More on Wrong-Way exposure

Introduction
This purpose of this note is to expand on some of the issues described in the paper we wrote for Risk magazine named “Wrong-way exposure” published in the July issue 1999. In order to appreciate these issues one needs to first read the “Wrong-way exposure” paper.

We will discuss five topics:
1. Some important right-way exposures.
2. Wrong way interest rate risk
3. Wrong way risks in currencies other than the counterparty’s currency
4. Residual value factors for different currencies.
5. Bootstrapping risk-neutral default probabilities from par spreads.

1. Important right way exposures
One sort of “right-way” risk is derivatives with local entities in local currency. Consider, for example, the same derivative transaction done with two corporations of identical credit quality. One is a local firm, the other not. Assume also that the expected derivative value in the local currency given default is the same for each counterparty. If expected credit loss is being measured in the local currency, we would then find no difference between the two transactions. However, if we measure it in dollars, the expected loss to the local firm is lower since the dollar value of the currency is likely to have depreciated. Thus the transaction done with a local entity is “right way” relative to an outside firm.

Based on this, the credit default swap spread on a given name should depend on the currency underlying the contract. For example, for Japanese corporations the spread on yen denominated credit default swaps should typically be lower than those which are dollar denominated since we expect yen value (and therefore loss of notional when valued in dollars) to be lower in the event of their default. This effect, among others, also applies to debt. The dollar denominated debt of the firm should, in general, carry a higher spread than debt denominated in its local currency.

2. Wrong way interest rate risk
The Risk paper focused on floating-to-floating cross currency swaps where the emphasis is on the wrong way risk associated with the local currency and not on other market rates. Here we outline a methodology for taking yield change into account. The approach can be generalized to apply to other market rates as well.

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1 This piece was written Jointly with Ronald Levin.
2 As in the paper we use dollars here for ease of reference. The same would apply to other major currencies.
3 Another factor driving up the spread is the fact that sovereign default may force the corporation into default on its foreign currency debt but not on its local currency debt.
**Fixed rate swaps** - We treat here swaps in which the counterparty receives a fixed rate in its local currency. This will have direct application to cases: 1) a fixed-floating swap which is entirely in the local currency, and 2) a fixed-floating swap in which the fixed leg is in the local currency and the floating leg is in dollars. In the first of these the only wrong-way adjustment relates to interest rates. In the second, the wrong way adjustment for currency will be compounded by an additional adjustment for interest rates.

The adjustment to expected value for interest rates conditional on counterparty default is more ambiguous than in the FX case. For some countries, an economic slow down and high corporate defaults may be associated with low interest rates. For others, this default risk may be associated with hyperinflation and its accompanying high interest rates. To determine the sign and magnitude of this difference in expected value, one should consider looking at historical data as well as macroeconomic analysis.

We write

\[ E[y(t,T)| \text{Def}] = y_F(t,T) + \Delta_d y \]

\(y_F(t,T)\) - Forward yield to maturity of the fixed leg for settlement at t. (This is determined with respect the swap yield curve.)

\(y(t,T)\) - Realized yield to maturity of the fixed leg for settlement at t.

\(\Delta_d y\) - expected yield change conditional on counterparty default. \(\Delta_d y\) may depend on time to maturity \((T-t)\).

The expected value of the fixed leg in units of local currency is then given by:

\[ E[B_{\text{fix}}(t)| \text{Def}] = B[r_{\text{fix}}, y_F(t,T) + \Delta_d y, t, T] \]

\(r_{\text{fix}}\) - Fixed interest rate for the fixed leg of the swap. The swap matures at T.

\(B[r_{\text{fix}}, y(t,T), t, T]\) - Value (in the local currency) of the fixed leg at t given a yield to maturity of \(y(t,T)\).

For case 1, above, in which the swap is entirely in the local currency, the expected swap value upon default is then given by:

\[ 1 - E[B_{\text{fix}}(t)| \text{Def}], \text{ per unit of local currency.} \]

We can then use this conditional expected swap value, together with a volatility estimate, to

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4 This will relate to a sovereign counterparty to the extent that the sovereign can default on obligations in its own currency.

5 The fact that both legs are in the local currency implies a “right way” adjustment as described in section 1, above.

6 Alternatively, we could have expressed the impact of default as a multiplicative adjustment.

7 This is per one unit of local currency notional. This expression ignores an adjustment for convexity due to differences between the conditional and unconditional yield distributions. However, this will generally be of second order.
determine expected exposure in the event of counterparty default, as described in the *Risk* paper.\(^8\)\(^9\)

We next consider case 2 in which the fixed leg is in the local currency but the floating leg is in dollars. To obtain the expected value of the fixed leg in dollars we need to multiply by the expected currency value conditional on default. For a sovereign default this will amount to \(RV \ast \overline{FX}(t)\) (see equation (1) in the *Risk* paper). To express this value per dollar notional we further need to divide by \(FX(0)\), the swap exchange rate. We can now express the expected dollar value of the local currency leg upon sovereign default (per dollar notional):

\[
E[S_{fix}(t) | \text{Def}] = 1 - E[B_{fix}(t) | \text{Def}] \ast RV \ast \overline{FX}(t) / \overline{FX}(0)
\]

This together with a volatility assumption for the market rates is then used to determine expected exposure, as described in the paper.

In the case of a corporate counterparty, we would need to compute, in a similar way, the expected exposure conditional on corporation but not sovereign default, and then weigh the respective exposures appropriately.

3. **Wrong way risks, in currencies other than the counterparty’s currency.**

The clearest wrong way examples involve deals in which we pay the local currency and receive dollars, as described in the paper. It is important to note, however, that there can be a significant wrong way FX effect even when the counterparty’s currency is not part of the transaction.

Consider for example a forward FX transaction on the currency country Y which takes place between a US bank and country X. If there is a strong dependence of the economy of Y on that of X, the expected value of currency Y will be considerably lower given default of X\(^10\). What we need here is an estimate of the residual value of currency Y given default of X.

One approach involves a simple correlation adjustment to \(RV_X\), the residual value factor for X. The expected reduction in the value of currency X conditional on its default is given by the factor \((1 - RV_X)\). The corresponding expected reduction in the value of

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\(^8\) For corporate counterparties we need to treat each type of default (i.e., whether sovereign has defaulted or only the corporation) separately, and weigh the respective expected exposures appropriately. (Note that the sovereign might not drive a corporation to default on its local debt, and thus the weights might be different than in the *Risk* Paper.)

\(^9\) Where appropriate we determine expected exposure to the counterparty on a netted basis.

\(^10\) We also need to consider cases in which there is no economic dependence between the two countries. We have seen recent situations in which the market reacts to severe problems in one emerging market credit by increasing its risk premium for all emerging market credits. This will then have an overall effect on emerging market currencies, credit spreads and interest rates.
currency $Y$ is this value multiplied by the correlation between the two currencies, and the ratio of the currency volatilities. The result is then

$$1 - RV_Y^X = \rho \frac{\sigma_{FX}^Y}{\sigma_{FX}^X} (1 - RV_X^X) \quad RV_Y^X = 1 - \rho \frac{\sigma_{FX}^Y}{\sigma_{FX}^X} (1 - RV_X)$$

where:

- $RV_Y^X$ - Residual value factor for the currency of $Y$ conditional on the default of $X$.
- $\sigma_{FX}^X$ - Annualized volatility of the value of currency $X$ with respect to dollars.
- $\rho$ - Correlation of the currencies with respect to dollars.

**Example:** If $RV_X = 40\%$, $\rho = 25\%$, and $\sigma_{FX}^Y = \sigma_{FX}^X$, then

$$RV_Y^X = 1 - (0.25)(1)(1-0.4) = 85\% .$$

Another approach uses the Merton based methodology we used in the paper to determine corporate residual value factors. Here we treat a sovereign default as if it were based on a movement of asset value. Let $Q_X(T)$ denote the cumulative default probability for the country over the time horizon $T$ (e.g., 4 years as in the paper). $Z_X$ denotes the size of the “asset” move, in units of standard deviations, which will drive $X$ into default. As in the paper we have: $Z_X = N^{-1}(Q_X(T) / 2)$. (Keep in mind that $Z_X$ is negative.) In addition we assume a correlation of 1 between the assets of $X$ and its currency. The expected size of the move in the currency of $Y$, in units of standard deviations, given default of $X$ is then just $\rho Z_X$. The expected size of this move in percent terms is then:

$$RV_Y^X = 1 - \rho Z_X \sigma_{FX}^Y \sqrt{T}$$

When the counterparty is a corporate rather than a sovereign, the knowledge of its default will, in general have an impact on the expected value of currency $Y$. One can follow the treatment of corporates, replacing the correlation between the corporate and its own currency ($X$) as well as the volatility $\sigma_{FX}^X$, by its correlation with currency $Y$, and the volatility $\sigma_{FX}^Y$. One possible way to come up with a correlation, is to multiply the correlation of the corporate and the dollar value of currency $X$ by the correlation between the currencies.

**4. Residual value factors as measured in different currencies:**

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11 It is important to note that the relevant correlation here is correlation in time of stress (i.e., sovereign default). The historical correlation, which is based on co-movement over calmer periods, may be of limited validity here.

12 This is effectively assuming that the correlation between the corporate and currency $Y$, is only due to the correlation between currencies $X$ and $Y$, and the correlation between currency $X$ and the corporation. If the corporate is an exporter where most of its revenue comes from country $Y$, such an assumption will not be valid.
Until now we discussed the relationship between a corporate or a sovereign and the value of a currency with respect to US dollars. In particular the study “Currency risk associated with sovereign defaults” (see www.jpmorgan.com/businesses/deres), analyzed the residual value of defaulted currencies, relative to US dollars.

Consider a FX trade in which country X receives its own currency and pays the currency of country Y. In the event of X defaulting the expected adjustment to its own currency, as measured in dollars, is based on $RV_X$, while the expected adjustment to the other currency is based on $RV_Y$. The residual value factor for currency X in units of the currency of Y is then estimated by: $RV_X / RV_Y$.

A second point worth mentioning is that since the values in the default study may not apply directly to high rated sovereigns with substantial economies. The default of such a sovereign will typically shock the world economy (US included). It is conceivable that in those cases the residual value of the defaulting currency will be higher than the one in the study since all economies will become weaker, and thus devaluation of the defaulting currency as measured in another currency will not be as severe.

5. Bootstrapping for risk-neutral default probabilities:
Here we describe the iterative technique for deriving risk neutral default probabilities, $q(t)$, from the combination of market rates and a recovery rate assumption. As described in the paper, $q(t)$ is the probability of surviving to time $t-1$ and then defaulting between $t-1$ and $t$. We assume that we are given the following:

- $Z(t)$ - the riskless discount factor for time $t$
- $C(T)$ - the risky par coupon for maturity $T$ per payment period
- $RR$ - the recovery rate

We also assume that if default occurs between $t-1$ and $t$ that the recovery rate is paid on the notional amount at time $t$. We take the notional amount to be equal to 1. The method below can easily be extended to handle more general assumptions.

$T=1$. For a maturity of 1 we have the following expression:

$$[1 + C(1)]Z(1)[1-q(1)] + Z(1)RRq(1) = 1$$

That is, the sum of the discounted no-default and default cash flows must add up to par. Solving for the default probability we get:

$$q(1) = [1 +C(1) - 1/Z(1)]/[1 +C(1) - RR]$$

$T>1$. The expression for the cash flows of a par bond of maturity $T$ is:

13 This is one of several possible assumptions. The methodology can be generalized to handle other assumptions as well.
(1) \[ C(T) \left[ \sum_{t=1}^{T-1} Z(t) * (1 - Q(t)) \right] + \left[ 1 + C(T) \right] Z(T) * [1 - Q(T)] + RR \sum_{t=1}^{T} Z(t) * q(t) = 1 \]

As described in the Risk paper, \( Q(t) \) is the probability that default occurs prior to \( t \).

The first term represents the scheduled payments prior to maturity (i.e., the coupons through to time \( T-1 \)). The second term represents the scheduled maturity payment (principal plus coupon). The last term represents the recovery payment upon default.

If define \( S_1(T) = \sum_{t=1}^{T} Z(t) * [1 - Q(t)] \) and \( S_2(T) = \sum_{t=1}^{T} Z(t) * q(t) \)

we can rewrite (1) as

\[ C(T) * S_1(T-1) + [1 + C(T)] * Z(T) * [1 - Q(T)] + RR * S_2(T) = 1 \]

If we substitute \( Q(T-1) + q(T) \) for \( Q(T) \) and \( S_2(T-1) + Z(T)*q(T) \) for \( S_2(T) \) we can rewrite this expression as:

\[ C(T) * S_1(T-1) + [1+C(T)] * Z(T) * [1-Q(T-1)-q(T)] + RR * S_2(T-1) + RR * Z(T) * q(T) = 1 \]

By induction, the only unknown expression in this equation is \( q(T) \). Solving, we get:

\[ q(T) = \frac{C(T) * S_1(T-1) + [1+C(T)] * Z(T) * [1-Q(T-1)] + RR * S_2(T-1) - 1}{Z(T) * (1+C(T) - RR)} \]

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