicate that both the standard primary as well as our extended option-motivated models are statistically significant in predicting bankruptcy. The significant primary option variables include the face value of the debt owed at maturity (lnB), the current market value of the firm´s assets (lnV), and the standard deviation (σ) of firm value changes (returns). The mean drift (growth rate) of firm value changes was not a significant predictor of business failure, while the risk-neutral probability of default $N(-d_2)$ was found to be a significant predictor.[31] Moreover, when these primary option variables are included along with the additional profitability, cash-flow/liquidity and interest related variables used to also capture the probability of intermediate default on due interest payments, the above primary option variables maintain their sign and significance. The latter results indicate that the related profitability and cash-flow/liquidity variables have incremental explanatory power beyond the primary option variables. The overall results are consistent with the predictions of (extended) option theory. Our theory-driven model has significant explanatory power and prediction ability in all years tested, providing a deeper understanding of the factors determining firm bankruptcy.

[31] These findings are in accordance with option valuation theory. Of course, the insignificance of the mean drift of firm value may be due to the general difficulty of predicting drifts (mean returns).

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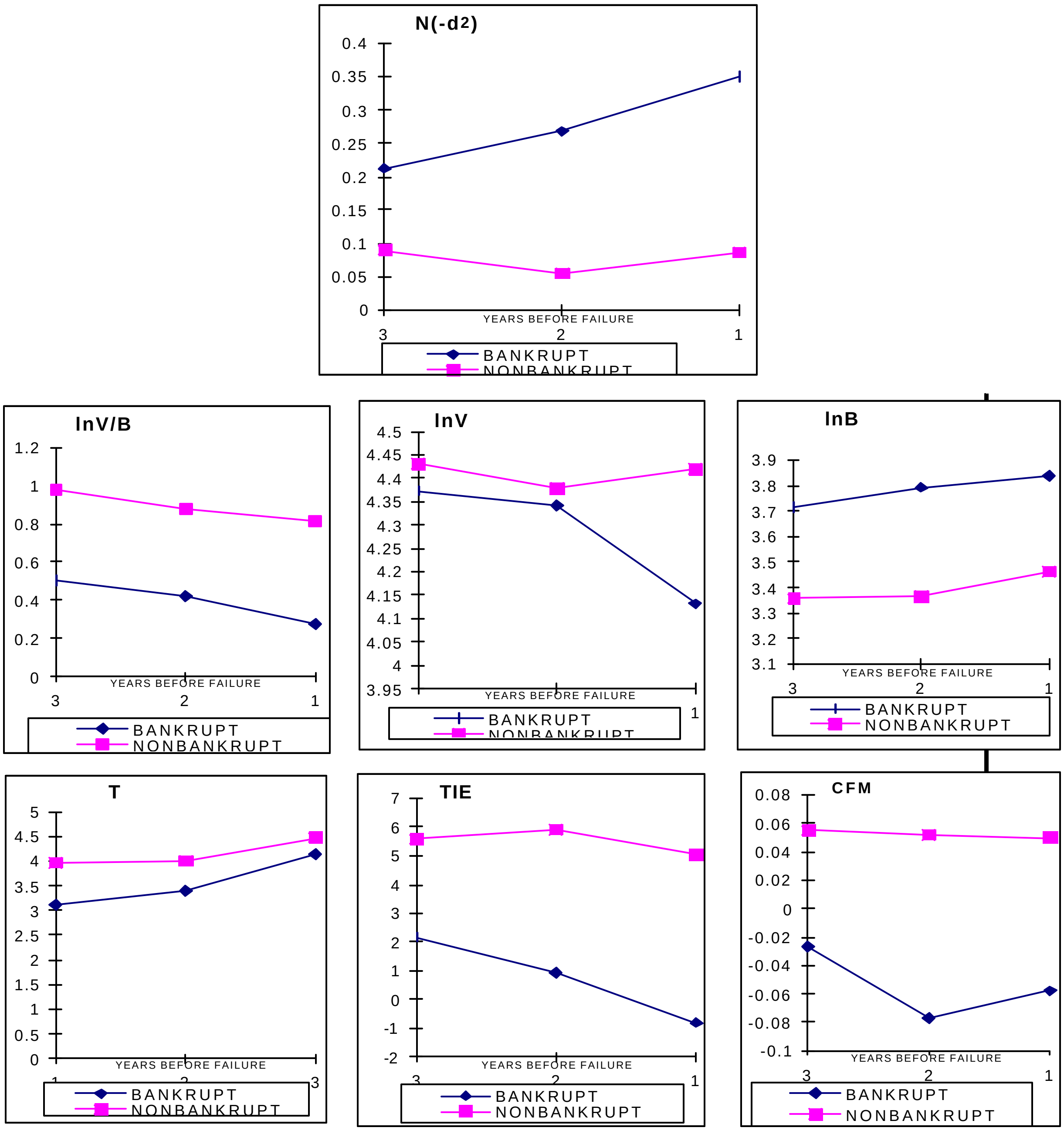
Figure 1
Trends of major option-motivated predictor variables
one, two and three years prior to business failure



Note: N(-d2): Probability of default (risk neutral); LnV: Log of current market firm value; lnB: Log of book value of total liabilities; lnV/B: log of current market firm value to book value of total liabilities; T: Average time to debt's maturity; CFM: cash flow margin ; TIE: Interest coverage.

Table 1

Descriptive statistics for option-motivated variables one, two and three years prior to business failure

		ONE YEAR BEFORE FAILURE				TWO YEARS BEFORE FAILURE				THREE YEARS BEFORE FAILURE			
OPTION-MOTIVATED VARIABLES	**SYMBOL**	Bankrupt	Non-Bankrupt	Paired t-test**	Wilcoxon Test**	Bankrupt	Non-Bankrupt	Paired t-test**	Wilcoxon test **	Bankrupt	Non-Bankrupt	Paired t-test**	Wilcoxon test **
A. Primary Option Variables													
Probability of default (risk neutral)	N(-d_2)	0.350*	0.087			0.267	0.056			0.213	0.090		
		(0.23)	*(0.2)*	*(0.000)*	*(0.000)*	*(0.23)*	*(0.18)*	*(0.000)*	*(0.000)*	*(0.23)*	*(0.17)*	*(0.000)*	*(0.000)*
Log of (firm market value/BV of debt)	Ln(V/B)	0.221	0.750			0.387	0.886			0.500	0.926		
		0.46	0.86	*(0.000)*	*(0.000)*	(0.72)	(0.74)	*(0.000)*	*(0.000)*	(0.88)	(0.73)	*(0.019*	*(0.001)*
Log of current market firm value	LnV	4.132	4.420			4.341	4.379			4.373	4.432		
		(1.33)	*(1.45)*	*(0.072)*	*(0.090)*	*(1.25)*	*(1.4)*	*(0.417)*	*(0.467)*	*(1.2)*	*(1.42)*	*(0.674)*	*(0.746)*
Log of book value of total liabilities	LnB	3.838	3.463			3.794	3.368			3.716	3.357		
		(1.47)	*(1.64)*	*(0.153)*	*(0.255)*	*(1.46)*	*(1.58)*	*(0.179)*	*(0.262)*	*(1.61)*	*(1.62)*	*(0.384)*	*(0.501)*
Std deviation of firm value changes	σ	0.249	0.270			0.262	0.277			0.290	0.280		
		(0.26)	*(0.14)*	*(0.086)*	*(0.347)*	*(0.29)*	*(0.15)*	*(0.130)*	*(0.384)*	*(0.3)*	*(0.15)*	*(0.069)*	*(0.223)*
Average time to debt's maturity	T	3.118	3.974			3.404	4.010			4.146	4.467		
		(3.24)	*(3.62)*	*(0.003)*	*(0.001)*	*(3.33)*	*(3.69)*	*(0.101)*	*(0.083)*	*(2.69)*	*(2.88)*	*(0.355)*	*(0.248)*
Interest rate minus firm payout	r-D	0.014	0.034			0.024	0.037			0.032	0.041		
		(0.06)	*(0.02)*	*(0.000)*	*(0.000)*	*(0.05)*	*(0.03)*	*(0.000)*	*(0.000)*	*(0.03)*	*(0.03)*	*(0.001)*	*(0.002)*
B. Extended Option Variables													
Interest coverage	TIE	-0.809	5.049			0.930	5.925			2.170	5.585		
		(19.7)	*(222.17)*	*(0.000)*	*(0.000)*	*(121.55)*	*(226.67)*	*(0.019)*	*(0.000)*	*(142.86)*	*(209.5)*	*(0.003)*	*(0.000)*
Profitability (losses) over last two years	PROFIT2N	1	0			0	0			0	0		
		(0.49)	*(0.34)*	*(0.000)*	*(0.000)*	*(0.49)*	*(0.34)*	*(0.000)*	*(0.000)*	*(0.47)*	*(0.31)*	*(0.000)*	*(0.000)*
Cash-flow margin	CFM	-0.058	0.05			-0.077	0.052			-0.027	0.055		
		(7.26)	*(0.12)*	*(0.113)*	*(0.000)*	*(8.75)*	*(0.17)*	*(0.079)*	*(0.000)*	*(15.23)*	*(0.26)*	*(0.063)*	*(0.000)*

* Median value (std. deviation in parenthesis).

** Test of the significance of the mean (median) differences: paired t-test (mean) / Wilcoxon test (median); p-value in parenthesis.

Table 2

Univariate logistic regression results for option-motivated variables one, two and three years prior to business failure

OPTION-MOTIVATED VARIABLES	SYMBOL	ONE YEAR BEFORE FAILURE				TWO YEARS BEFORE FAILURE				THREE YEARS BEFORE FAILURE			
		Coefficient	P-value	Pseudo -R^2	Prediction	Coefficient	P-value	Pseudo -R^2	Prediction	Coefficient	P-value	Pseudo -R^2	Prediction
A. Primary Option Variables													
Probability of default (risk neutral)	$N(-d_2)$	4.722	0.000	16.3%	69.8%	3.629	0.000	9.3%	65.1%	3.436	0.000	7.7%	62.2%
Log of (firm market value/BV of debt)	Ln(V/B)	-1.788	0.000	14.6%	71.6%	-0.753	0.000	4.8%	65.8%	-0.377	0.016	1.6%	61.5%
Log of current market firm value	lnV	-0.157	0.073	0.8%	56.8%	-0.073	0.420	0.2%	50.0%	-0.039	0.670	0.0%	52.2%
Log of book value of total liabilities	lnB	0.114	0.145	0.5%	52.5%	0.108	0.173	0.5%	55.0%	0.067	0.370	0.2%	54.3%
Std deviation of firm value changes	σ	1.102	0.081	0.8%	51.0%	0.905	0.120	0.7%	51.8%	1.091	0.065	1.0%	50.7%
Average time to debt's maturity	T	-0.108	0.004	2.3%	56.1%	-0.054	0.124	0.6%	50.4%	-0.042	0.335	0.2%	51.8%
Interest rate minus firm payout	r-D	-28.921	0.000	10.6%	66.2%	-20.384	0.000	6.0%	58.6%	-15.387	0.001	3.5%	57.6%
B. Extended Option Variables													
Interest coverage	TIE	-0.215	0.000	28.0%	80.9%	-0.002	0.039	1.6%	57.2%	-0.003	0.021	2.9%	56.8%
Profitability (losses) over last two years	PROFIT2N	2.269	0.000	17.6%	73.0%	1.571	0.000	7.8%	64.4%	1.343	0.000	4.9%	60.4%
Cash-flow margin	CFM	-2.187	0.000	8.3%	70.1%	-4.663	0.000	17.0%	73.7%	-2.773	0.003	14.1%	67.3%

Table 3: Multivariate logistic regression results for option-motivated variables one, two and three years prior to business failure

PANEL A: ONE YEAR PRIOR TO FAILURE

MODEL	N(-d2)***	lnV	lnB	ó	Ô	r-D	TIE	PROFIT2N	CFM	CONSTANT	Model signif test	Pseudo-R^2	Training results**	Testing results**
1		-3.204 *	3.409	7.461	-0.147	-5.024				-0.314		29.3%	78.7	78.0
		(0.000)	*(0.000)*	*(0.000)*	*(0.010)*	*(0.288)*				*(0.736)*	(0.000)			
1'	4.269									-1.096		13.2%	69.5	70.1
	(0.000)									*(0.000)*	(0.000)			
2		-2.272	2.567	6.824	-0.170	1.098	-0.117	1.684	-1.811	-1.421		45.5%	84.8	85.1
		(0.014)	*(0.005)*	*(0.006)*	*(0.009)*	*(0.826)*	*(0.068)*	*(0.002)*	*(0.010)*	*(0.226)*	(0.000)			
2'	1.634						-0.171	1.568	-1.233	-0.673		38.3%	79.9	81.6
	(0.124)						*(0.003)*	*(0.001)*	*(0.045)*	*(0.107)*	(0.000)			
3		-3.300	3.556	7.815	-0.179	1.436		2.011	-2.212	-1.288		42.8%	84.8	82.5
		(0.000)	*(0.000)*	*(0.001)*	*(0.007)*	*(0.724)*		*(0.000)*	*(0.000)*	*(0.246)*	(0.000)			
3'	3.045							2.229	-1.658	-1.542		29.9%	76.2	78.1
	(0.002)							*(0.000)*	*(0.006)*	*(0.000)*	(0.000)			

PANEL C: TWO YEARS BEFORE FAILURE

MODEL	N(-d2)	lnV	lnB	ó	Ô	r-D	TIE	PROFIT2N	CFM	CONSTANT	Model sig test	Pseudo-R^2	Training **	Testing **
1		-1.545	1.731	5.562	-0.084	-7.453				-0.574		16.4%	72.0	66.7
		(0.000)	*(0.000)*	*(0.000)*	*(0.098)*	*(0.215)*				*(0.499)*	(0.000)			
1'	4.000									-0.837		9.9%	67.1	62.3
	(0.000)									*(0.001)*	(0.000)			
2		-1.551	1.708	4.501	-0.088	12.071	-0.017	1.478	-2.474	-1.170		30.6%	78.1	66.7
		(0.003)	*(0.001)*	*(0.010)*	*(0.129)*	*(0.095)*	*(0.275)*	*(0.006)*	*(0.004)*	*(0.233)*	(0.000)			
2'	1.936						-0.022	1.218	-2.423	-0.658		25.8%	75.0	69.3
	(0.068)						*(0.150)*	*(0.018)*	*(0.005)*	*(0.035)*	(0.000)			
3		-1.733	1.888	4.570	-0.088	11.035		1.482	-2.586	-1.151		29.2%	79.9	70.1
		(0.001)	*(0.000)*	*(0.005)*	*(0.124)*	*(0.154)*		*(0.005)*	*(0.003)*	*(0.231)*	(0.000)			
3'	2.409							1.281	-2.797	-0.943		23.6%	75.6	72.8
	(0.018)							*(0.010)*	*(0.002)*	*(0.001)*	(0.000)			

PANEL C: THREE YEARS BEFORE FAILURE

MODEL	N(-d2)	lnV	lnB	ó	Ô	r-D	TIE	PROFIT2N	CFM	CONSTANT	Model sig test	Pseudo-R^2	Training **	Testing **
1		-0.763	0.827	4.631	-0.065	-12.065				-0.270		9.5%	64.0	65.8
		(0.024)	*(0.011)*	*(0.001)*	*(0.275)*	*(0.091)*				*(0.753)*	(0.001)			
1'	3.174									-0.645		6.3%	61.0	65.7
	(0.001)									*(0.007)*	(0.000)			
2		-1.270	1.235	2.982	-0.062	5.934	-0.002	1.389	-2.414	0.032		24.9%	72.6	71.0
		(0.005)	*(0.004)*	*(0.044)*	*(0.358)*	*(0.510)*	*(0.377)*	*(0.012)*	*(0.007)*	*(0.973)*	(0.000)			
2'	1.588						-0.002	1.417	-1.807	-0.624		20.8%	71.3	67.6
	(0.132)						*(0.273)*	*(0.009)*	*(0.013)*	*(0.021)*	(0.000)			
3		-1.322	1.282	2.986	-0.059	4.657		1.429	-2.537	0.072		24.4%	71.3	71.0
		(0.003)	*(0.003)*	*(0.042)*	*(0.379)*	*(0.606)*		*(0.010)*	*(0.005)*	*(0.940)*	(0.000)			
3'	1.779							1.474	-1.925	-0.736		19.9%	71.3	68.5
	(0.090)							*(0.006)*	*(0.013)*	*(0.005)*	(0.000)			

*Coefficient on top line, p-value in parenthesis; p-value corresponds to the Chi-square test for the significance of individual coefficients based on the -2 log-likelihood test.

** The training and testing samples consist of 164 and 114 firms over the period 1983-90 and 1991-94, respectively. *** N(-d2): probability of default (risk neutral), ln(V/B): log of the firm market value to book value of debt, lnV: log of current market firm value, lnB: log of book value of total liabilities, ó : std deviation of the firm value changes, T: average time to debt´s maturity, r-D: interest rate minus firm payout, TIE: interest coverage, PROFIT2N: profitability (losses) over the past two years, CFM: cash flow margin.