MONTE CARLO SIMULATION OF ECONOMIC CAPITAL REQUIREMENT & DEFAULT PROTECTION PREMIUM

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The paper presents a simulation framework for measuring and managing the default risk of a loan portfolio. Through the dependency of counterparty default on a systematic risk factor, we explore the economic capital requirement for a hypothetical credit portfolio. The study employs bivariate standard normal distribution for mapping asset return correlations into default correlations. Monte Carlo simulations are employed to approximate the loss distribution and estimate various risk measures. The analysis performed shows that the Asymptotic Single Risk Factor (ASRF) model is a fast way for generating heavy tailed credit loss distributions. Furthermore, we report complete analytic derivation of Basel II-IRB risk weight functions. The paper also comments on the pricing of single-period Portfolio Default Swaps.

Keywords: Credit VaR, Economic Capital, Expected Shortfall, Portfolio Default Swap

JEL classification codes: G13, G21

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Introduction:

Credit risk is the dominant source of risk for commercial banks and also the subject of extensive research over the past few decades. It is typically defined as the risk of loss resulting from failure of borrowers to honor their payments. Recent advances in credit risk analytics have led to the proliferation of a new breed of sophisticated portfolio risk models. These models play an increasingly important role in banks' risk management strategy. The calculation of potential losses and of the required capital cushion is imperative in the competitive lending business since capital is one of the most expensive resources. In contrast to VaR models for market risks, credit risk models focus in general on analysis of the effects of credit rating transitions on changes in the value of a credit portfolio. The portfolio approach incorporates concentration risk through the modeling of default correlations.

The idea that a bank's capital should be related to the "riskiness" of its assets enjoys widespread support. This idea underlies many banks' internal decisions about capital and is central to the current proposals for reform of the Basel Capital Accord. The June 2004 document of the Basel Committee on Banking Supervision (BCBS), *International Convergence of Capital Measurement and Capital Standards: a Revised Framework*, follows a series of three consultative papers on the New Basel Capital Accord (Basel II) stretching back to 1999.The Accord proposes two basic approaches for computing capital requirements for credit risk, viz, the Standardised Approach and the Internal Ratings-Based (IRB) Approach (further divided into Foundation IRB and Advanced IRB).The capital requirement calculated on the basis of any of these two approaches is termed as *Regulatory Capital*. On the contrary, *Economic Capital* is usually defined as the capital level that is required to cover the bank's future losses at a given confidence level. Quite analogous to market risk VaR models, internal credit risk models & Monte Carlo simulations are employed for estimating the economic capital requirement. It is generally assumed that it is the role of loan-loss reserves and provisions to cover *Expected Losses*, whereas, bank capital should cover *Unexpected Losses* incurred over and above the expected loss level. Thus, required economic capital is the amount over and above expected losses necessary to achieve the *Target Insolvency Rate* (0.1% under IRB Approach)[1]. In Chart 1, for a target insolvency rate equal to the shaded area, the required economic capital equals the distance between the two lines. In practice, the target insolvency rate is usually chosen to be consistent with the bank's desired credit rating. For example, if the desired credit rating is AA, the target insolvency rate should equal the historical one-year average default rate observed for AA-rated firms. Economic capital determination also forms the basis for computing the *Risk-Adjusted Return On Capital (RAROC)* for the bank. A stated goal of the New Basel Accord is to keep the overall level of capital in the global banking system from changing significantly, assuming the same degree of risk; however, that does not mean that the capital levels of each bank will remain unchanged (Saidenberg & Schuermann,2003).The Accord also attempts to end the *Regulatory Capital Arbitrage*, in which the most sophisticated institutions shift exposures off their balance sheets to avoid an overly stringent regulatory capital charge.

With this backdrop, the present study may essentially be considered as a simulation exercise in portfolio credit risk modeling. A key fact that we establish at the

[1]BCBS increased the confidence level in risk weight functions from 0.995 to 0.999 in November 2001.

outset is that the simulation framework presented in this paper can very well be generalized for internally-rated actual bank portfolio, provided, the bank has collected data on key borrower and facility characteristics. In the absence of such bank proprietary data, we hereby attempt to determine the economic capital required, over 1-year horizon, against a hypothetical credit portfolio with the following stylized features;

a) The portfolio comprises of 500 unsecured corporate exposures

b) Exposure at Default (EAD) per exposure is one rupee

c) Loss Given Default (LGD) per exposure is assumed equal[2]

d) The exposures are assigned CRISIL's facility-wise long-term ratings (AAA, AA, A, BBB, BB, B and C)

e) The distribution of 500 exposures across various rating grades is based on CRISIL's 1-year average transition matrix for the period 1992-2005 (refer Tables 1,2 & Chart 2)

f) Default-Mode paradigm is employed for scenario generation[3] because absence of credit curve in India obstructs the MTM framework.

The study employs bivariate standard normal distribution[4] for translating asset return correlations into default correlations. It offers a comparative anatomy of two especially influential returns simulation techniques, viz, Asymptotic Single Risk Factor model (Vasicek,1987) and J.P.Morgan's RiskMetrics algorithm to simulate correlated normal random variables based on Singular Value Decomposition (SVD). These simulations

[2]Under the IRB Foundation approach, senior unsecured claims on corporates are assigned 45% LGD and subordinated unsecured claims are assigned 75% LGD. In the presence of collateral, LGD is adjusted downwards based on Collateral Adjustment formula. However, in this paper, Credit VaR has been computed based on three different *constant LGD rates* (45%, 70% & 100%). The results of Bakshi et al. (2001) indicated that a model with a stochastic recovery rate performs equally well as a model with a constant recovery rate. Also note that the words Obligors, Exposures and Assets have been used interchangeably.

provide the vital structure for generating credit portfolio losses. Based on the loss distributions, Credit Value at Risk (VaR) and economic capital requirement at 99.9% confidence level is worked out. The results are subsequently evaluated against the capital required under IRB Approach of Basel II Accord. The paper also sheds some light on the issue of credit risk transfer. It comments on the cost of hedging the default risk of the simulated portfolio through the use of single–period Portfolio Default Swaps (PDS). Assuming another restricted portfolio of 50 assets, the capital multiplier (CM) implied under Internal Analytical Model at $VaR_{99.9\%}$ has been calibrated as well. In Appendix 1, we produce entire analytic derivation of IRB risk weight functions. The key contribution of this paper is that it provides a statistically rigorous and easily interpretable method for computing asset correlations between any two rated obligors, which in turn facilitates modelling of default correlations and simulation of portfolio losses. In addition, the method can straightforwardly compute correlations for banks' internally rated SME exposures and other non-traded firms, for which multifactor correlation approach (eg. CreditMetrics & KMV) is impracticable.

The layout of the paper is as follows; Section 2 provides literature review, Section 3 sets out the modelling framework for generating loss distribution & pricing of Portfolio Default Swaps, Section 4 reports the results and Section 5 concludes.

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3 Banks generally adopt either a Default-Mode (DM) paradigm or a Mark-to-Market (MTM) paradigm for defining credit losses. Default-Mode is sometimes called a "binomial" model because only two outcomes are relevant: non-default and default. It ignores the gains or losses arising due to rating changes. As against this, Mark-to-Market paradigm incorporates the impact of changes in the creditworthiness of borrowers, short of default. J.P.Morgan's *CreditMetrics* is based on MTM framework, whereas, Credit Suisse Financial Product's *CreditRisk*$^{+}$ is DM driven.

4 Bivariate Normal CDF add-in for MS Excel is freely downloadable on the internet.

2) Literature Review:

Over the past several years, number of researchers have examined the modelling of Credit VaR and pricing of credit derivatives. Several papers addressing these issues are available on the internet. However, the received literature has mainly been located in the context of developed economies. See for example, Koyluoglu, Hickman (1998),Nishiguchi, Kawai, Sazaki(1998), Jones and Mingo(1998), Ong (1999), Finger (1999), Schönbucher (2000), Wilde (2001) and Xiao (2002) for calibration of economic capital requirement. Similar studies undertaken within emerging economies are virtually non- existent. BCBS, in its April 1999 publication, reported the results of the survey undertaken of the current practices and issues in credit risk modeling. The survey was based on the modelling practices at 20 large international banks located in 10 countries. It identified data limitations and methodological inconsistency as two major shortcomings of the current modeling practices. Also refer the survey results of the US Federal Reserve Task Force on Internal Credit Risk Models (May 1998). In July 2005, BCBS released an explanatory note on the Basel II IRB risk weight functions. It focuses on explaining the Basel II risk weight formulas in a non-technical way by describing the economic foundations as well as the underlying mathematical model and its input parameters. Procyclicality of economic capital is yet another issue that comes up a great deal in the discussions on IRB approach. Lowe (2002) and Allen, Saunders (2003) offer an excellent literature review on the same. They forcefully argue that a system of risk-based capital requirements is likely to deliver large changes in minimum requirements over the business cycle, particularly if risk measurement is based on market prices. Carlos, Céspedes (2002),Jacobs (2004), d-Fine Consulting (2004) and Cornford (2005) explore the issues and challenges in implementing Basel II Accord. They expect the implementation to be a large-scale exercise, making major demands on bank supervisors and requiring extensive technical assistance, especially for developing countries.

Default correlations play a crucial role in virtually all fields of credit risk analysis. Many papers have explored and grappled with the complexities of correlated defaults. For empirical evidence on modelling of joint defaults and firms' correlations, refer Li (2000), Servigny, Renault (2003), Dullmann, Scheule (2003) and Hahnenstein (2004). Lopez (2002) assesses the validity of the assumption of inverse relationship between asset correlation and default probability underlying the IRB capital rules. Bangia, Diebold, Schuermann (2000) and Fraser (2000) discuss the basic ideas in stress testing credit portfolio models. Ranciere(2001) and Minton, Stulz, Williamson (2005) assess the credit derivatives market structure in emerging economies and US. For structuring and pricing of these products, refer Schönbucher (2003), Bomfim (2005) and Elizalde (2005). O'Kane, McAdie (2001) and Houweling, Vorst (2005) present empirical evidence on the pricing of credit default swaps. Their findings fully document the occasional mispricing and arbitrage opportunities that arise in this market.

3) Modelling Framework:

This section is divided into three sub-sections. The first section supplies the necessary theoretical context to our exercise, the second section illustrates the simulation framework, and the third assesses estimation of single-period *Portfolio Default Swap* (PDS) Premium.

(3.1) 'Asymptotic Single Risk Factor' Model:

We present below the modelling framework for simulating credit portfolio losses. While being analytically tractable, it builds on the seminal work of Merton (1974) and Vasicek (1987). Vasicek model leads to simple analytic asymptotic approximation of loss distribution and the VaR. The approximation works very well when the portfolio is of large size and there is no exposure concentration, i.e., the portfolio is not dominated by a few loans. In this model, the end-of-period borrower's state (Default or No-default) is driven by an unobserved latent random variable R_i. In particular, for a hypothetical firm i, default occurs if the firm's asset return R_i falls below a given threshold C_i at a given time t. Although R_i is unobservable, the firm's asset value V_i may be derived through the application of Merton model (Kulkarni *et al*, 2005, for Indian study). Hence, firm i defaults at time t if and only if,

$$R_{i,t} < C_{i,t} \qquad \text{……….1}$$

The returns have been normalized, thus,

$$R_{i,t} \equiv \frac{\dot{R}_{i,t} - \mu_i}{\sigma_i} \qquad \text{……….2}$$

where $\dot{R}_i$ are "raw" non-standardized returns, whereas, μ_i & σ_i are the mean and standard deviation of $\dot{R}_i$.

$$R_{i,t} \sim N(0, 1) \qquad \text{……….3}$$

Note that, by working with standardized returns, all the firms in the portfolio have the standard normal return distribution. The unconditional default probability for any firm is given by,

$$W_i = \Pr(R_i < C_i) = N(C_i) \qquad \text{……….4}$$

where N(.) is the cumulative standard normal distribution function.

The unconditional default probability W_i may very well be approximated on the basis of average 1-year default rate reported by agency rating transition matrix. Based on W_i, the return thresholds C_i can be extracted through equation 4,

$$C_i = N^{-1}(W_i) \qquad \text{……….5}$$

Asymptotic Single Risk Factor (ASRF) model assumes that the firm's asset returns are function of a single systematic factor X and a firm-specific idiosyncratic factor ε_i

$$R_{i,t} = \sqrt{\beta_i} X_t + \sqrt{1-\beta_i}\,\varepsilon_{i,t} \qquad \text{……….6}$$

The systematic factor could represent the state of the economy. It affects all the firms in the portfolio through the sensitivity coefficient $\sqrt{\beta}$ .As β increases, the returns are principally influenced by X; as β decreases, idiosyncratic risk prevails. In the limit, as β tends to zero, the returns are completely driven by the firm-specific risk factor ε_i. To be consistent with the definition of R_i as a standard normal random variable, X and ε_i are standardized as well; they have zero mean and unit variance.

$$X_t \sim N(0,1) \qquad \text{……….7}$$

$$\varepsilon_{i,t} \sim N(0,1) \qquad \text{……….8}$$

We further assume that X_t and $\varepsilon_{i,t}$ are uncorrelated and that they are serially independent; hence,

$$Cov(X_t, \varepsilon_{i,t}) = Cov(X_t, X_{t-s}) = Cov(\varepsilon_{i,t}, \varepsilon_{i,t-s}) = 0 \qquad \text{……….9}$$

It is easy to verify that equations 6 to 9 jointly establish equation 3. In light of equation 6, we can show that β plays a crucial role in capturing the degree of asset return correlation between any two given firms. The unconditional correlation, i.e., the correlation coefficient independent of the value taken by the systematic factor X, and the unconditional covariance between returns R_i & R_j can be expressed as,

$$Corr(R_{i,t}, R_{j,t}) = Cov(R_{i,t}, R_{j,t}) = \sqrt{\beta_i}\sqrt{\beta_j} \qquad \text{……….10}$$

Therefore, the degree of asset correlation is determined by the sensitivity of R_i to the systematic factor X. In summary, a low correlation implies that the credit problems faced by borrowers are largely independent of each other; they are due to the idiosyncratic risk a particular borrower is exposed to. On the other hand, a higher asset correlation would suggest that credit difficulties occur simultaneously among borrowers in response to a systematic risk factor, such as general economic conditions. The conditional mean, variance and covariance of the asset returns, given any specific realization of X are,

$$E(R_{i,t} | X_t) = \sqrt{\beta_i} X_t \qquad \text{……….11}$$

$$Var(R_{i,t} | X_t) = 1 - \beta_i \qquad \text{……….12}$$

$$Cov(R_{i,t}, R_{j,t} \mid X_t) = 0 \qquad \text{……….13}$$

Equation 13 implies,

$$Corr(R_{i,t}, R_{j,t} \mid X_t) = 0 \qquad \text{……….14}$$

The conditional default probability for a firm may be written as,

$$W_i(X_t) = \Pr\left(R_i < C_i \mid X_t\right) = \Pr\left(\varepsilon_{i,t} < \frac{C_i - \sqrt{\beta_i} X_t}{\sqrt{1-\beta_i}} \middle| X_t\right) \qquad \text{……….15}$$

wherein, X_t is a realized value.

The unconditional probability of joint default between firms i and j is,

$$W_{i\&j} = \Pr\left(R_i < C_i \;\&\; R_j < C_j\right) = \hat{N}\left(C_i, C_j, \sqrt{\beta_i}\sqrt{\beta_j}\right) \qquad \text{……….16}$$

where, $\hat{N}(.)$ is the cumulative distribution function of the bivariate standard normal distribution. After developing this view, from Schonbucher (2000), we can now write out the default correlation between firms i and j as,

$$\rho_{ij} = \frac{\hat{N}\left(C_i, C_j, \sqrt{\beta_i}\sqrt{\beta_j}\right) - N(C_i)N(C_j)}{\sqrt{N(C_i)(1-N(C_i))}\sqrt{N(C_j)(1-N(C_j))}} \qquad \text{……….17}$$

Thus, default correlation is a function of default thresholds C_i and asset correlation $\sqrt{\beta_i}\sqrt{\beta_j}$. The CreditMetrics methodology constructs estimates of the firms' individual asset correlations from market indices by means of a multifactor model. Within this approach, both country and industry weights are assigned to each firm according to its *Degree of Participation*. For a portfolio comprising of n obligors, we require empirical estimates of n (n – 1) / 2 pairwise asset correlations. This produces 4950 asset correlation estimates for a meager 100-asset portfolio; however, almost all the commercial banks hold credit portfolios reasonably larger than this. Moreover, the problem of using equity correlations as a proxy for asset correlations has already been recognized as a potential drawback by the model's inventors themselves. It has recently been attacked on theoretical and empirical grounds by Zeng and Zhang (2002) of KMV. Besides, Basel II doesn't truly encourage full correlation modelling, such as Creditmetrics and KMV multifactor models, and neither does it insist on banks to generate their own internal estimates of asset correlations .This is essentially due to both the technical challenges involved in reliably deriving and validating these estimates for specific asset classes and the desire for consistency. Owing to these shortcomings, multifactor correlation modelling seems ill suited for the present study. Moreover, the study being purely a simulation exercise, and not based on actual credit portfolio data and equity prices, multifactor approach is infeasible as well. We therefore employ the Basel II IRB, June 2004 formula itself for estimating asset correlation based on average (unconditional) default probability of various rating grades. This formula is grounded primarily on the empirical studies undertaken by the Basel Committee. The credibility of this approach has a critical bearing on the following two assumptions: first, all firms within the same rating class have the same default rate, and hence, the same asset correlation (R). And second, that the asset correlation is inversely related to probability of default (PD). Intuitively, this can be explained as follows: the higher the PD, the higher the idiosyncratic risk component of a borrower. And so, the default risk depends less on the overall state of the economy and more on individual risk drivers.

$$R = 0.12\left(\frac{1-e^{-50*PD}}{1-e^{-50}}\right) + 0.24\left(1-\frac{1-e^{-50*PD}}{1-e^{-50}}\right) \qquad \text{..........18}$$

Basel II risk weight functions implicitly assume an *infinitely-granular homogeneous portfolio,* wherein, all the obligors falling within a given risk-class have similar PD. Thus, from equation 18, all such obligors are assigned similar correlation coefficient. R is an input in the IRB risk weight function, which itself has its origin in Vasicek's homogeneous portfolio framework. Consequently, *R may be interpreted as the asset correlation between any two firms falling in the same rating-bucket.* As an illustration, let the given risk bucket comprise of N homogeneous obligors. All such obligors hold similar credit rating and PD. From equation 10, the asset correlation between obligor N1 and any other obligor Ni (i≠1) may be written as,

$$Corr_{N1,Ni(i\neq 1)} = \sqrt{\beta_{N1}}\sqrt{\beta_{Ni(i\neq 1)}} \qquad \text{……….19}$$

Since the obligors belong to the same risk category, from equation 18, they have equal asset correlation coefficient,

$$\therefore R_{N1} = R_{Ni(i\neq 1)} = R \qquad \text{……….20}$$

Being homogeneous, they possess similar sensitivity ($\sqrt{\beta}$) to the systematic risk factor X.

$$\therefore \sqrt{\beta}_{N1} = \sqrt{\beta}_{Ni(i\neq 1)} = \sqrt{\beta} \qquad \text{……….21}$$

From equations 19, 20 & 21,

$$Corr_{N1,Ni(i\neq 1)} = \sqrt{\beta}\sqrt{\beta} = \beta = R \qquad \text{……….22}$$

$$\therefore \sqrt{\beta} = \sqrt{R} \qquad \text{……….23}$$

On similar lines, the asset correlation between any two firms i and j representing two different rating grades, say A and B respectively , may be expressed as a product of their sensitivity coefficients,

$$Corr_{Ai,Bj} = \sqrt{\beta_{Ai}}\sqrt{\beta_{Bj}} \qquad \text{……….24}$$

where, $\sqrt{\beta_{Ai}} = \sqrt{R_{Ai}}$ ……….25

$$\sqrt{\beta_{Bj}} = \sqrt{R_{Bj}} \qquad \text{……….26}$$

In this paper, since 500 assets have been allocated over seven different rating grades, we get 28 distinctive ratings-based asset correlation estimates between any two exposures in the portfolio; this is inclusive of the seven estimates for exposures with similar ratings (refer diagonal elements of Table 4). These correlations provide the foundation for simulating portfolio losses.

(3.2) Determination of Economic Capital Requirement:

Basel II document provides the risk weight functions for calibrating the capital requirement under IRB approach. The output is dependent on the estimates of PD, LGD, EAD, asset correlations, effective maturity & sales turnover (Refer Appendix 1). The resulting capital requirement is termed as *Regulatory Capital*. On the contrary, determination of *Economic Capital* necessitates developing an internal model for simulating credit portfolio losses. In this paper, the two distinct simulation techniques that we look into are the ASRF model (equation 6) and RiskMetrics (1996) algorithm to simulate correlated normal random variables based on Singular Value Decomposition (SVD) of asset returns covariance matrix.

(3.2.1) Simulations based on ASRF model:

Asset return simulation, being a prerequisite for generating portfolio loss distribution, is performed through the following steps;

Step 1) Assign values to the key model parameters, C_i & $\sqrt{\beta_i}$. For instance, Equation 25 gives value of $\sqrt{\beta_i}$ for each rating grade, whereas, equation 5 gives C_i.

Step 2) For each scenario, generate 501 standard normal random values representing one draw for the systematic factor X & 500 draws for the idiosyncratic factor ε_i per exposure. This translates into total 32.8155 million random draws for the 500 assets portfolio over 65,500 scenarios.

Step3) Through equation 6, compute asset return R_i for each exposure (per scenario) & compare it against threshold C_i to record the occurrence of default.

Step4) Once total defaults have been counted and recorded over each scenario generated, the portfolio Expected Loss (EL_P) and frequency distribution for the *Total Defaults per Scenario* (0 to 500) is computed. This necessitates construction of "bins" per default loss-size. Actual loss distributions are estimated assuming three different constant LGD rates, i.e., 100%, 70% and 45%.

Step 5) Based on the frequency distribution of simulated losses, VaR at 99.9% confidence level is worked out. This figure goes into the estimation of Economic Capital.

$$EC = VaR_{99.9\%} - EL_P \qquad \text{..........27}$$

The output from the above equation is subsequently evaluated against the capital required under IRB Approach of Basel II Accord.

Expected Shortfall (ES):

In the literature on risk management, VaR has shown itself to be a very useful risk measure. However, it is also now widely accepted that VaR is not the finest risk measure available (Acerbi, Tasche(2002); Rockafellar,Uryasev(2002)). VaR is often criticized for its failure to reflect the severity of losses in the worst scenarios in which the loss exceeds VaR. In other words, VaR is not a measure of the heaviness of the tail of the distribution. A novel theoretical development in recent years is the Expected Shortfall (ES), a measure that is sometimes also known as Conditional VaR or Tail VaR . ES is the expected value of the losses, L, if we get a loss in excess of VaR. The VaR tells us the most we can expect to lose if a bad (i. e., tail) event does *not* occur. However, in direct opposition to this view, ES tell us what we can expect to lose if a tail event *does* occur. It reflects the losses incurred in the tail region (beyond VaR) of the loss distribution. ES is preferable to VaR because it satisfies the highly desirable properties of coherence, which the VaR, in general, does not. The presence of *heavy tailed loss distribution* significantly elevates the ratio of ES to VaR. Thus, ES being more conservative a risk measure than VaR, we do report it in the paper.

$$ES_{\alpha}(L) = E\left[L \middle| L \geq VaR_{\alpha}\right] \qquad \text{.........28}$$

(3.2.2) Simulations based on RiskMetrics SVD Algorithm:

RiskMetrics Document, in its Appendix E, introduced three algorithms for simulating correlated normal random variables from a specified covariance matrix ∑; it is square and symmetric. The algorithms were Cholesky Decomposition (CD), Eigenvalue Decomposition (ED) and the Singular Value Decomposition (SVD). CD is efficient when ∑ is positive definite. However, CD is not applicable for positive semi-definite matrices. ED and SVD, while computationally more intensive, are useful when ∑ is positive semi-definite. The decomposition is necessary in order to reproduce the original matrix ∑ from the simulated returns; the reproduced covariance matrix ∑* should closely match original matrix ∑. Further discussion of this issue is available in Ong (1999) and d Fine (2004). In the present study, we employ SVD for simulating correlated asset returns[5]. *However, for the sake of brevity, the base portfolio size has been restricted only to 10% of the original portfolio, i.e., total 50 exposures allocated across the seven rating grades (refer Table 14).* Economic Capital estimated through SVD algorithm is further compared against similar estimate obtained under ASRF model simulation framework of Section (3.2.1) for the restricted portfolio. The capital multiplier (CM) implied under Internal Analytical Model at $VaR_{99.9\%}$ is calibrated as well for this 50-asset portfolio (refer Appendix 2).

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[5] Results on matrix decomposition available on request from the author.

(3.3) Estimation of single-period PDS Premium.

Credit derivatives permit the transfer of credit risk without the need to liquidate positions in the underlying instruments. Internationally, the rapidly expanding credit derivatives market has created a large set of instruments for credit risk management. An increasingly popular product is the Credit Default Swap (CDS), wherein, one party purchases credit protection from another to cover the credit loss of an asset following a credit event. The credit default premium is paid at periodic intervals throughout the swap tenor. Since banks hold very large loan portfolios, they would find it rather economical to buy protection through a single Portfolio Default Swap (PDS) as against multiple CDS. The reason for this belief is essentially the default correlation structure embedded in the credit portfolio; the correlation structure provides diversification benefits to the counterparties. PDS offers protection for a given fraction (loss tranche) of the total portfolio value. The credit losses incurred beyond the tranche are borne by the bank itself[6].

Estimation of protection premium entails modelling various credit risk parameters, such as, PD, LGD & default correlation. The hazard rate approach, based on the seminal work of Jarrow and Turnbull (1995), has gained prominence in credit derivatives literature. This approach employs market data on credit spreads or CDS spreads for calibrating Protection Premium. However, the absence of CDS market, coupled with illiquid corporate bond market in India (Sharma & Sinha, 2005) pre-empts

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[6] In case, the protection is bought only against any first default in the portfolio, the default swap is termed as 'First-to-Default' Swap. On similar lines, one can structure 'Second-to-Default', 'Third -to-Default' or any 'Nth -to-Default' swap. Also note that RBI had issued draft guidelines on introduction of credit derivatives in India in March 2003.

the possibility of employing hazard rate approach in this study. Therefore, the premium has to be derived through the simulated loss distribution of protected tranche. This distribution is flat at the tranche-boundaries owing to the possibility of portfolio loss exceeding the loss tranche, and this feature becomes more pronounced when the tranche is narrowed. Ours is only a one-period approach and it assumes that defaults occur only at the year-end (this simplifies the treatment of accrued premium). It ignores the survival probability of individual credits beyond one year. In other words, the PDS doesn't offer default protection beyond one-year. We further assume away counterparty credit risk, or that, it gets managed through collateral requirement. For the 500 assets portfolio, we report First-Loss Protection Premium over eight different First-Loss Tranches. However, Second-Loss Tranche is prefixed to the size of twenty defaults, observed beyond each of the First-Loss Tranches (Refer Tables 19 & 20).

The methodology employed for pricing the single-period PDS is as under,

Step1) Simulate the ASRF model-based loss distribution (100%LGD) for the 500 assets portfolio.

Step2) Subject to the size of First-Loss Tranche, allocate the losses to the First-Loss Protection Seller.

Example: The size of First-Loss Tranche is 30 defaults. For each default-bin from 1 to 30, allocate the entire value of actual defaults observed & losses incurred to the First-Loss Protection Seller. However, when defaults exceed 30 credits, the First-Loss Protection Seller is to bear losses only to the extent of 30; the excess being borne by the Protection Buyer himself or by the Second -Loss Protection Seller (if any).

Step 3) Generate loss distribution for the First Loss Tranche

Step 4) Compute expected value of the losses to be incurred by the First-Loss Protection Seller

$$EL_{FirstLoss} = \sum_{i=1}^{N} \left(\frac{f_i \times L_i}{\sum f} \right) \qquad \text{……….29}$$

where, L_i depicts the losses allocated to the First Loss Tranche (from Step 3) , whereas, f_i represents the frequency statistic for L_i

Step 5) The Protection Premium *payable upfront* to the Protection Seller is computed as a percentage of the size of First-Loss Tranche,

$$\Pr emium = \frac{EL_{FirstLoss} \times D}{\text{SizeofFirst-Loss Tranche,}} \times 100 \qquad \text{……….30}$$

where D is the discount factor[7] acceptable to both the counterparties.

Step 6) Analogous to this, estimation of Second-Loss Protection Premium entails repeating the steps 2 to 5

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[7]For the purpose of computational ease, the discount rate of 10% was employed in the study

4) Results:

Table 1 presents Crisil's 1-year Average Rating Transition Matrix over the period 1992-2005. This matrix is based on *facility-wise* long-term ratings of corporate bonds/debentures[8]. Crisil doesn't factor in the security provided (if any) or expected recovery in its bond ratings. The last column of the matrix reports the average default rate observed for various rating grades. One important fact to be pointed out is that historical default rate is virtually zero for AAA & AA ratings. However, Basel II Accord prescribes a minimum default rate of 0.03% for obligors with the best credit quality. We therefore employ the floor rate of 0.03% for AAA & AA exposures. Also note that the default rate for B rating marginally exceeds that for C. Although this seems counter-intuitive, we take account of this evidence in the analytical framework. From Table 2 & Chart 2, it is evident that the distribution of exposures across various rating grades is skewed towards the investment-grade ratings of AAA, AA, A & BBB. This fact need not truly reflect the actual rating-wise exposure distribution observed at various commercial banks. However, notwithstanding this limitation, we allocate the 500 assets portfolio based on Crisil's rating distribution because any study on rating-wise distribution of exposures at Indian banks is non-existent. Also refer Table 3 for the unconditional (average) default probability & return thresholds computed for various rating grades.

The interrelationship between asset correlation & default correlation being a key objective of this study, Tables 4 to 7 & Chart 3 elaborate on this linkage. In Table 4, we report the asset correlation between any two rated exposures; the correlation coefficient is inversely related to the exposure's PD (refer equation 18 of Section 3). The

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[8]In the absence of data on *Obligor Ratings* in India, *Facility-wise Ratings* was the only alternative available for proceeding with the present study.

diagonal elements, in particular, emphasize this fact. Based on bivariate standard normal distribution, Table 5 reports the estimates for joint default probability (JDP) between the exposures. The figures are relatively high for the exposures rated at lower grades; the JDP seems positively related to the exposures' individual PD levels. On similar lines, from Table 6, default correlation between any two exposures is an increasing function of the individual PD estimates. A related implication is that for any two exposures which are rated alike (diagonal elements), asset correlation & default correlation are inversely related (refer Chart 3). If the results are to be taken at face value, they suggest that at lower ratings & higher individual PD levels, *asset correlation is low, whereas, default correlation is high*. This anomaly may be scrutinized through the following illustration;

Example: Table 7 reports the default correlation between two firms with identical ratings; both are either AAA or BBB. The lower PD for AAA firms translates into higher asset correlation coefficient under IRB approach. On the contrary, for BBB firms, the value is relatively low. From equation 16 of Section 3, JDP is jointly determined by asset correlation coefficient & individual PD levels. Lower asset correlation for BBB rated firms should theoretically produce lower JDP & lower default correlation coefficient, vis-à-vis, AAA rated firms. However, this effect is nullified & largely reversed by the historical default rate of BBB firms, which is quite high. Consequently, JDP & default correlation estimate for BBB firms is higher than that for AAA firms, although the asset correlation is quite low for the former.

Furthermore, the correlation between default probabilities is always less than the correlation between asset values. The IRB risk weight functions employ estimates of asset correlation, & not default correlation, for calibrating capital requirement. Hence, *low rated exposures which carry smaller asset correlation parameter, require higher allocation of bank capital owing to their PD & not due to their higher implied default correlation parameter*. This vital conclusion is the terminus of the above analysis.

Tables 8 to 11 & Chart 4 summarize the results of the ASRF model based simulation exercise performed for the 500 assets portfolio. Credit VaR, Expected Shortfall (ES) & Economic Capital (EC) requirement at 99.9% confidence level is reported at different LGD rates, i.e., 100%, 70% & 45%. Assuming 70% constant LGD, which was incidentally proposed in *S & P Asia-Pacific Banking Outlook 2004* for Indian credit assets, EC requirement works out to 8.516% of total exposure. It declines still further to 5.48% assuming 45% LGD. However, in case the bank intends to attain AA rating at 99.97% confidence level (0.03% default rate from Table 3), it has to maintain capital at 11.363% (70% LGD). To carry the same argument further, presence of *extreme events / fat tail* in the loss distribution manifests itself in the form of significant ES to VaR ratio (refer Charts 5 to 7). In the study, this ratio almost equals 1.54 as reported in Table 13. The reason for this can be traced to the simulation trials having generated scenarios with default sizes as high as 466,468,477,480 & 486 in the 500 asset portfolio. The skewness parameter of 7.022 & kurtosis of 235.46 further underline the presence of fat tail (refer Table 8). Assuming Beta distribution for the percentage credit losses, the calibrated parameters α & β are reported in Table 10. From Table 9, the capital requirement at $VaR_{99.9\%}$ (70% LGD) works out to Rs.42.5845. As against this, the IRB risk weight functions produce capital requirement @ 9% of RWA, of Rs.47.2174 at the same LGD level (refer Table 12). Thus, the simulation exercise does produce sizeable capital savings as against IRB approach. The results need not significantly differ in case the credit portfolio is tilted towards the speculative-grade ratings of BB,B & C[9]. In conclusion, note that the 95%confidence interval around *Expected Loss (EL)* undoubtedly underlines the accuracy of the estimation results[10]. At 70% LGD, the EL of Rs.9.9155 lies within a narrow band of Rs.9.8562 & Rs.9.9747. This result therefore inspires confidence in our estimates. The estimates are evidently free of the *noise* stemming from Monte Carlo simulations.

ASRF model & J.P.Morgan's RiskMetrics SVD algorithm are the two prominent return- simulation techniques available for generating credit loss distribution. A priori, the variation between $VaR_{99.9\%}$ estimates produced under these two approaches need not be substantial. Likewise, the capital requirements should be near identical. The results addressing this issue are produced in Tables 14 & 15, restricting the size of base portfolio to 50 assets & LGD to 100%. As is evident from the results, VaR & subsequent capital requirements are not truly equivalent under the two methods. However, the difference between EL calibrated under these two techniques is statistically insignificant at 95% confidence level. Whereas, the ASRF model generates Expected Shortfall of Rs.11.08, the estimate is Rs.10.05 under SVD algorithm. On this ground, the ratio of ES to VaR equals 1.144 for ASRF & 1.077 under SVD technique. Again, this divergence between the results largely emanates from the *fat tail* produced through ASRF model based simulations. For the 50-asset portfolio, this technique is relatively successful in producing default scenarios as large as 44, 47, 49 & 50. The SVD algorithm, however,

[9]The IRB risk weight function is positively sloped, although relatively flat, for very low quality credits.

[10]CreditMetrics Document, in Appendix B, proposes a methodology to construct confidence intervals around the mean value of simulated portfolio losses. Thus, after generating *N* scenarios, we may say that we are 95% confident that the true mean value of portfolio losses lies between $\mu-1.96(\sigma/\sqrt{N})$ & $\mu+1.96(\sigma/\sqrt{N})$. $\sigma/\sqrt{N}$ is the standard error; the bands will tighten as *N* increases.

falls short of generating any default size larger than 21 & 25. Chart 11, in particular, portrays the tails of these simulated loss distributions. The upshot of this analysis is that the risk measures produced under ASRF model score high on the ground of conservatism. Moreover, given the massive size of credit portfolios which banks actually hold & also the constraints on computing power, ASRF model should be the preferred simulation technique. Besides, decomposition of covariance matrix under SVD could turn out to be highly unwieldy for large portfolios. Even so, inability to generate *heavy tailed distribution* is perhaps the supreme disadvantage of SVD method.

For the 50-asset portfolio, capital multiplier (CM) implied under Internal Analytical Model at $VaR_{99.9\%}$ is reported in Table 16. It equals 5.2540 for ASRF model based VaR, whereas, 5.0186 for RiskMetrics SVD algorithm based VaR. Also note that the capital requirement under Internal Analytical Model is almost equal to the respective estimates from simulation techniques (refer Chart 8). Hence, Monte Carlo simulations seem to reasonably duplicate the Internal Analytical Model results. In the final analysis, Table 18 & Chart 12 shed some light on the benefits of diversification effect embedded in much larger credit portfolios. This fact gets reflected in the ratio of economic capital to total exposure, which is 16.144% for 50-asset portfolio & barely 12.167% for 500 assets portfolio. However, for the 50-asset portfolio, capital requirement of 14.001% calibrated under IRB Approach is far less than that under simulation techniques & Internal Analytical Model (refer Chart 8). But the finding gets reversed for the 500 assets portfolio (refer Chart 4).

In the conventional analysis, credit derivative premium is modelled through loss distribution based on default probabilities & recovery rate of the reference asset/portfolio. An alternative approach, commonly referred to as *reduced-form*, is to rely on the market data on credit spreads of traded corporate bonds. However, the pursuit of this line of research gets seriously circumscribed in our study on account of the fact that Indian private debt market is largely illiquid beyond investment-grade credits. We therefore rely on the former approach of modelling loss distributions for the protection seller over one-year horizon. The results on measurement of single-period *Portfolio Default Swap* premium (First-Loss Tranche & Second-Loss Tranche), receivable upfront by the protection seller, on the 500 assets portfolio (100% LGD) are reported in Tables 19 & 20. In the analysis, First-Loss Tranche was progressively increased as a given percentage of total portfolio, but the Second-Loss Tranche was preset at 20 defaults observed beyond each of the First-Loss Tranches. As is evident from Chart 13, the percentage of First-Loss protection premium declines rapidly with gradual increase in tranch size. Similar are the dynamics for percentage of Second-Loss premium. However, it is interesting to note that the premium value, when denominated in rupees , increases for First-Loss Tranche & declines for Second-Loss Tranche at distant levels (refer Chart 14). A promising candidate for explaining this result is the shape of the loss distribution, which is positively skewed. Consequently, the probability of observing extreme losses beyond EL level are negligible but non-zero. And so, the premium charged for relatively larger First-Loss tranches increases in amount, but declines in proportion. Second-Loss premium however declines on both the counts, because at farther levels, the Second-Loss protection tranche could hardly get invoked. Also refer Charts 15 to 17 for the loss distribution of First-Loss Tranche & Second-Loss Tranche over the given tranche size.

5) Conclusion:

Portfolio credit risk modelling is the tool for estimating required capital reserves against loan portfolio. In this study, Monte Carlo simulation approach was presented for analyzing and measuring credit risk in a number of ways. Given a hypothetical credit portfolio of 500 assets, assuming different LGD rates, ASRF model-based loss distributions were generated. Based on this, Credit VaR and Economic Capital requirement at 99.9% confidence level was worked out. The estimate is 8.516% of total exposure at 70% LGD. The simulation trials generated scenarios with default sizes as high as 466,468,477,480 and 486 in the 500 asset portfolio. Consequently, the ratio of Expected Shortfall to VaR turned out to be quite high, at around 1.54. Keeping in view that credit loss distributions are quite often *heavy tailed*, coupled with the fact that the tail region is most relevant for risk management, the results provide an interesting insight into the ongoing debate on the suitability of VaR as a risk measure. Be that as it may, the simulation exercise did succeed in producing sizeable capital savings as against IRB approach. For various rating grades, the study also mapped asset correlations into default correlations through bivariate standard normal distribution. The analysis concluded that the default correlation is positively related with the firms' PD. However, the function is inverse for asset correlation & PD. Furthermore, the correlation between default probabilities is always less than the correlation between asset values.

The paper further probed into the comparative anatomy of two distinct returns simulation techniques; ASRF model and J.P.Morgan RiskMetrics algorithm to simulate correlated normal random variables through Singular Value Decomposition (SVD). The assessment was done for a restricted portfolio of 50 exposures. The results suggested that the risk measures produced under ASRF model simulations are relatively conservative. Moreover, given the massive size of credit portfolios which banks actually hold and also the constraints on computing power, ASRF model should be the preferred simulation technique. Besides, matrix decomposition under SVD could turn out to be highly unwieldy for large portfolios. Even so, inability to generate *heavy tailed distribution* is perhaps the supreme disadvantage of SVD method. For the 50-asset portfolio, capital multiplier (CM) implied under Internal Analytical Model at $VaR_{99.9\%}$ is around 5.25. Also, Monte Carlo simulations seem to reasonably duplicate the Internal Analytical Model results.

In the concluding section, the paper evaluated the issue of credit risk transfer. It estimated default protection premium for the 500 assets portfolio through the use of single–period Portfolio Default Swaps over two loss tranches, viz., First-Loss and Second-Loss Tranche. The percentage of First-Loss protection premium declined rapidly with gradual increase in tranche size. Similar results were observed for percentage of Second-Loss premium. However, given the premium value in rupees, the cost increased for First-Loss tranche and declined for Second-Loss tranche at distant levels. This result has been attributed to the shape of the loss distribution, which is positively skewed.

Taken as a whole, the analytics presented in this paper elucidate the mechanics of calibrating *Economic Capital* within a simulation framework. Since any study on portfolio credit risk undertaken within developing economies is virtually non-existent, the present study could serve as a modest attempt to fill this gap. We hope that this paper will kindle research interest in a number of other related issues, such as, stress testing credit portfolio models and estimation of multi-period default protection premium.

Tables:

Table 1: CRISIL's 1-Year Average Rating Transition Matrix over 1992-2005 (%)

Sample Size	From\To	AAA	AA	A	BBB	BB	B	C	D
508	AAA	**97.24**	2.76	0	0	0	0	0	**0**
1305	AA	2.45	**89.66**	6.74	0.61	0.38	0.15	0	**0**
1401	A	0	3.78	**82.37**	7.42	4.50	0.21	0.71	**1**
617	BBB	0	0.32	5.67	**73.26**	14.10	1.30	1.94	**3.40**
336	BB	0	0.60	0	1.79	**75.00**	1.79	5.36	**15.48**
34	B	0	0	0	5.88	0	**55.88**	8.82	**29.41**
81	C	0	0	0	1.23	0	0	**70.37**	**28.40**

(Source: "Insight" CRISIL Default Study, April 2006)

Table 2: Rating-wise allocation of 500-assets portfolio

Rating	Sample size (Crisil Matrix)	Actual %	Allocated %	Within 500
AAA	508	11.86%	10%	50
AA	1305	30.48%	30%	150
A	1401	32.72%	35%	175
BBB	617	14.41%	15%	75
BB	336	7.85%	7%	35
B	34	0.79%	1%	5
C	81	1.89%	2%	10
	4282	**100%**	**100%**	**500**

Table3: Unconditional PD & Return Thresholds

Rating	PD	Return Threshold	Asset Correlation (IRB Formula)
AAA	0.03%	-3.432	0.23821
AA	0.03%	-3.432	0.23821
A	1.00%	-2.326	0.19278
BBB	3.40%	-1.825	0.14192
BB	15.48%	-1.016	0.12005
B	29.41%	-0.541	0.12000
C	28.40%	-0.571	0.12000

Table4: Rating-wise *Asset Correlation* (in decimals)

	AAA	AA	A	BBB	BB	B	C
AAA	**0.23821**	0.23821	0.21430	0.18387	0.16911	0.16907	0.16907
AA	0.23821	**0.23821**	0.21430	0.18387	0.16911	0.16907	0.16907
A	0.21430	0.21430	**0.19278**	0.16541	0.15213	0.15210	0.15210
BBB	0.18387	0.18387	0.16541	**0.14192**	0.13053	0.13050	0.13050
BB	0.16911	0.16911	0.15213	0.13053	**0.12005**	0.12003	0.12003
B	0.16907	0.16907	0.15210	0.13050	0.12003	**0.12000**	0.12000
C	0.16907	0.16907	0.15210	0.13050	0.12003	0.12000	**0.12000**

Table 5: Rating-wise *Joint Default Probability (JDP)*

	AAA	AA	A	BBB	BB	B	C
AAA	**0.00000130**	0.00000130	0.00001720	0.00003610	0.00010270	0.00016090	0.00015730
AA	0.00000130	**0.00000130**	0.00001720	0.00003610	0.00010270	0.00016090	0.00015730
A	0.00001720	0.00001720	**0.00032440**	0.00080230	0.00268110	0.00444860	0.00433060
BBB	0.00003610	0.00003610	0.00080230	**0.00216500**	0.00788550	0.01358940	0.01319930
BB	0.00010270	0.00010270	0.00268110	0.00788550	**0.03118520**	0.05569740	0.05398330
B	0.00016090	0.00016090	0.00444860	0.01358940	0.05569740	**0.10101650**	0.09781790
C	0.00015730	0.00015730	0.00433060	0.01319930	0.05398330	0.09781790	**0.09472580**

Table 6: Rating-wise *Default Correlation* (in decimals)

	AAA	AA	A	BBB	BB	B	C
AAA	**0.00403**	0.00403	0.00824	0.00825	0.00898	0.00921	0.00923
AA	0.00403	**0.00403**	0.00824	0.00825	0.00898	0.00921	0.00923
A	0.00824	0.00824	**0.02267**	0.02564	0.03148	0.03325	0.03322
BBB	0.00825	0.00825	0.02564	**0.03072**	0.04000	0.04348	0.04336
BB	0.00898	0.00898	0.03148	0.04000	**0.05520**	0.06171	0.06143
B	0.00921	0.00921	0.03325	0.04348	0.06171	**0.06995**	0.06957
C	0.00923	0.00923	0.03322	0.04336	0.06143	0.06957	**0.06919**

Table7: Comparative Analysis

	Both firms: AAA	Both firms: BBB
PD	0.03%	3.4%
Asset correlation	0.23821	0.14192
JDP	0.000001300	0.002165000
Default correlation	0.00403	0.03072

Table 8: Descriptive Statistics: Loss distributions (in Rs; EAD: 500)

	100% LGD	70% LGD	45%LGD
Mean	14.16508	9.915559	6.374288
Median	12	8.4	5.4
Mode	8	5.6	3.6
Standard Deviation	11.05378	7.737644	4.9742
Kurtosis	235.4678	235.4678	235.4678
Skewness	7.022826	7.022826	7.022826
Maximum	486	340.2	218.7
Minimum	0	0	0

Table 9: Economic Capital at 99.90% confidence level (500 assets)

	100% LGD	70% LGD	45%LGD
Expected Loss (EL)	14.165	9.9155	6.40
EL Upper Limit: 95% confidence	14.249	9.9747	6.438
EL Lower Limit: 95% confidence	14.080	9.8562	6.361
Credit VaR at 99.90%	75	52.50	33.80
Economic Capital at 99.90%	60.835	42.5845	27.4
Economic Capital (% of 500)	**12.167%**	**8.516%**	**5.48%**

Table 10: Beta Distribution parameters-
Calibrated through the % Loss distributions (as % of 500)

	100% LGD	70% LGD	45%LGD
Mean % loss (μ)	0.02833	0.019831	0.012748
Variance	0.000489	0.000239	0.000098
α	1.567312	1.589768	1.60848
β	53.7557	78.5755	124.5610

$$\alpha = \left[\mu^2 \times (1-\mu) / \sigma_\mu^2\right] - \mu$$

$$\beta = \alpha \times \left[\left(\frac{1}{\mu}\right) - 1\right]$$

Table 11: Economic Capital at 99.97% confidence level (500 assets)
(Target Rating: AA)

	100% LGD	70% LGD	45%LGD
Credit VaR at 99.97%	95.333	66.733	42.933
Economic Capital at 99.97%	81.168	56.8175	36.533
Economic Capital (% of 500)	**16.233%**	**11.363%**	**7.306%**

Table 12: Capital Requirement under IRB Approach (500 assets)

	100% LGD	70% LGD	45%LGD
IRB Approach: **Risk Weighted Assets (RWA)**	749.4838	524.6385	337.2676
IRB Approach: **Capital Required@ 9% of RWA (Rs.)**	67.4535	47.2174	30.3540
Capital Requirement (% of 500)	**13.491%**	**9.443%**	**6.071%**

Table 13: Expected Shortfall (ES) at 99.90% confidence level (500 assets)

	LGD 100%	LGD 70%	LGD 45%
Expected Shortfall	115.53	80.8753	52.0144
Ratio of ES to Credit VaR	**1.54**		

Table 14: Rating-wise allocation of 50 assets portfolio

Rating	Within 50	Allocated %
AAA	5	10.00%
AA	15	30.00%
A	17	34.00%
BBB	7	14.00%
BB	4	8.00%
B	1	2.00%
C	1	2.00%
	50	**100.00%**

Table 15: Simulation results for 50 assets portfolio (ASRF model v/s RiskMetrics SVD Algorithm)

	ASRF	RiskMetrics SVD
Portfolio size	50	50
LGD	100%	100%
Expected Loss (EL)*	1.6168	1.6177
EL Upper Limit: 95% confidence	1.6290	1.6296
EL Lower Limit: 95% confidence	1.6046	1.6058
Credit VaR at 99.90%	9.689	9.327
Economic Capital at 99.90%	8.072	7.709
Economic Capital (% of 50)	16.144%	15.419%
Expected Shortfall	11.08	10.05
Ratio of ES to Credit VaR	**1.144**	**1.077**

*The difference between EL calibrated under the two techniques is *statistically insignificant* at 95% confidence level. The z variate is -0.1001. For test details, refer Chapter 9, Levin & Rubin (2004).

Table 16: Internal Analytical Model (50 assets)

LGD : 100%		
Confidence Level: 99.9%		
Portfolio EL : 1.6113		
Portfolio UL : 1.5374		
	ASRF	**RiskMetrics SVD**
Credit VaR at 99.90%	9.689	9.327
Economic Capital Required	8.0777	7.7157
Economic Capital (%)	16.155%	15.431%
Implied Capital Multiplier	**5.2540**	**5.0186**

Table 17: Capital Requirement under IRB Approach (50 assets)

	100% LGD
IRB Approach: **Risk Weighted Assets (RWA)**	77.78504
IRB Approach: **Capital Required@ 9% of RWA (Rs.)**	7.000654
Capital Requirement (% of 50)	14.001%

Table 18: Capital requirement as % of EAD (LGD: 100%)

	EAD: Rs. 500	**EAD: Rs. 50**
ASRF model simulations	12 .167%	16.144%
RiskMetrics SVD simulations	------	15.419%
IRB Approach	13.491%	14.001%

Table19: *First-Loss Tranche* Premium (100% LGD)

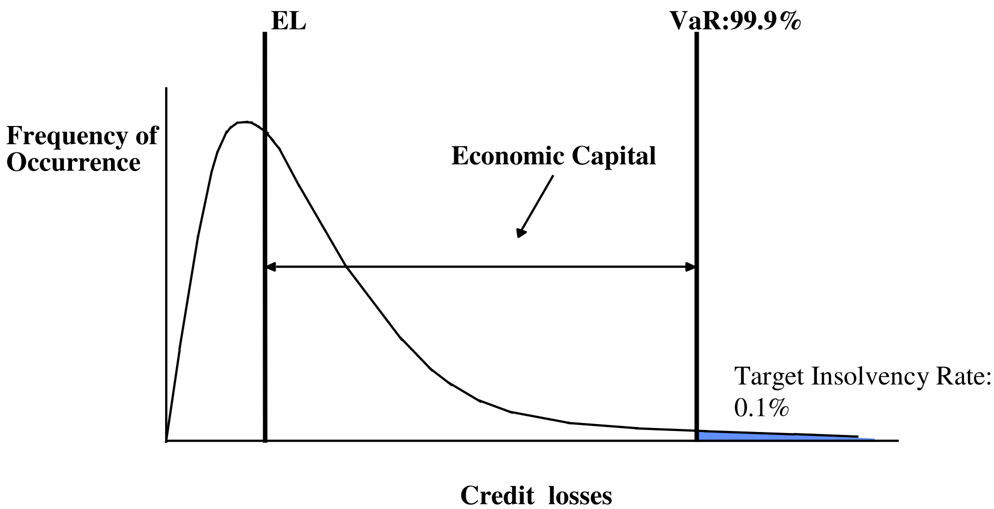
Tranche Size	**Premium in Rs.**	**Premium (% of Tranche size)**
1	0.905	90.473
5	4.308	86.165
10	7.584	75.836
20	10.975	54.873
30	12.168	40.561
40	12.590	31.475
50	12.746	25.493
60	12.809	21.348

Table20: *Second-Loss Tranche* Premium (100% LGD)

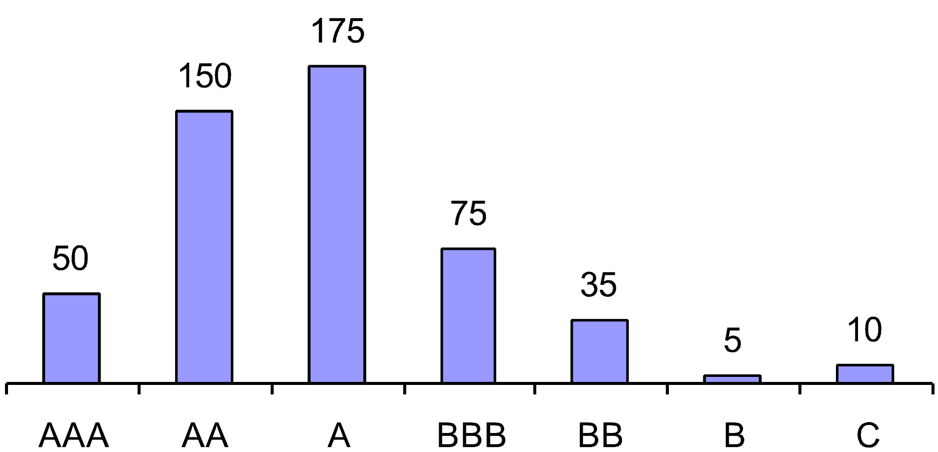
Tranche (Size 20)	**Premium in Rs.**	**Premium (% of Tranche size 20)**
2 to 21	10.253	51.267
6 to 25	7.418	37.091
11 to 30	4.585	22.923
21 to 40	1.616	8.082
31 to 50	0.578	2.891
41 to 60	0.219	1.095
51 to 70	0.086	0.432
61 to 80	0.034	0.168

Charts:

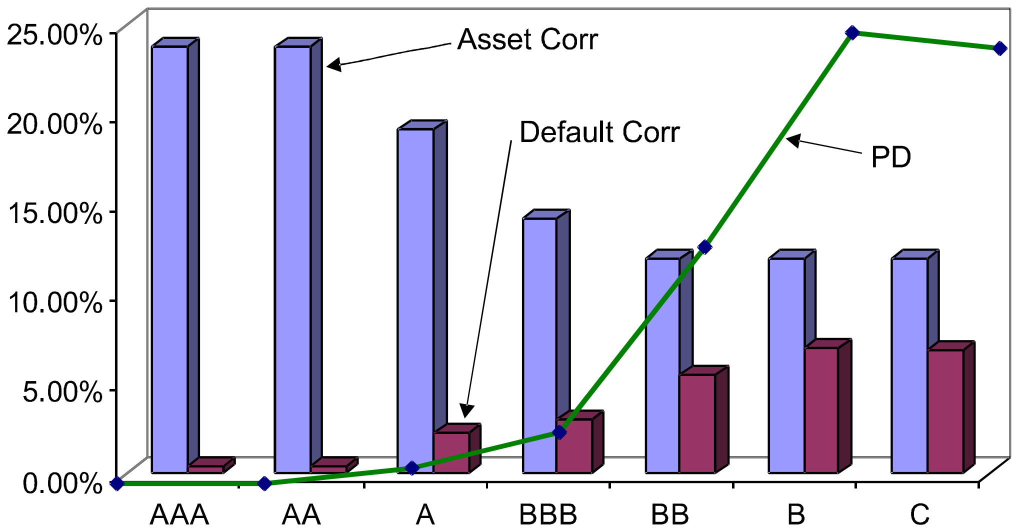
1) Economic Capital for credit portfolio



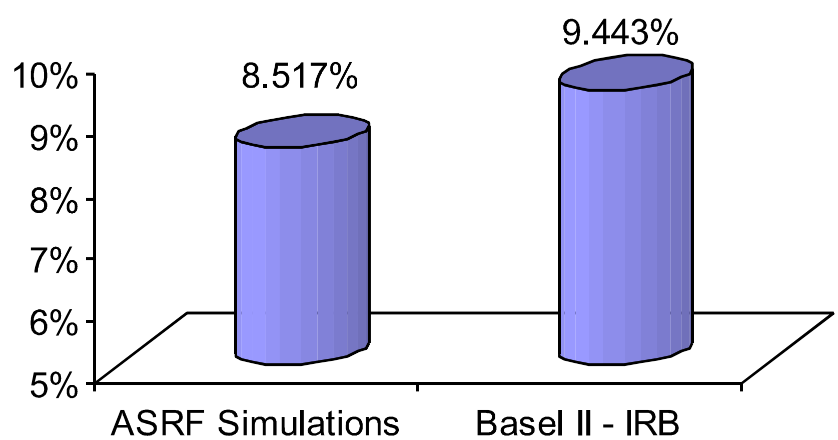
2) Rating-wise distribution of 500 assets portfolio



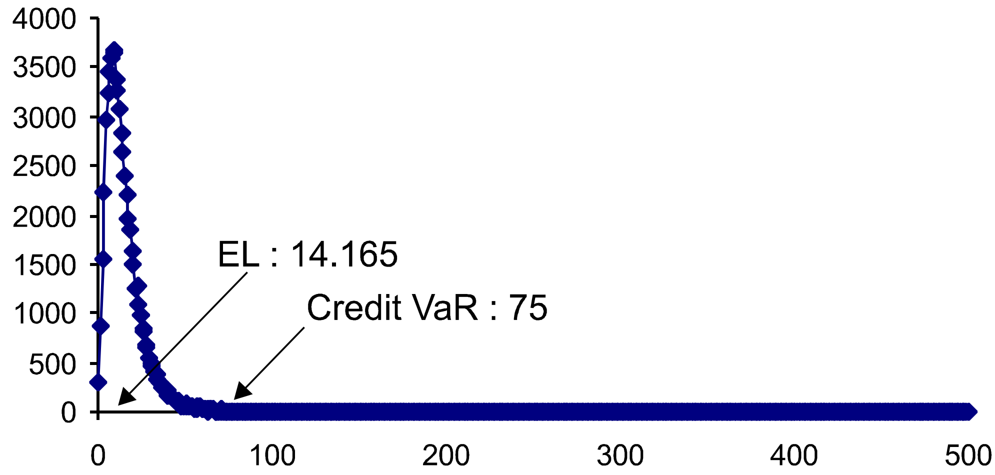
3) PD estimates & Correlation within obligors falling in the same 'Rating-Bucket'



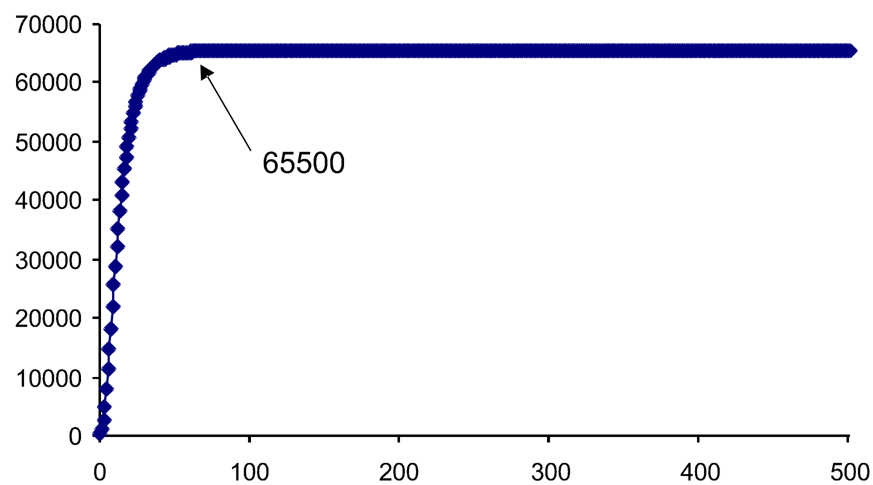
4) Capital Required for 500 assets portfolio (70% LGD)



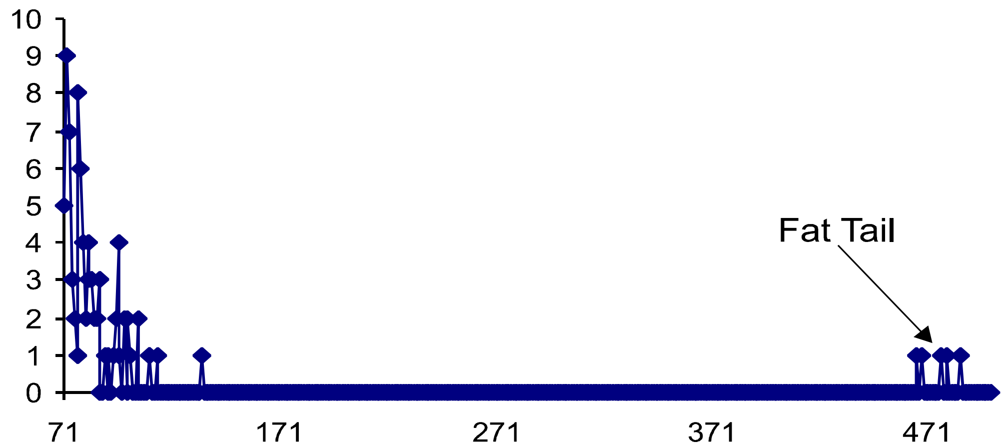
5) Frequency distribution of credit losses based on ASRF model: 500 assets portfolio (100% LGD)



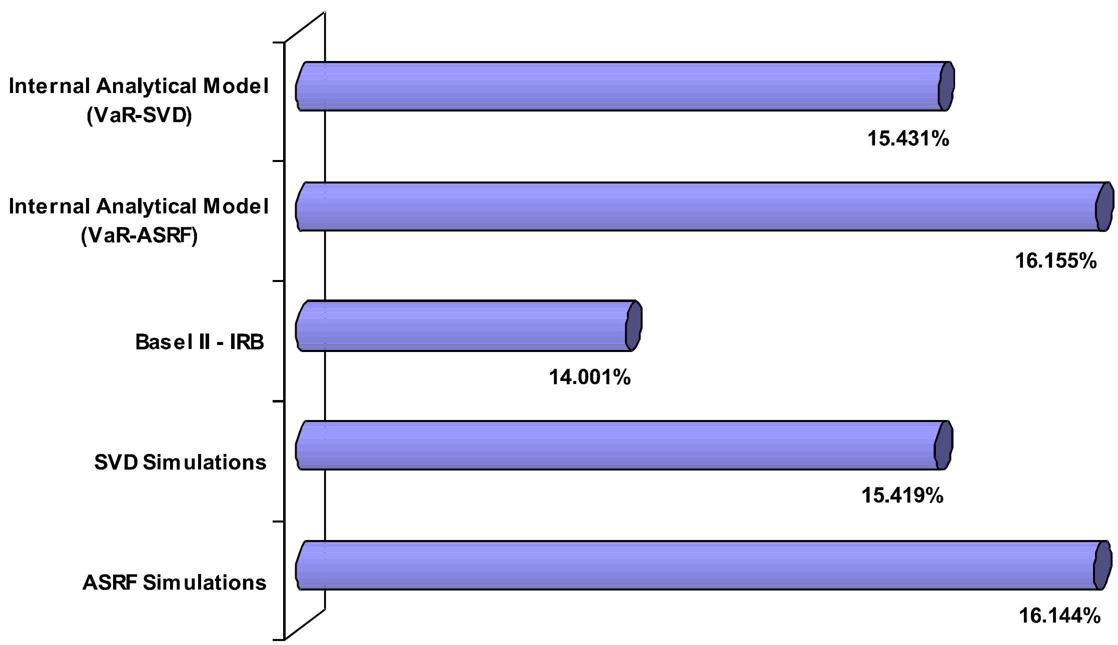
6) Cumulative loss distribution: 500 assets portfolio (100% LGD)



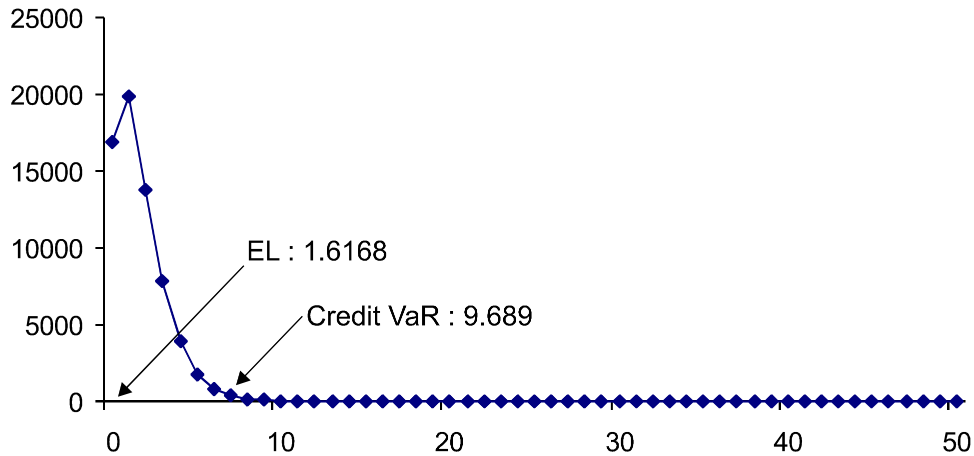
7) Tail losses in the 500 assets portfolio (100% LGD)



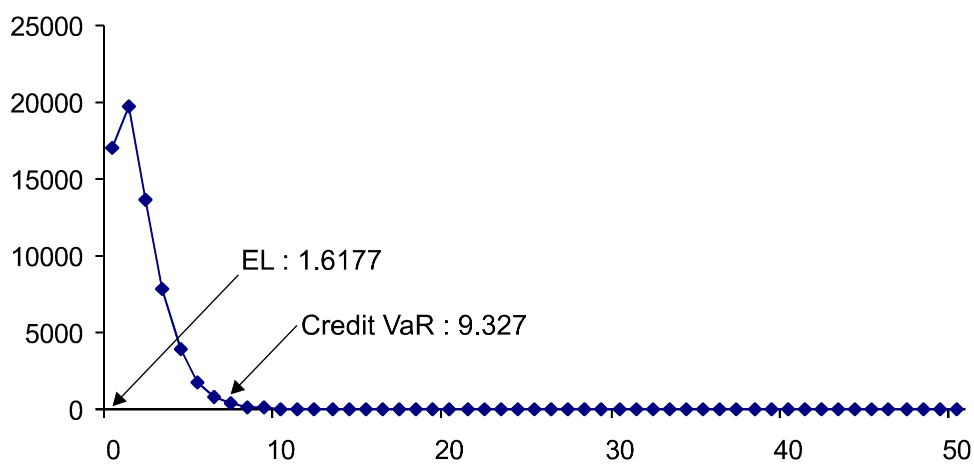
8) Capital Required for 50 assets portfolio (100% LGD)



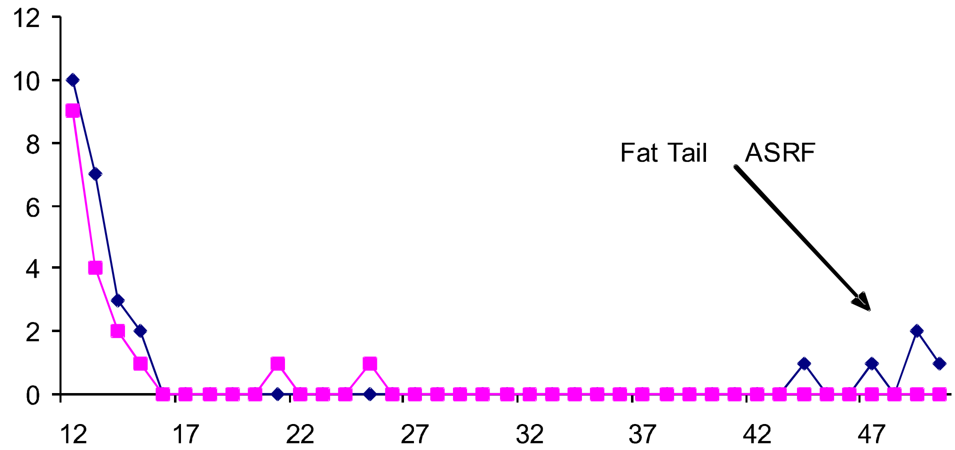
9) ASRF model - based loss distribution: 50 assets portfolio (100% LGD)



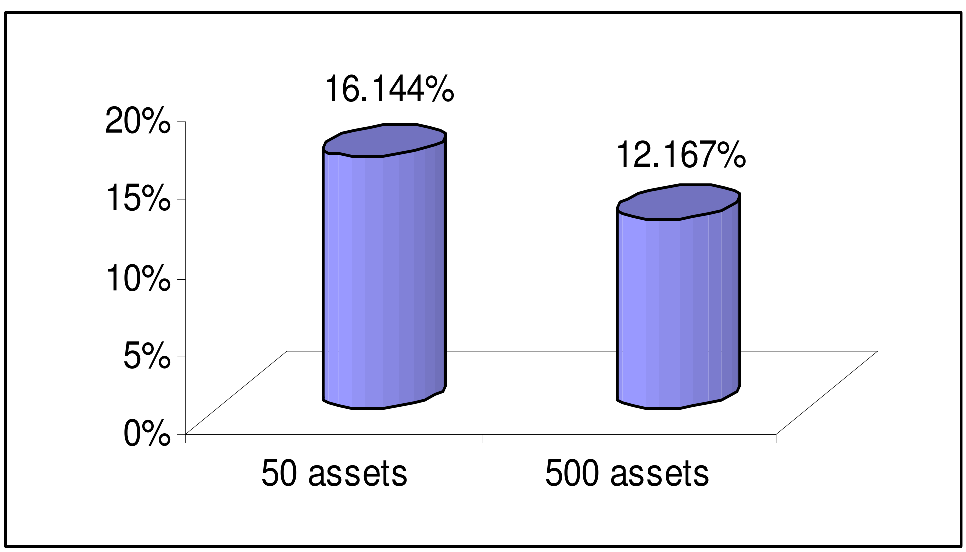
10) RiskMetrics SVD- based loss distribution: 50 assets portfolio (100% LGD)



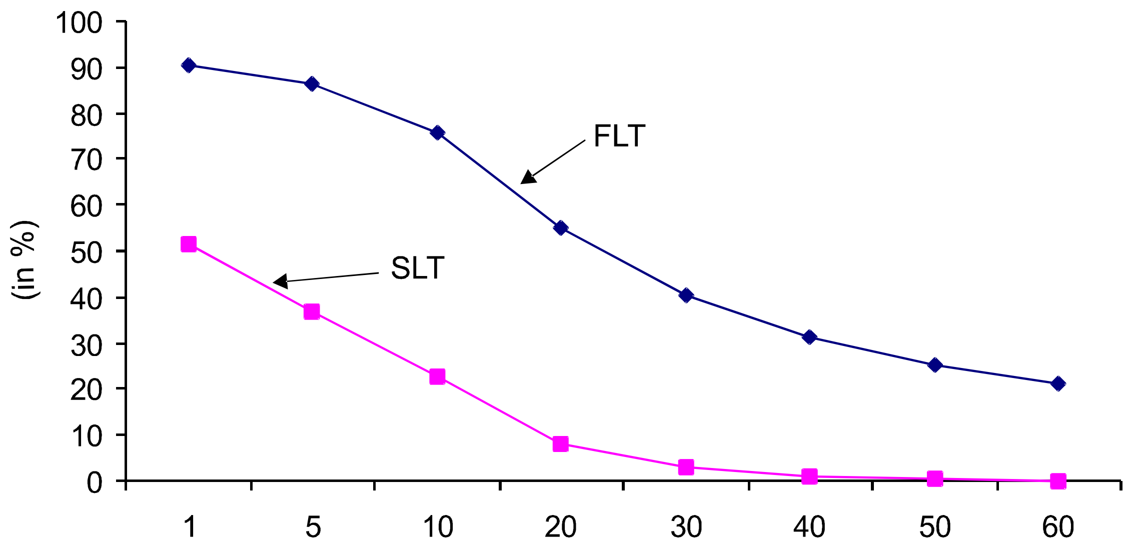
11) Tail losses in the 50 assets portfolio (ASRF model v/s SVD simulations)



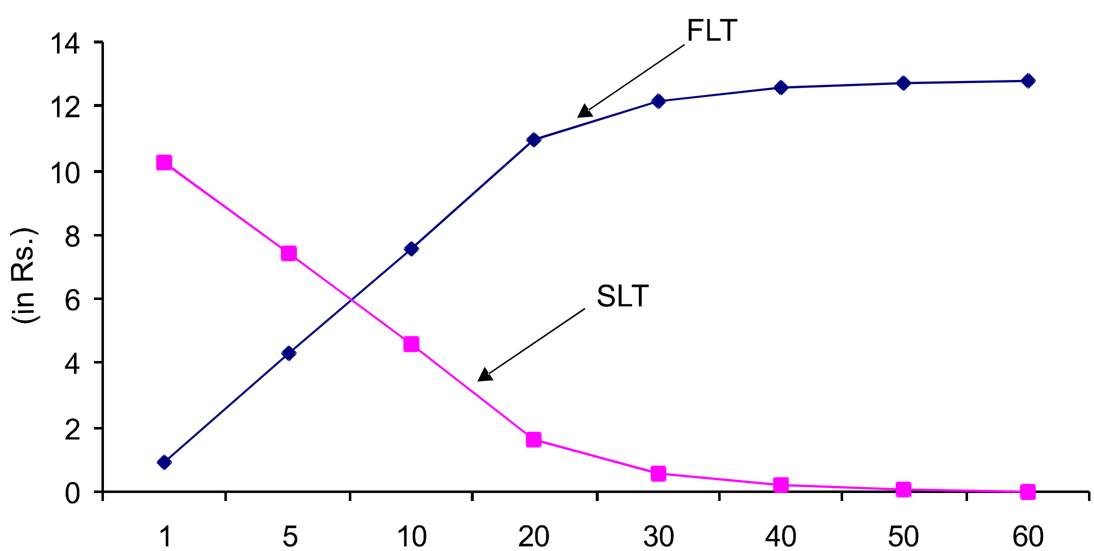
12) Capital required under ASRF model-based simulations (100% LGD): 500 assets v/s 50 assets portfolio



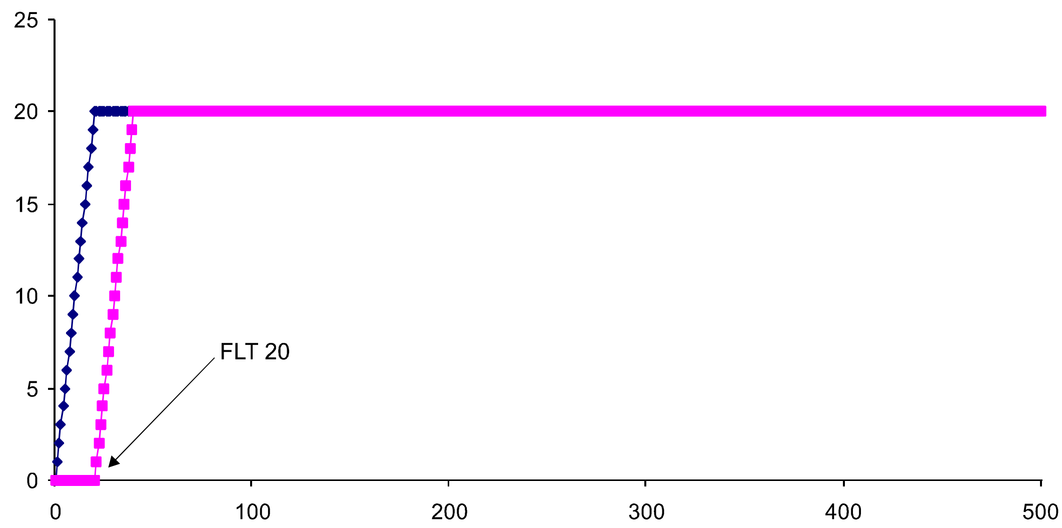
13) Portfolio Default Swap (PDS) Premium as % of Tranche size (100% LGD)



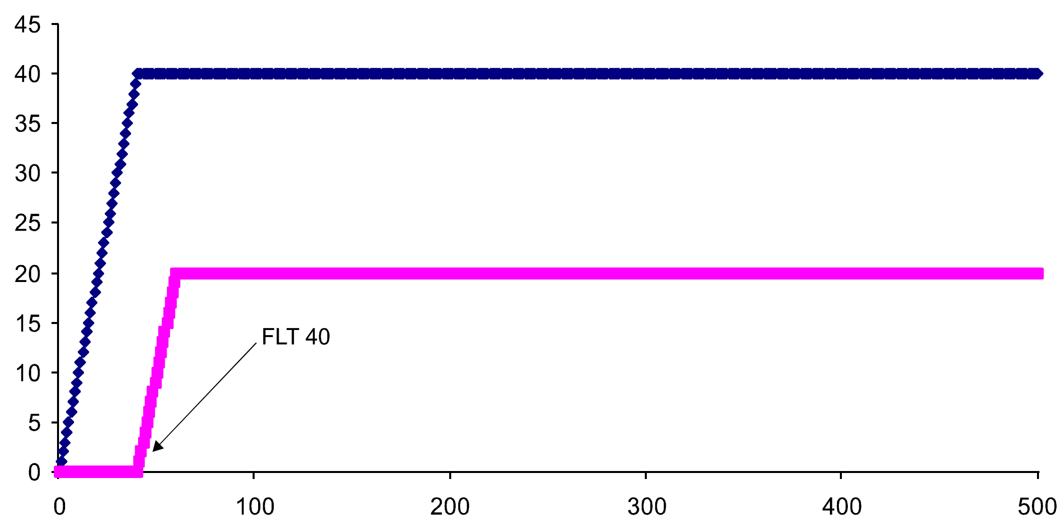
14) PDS Premium in Rupees (100% LGD).



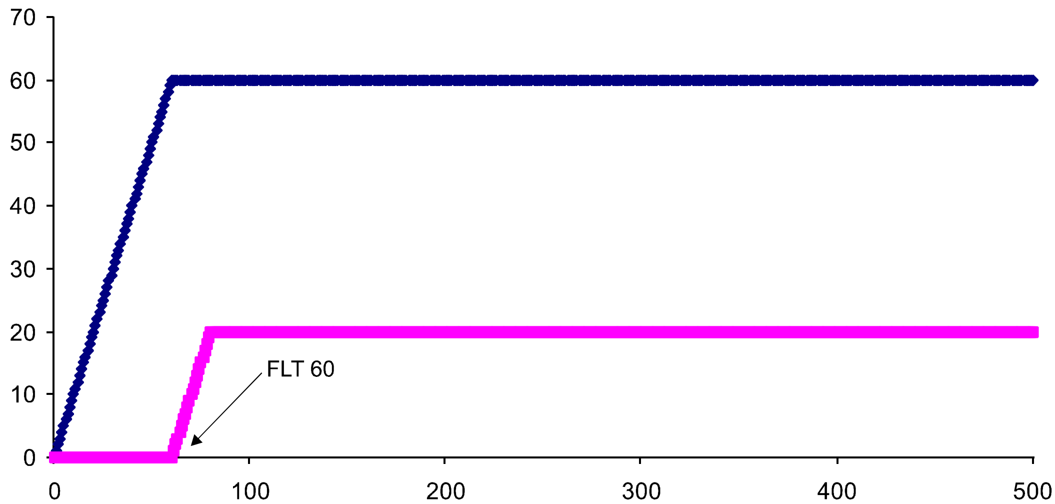
15) Loss distribution for PDS First Loss Tranche:20 & Second Loss Tranche: 21to 40



16) Loss distribution for PDS First Loss Tranche:40 & Second Loss Tranche: 41to 60



17) Loss distribution for PDS First Loss Tranche: 60 & Second Loss Tranche: 61to 80



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Appendix 1: Derivation of Basel II IRB risk weight function

The ASRF model, coupled with *Homogeneous Portfolio* assumption, forms the basis for IRB risk weight functions. The model assumes that for all the assets in the credit portfolio,

$\beta_i = \beta$, $C_i = C$ & $LGD_i = 100\%$

Secondly, the model assumes that all the assets in the portfolio are equally weighted. This guarantees that in a portfolio with N assets, the probability of observing k defaults is equivalent to the probability of observing k/N percentage of default losses.

Under Default Mode paradigm, the number of defaults (k) in the portfolio follow binomial distribution for a given value of systematic risk factor X. Thus, if we let L denote the fraction of defaults (percentage of default losses), then we may write the conditional probability of observing a given loss level as,

$$\Pr\left(L = \frac{k}{N} \middle| X \right) = \frac{N!}{k!(N-k)!} W(X)^k [1 - W(X)]^{N-k} \qquad \text{.......A.1.1}$$

where, W(X) is the conditional default probability, as defined in equation 15 of Section 3

The major thrust of *Large-Portfolio Approximation* framework (Vasicek,1987) is that for an infinitely large homogeneous portfolio, through the *Law of Large Numbers,*

$$\theta \equiv E\left(L = \frac{k}{N} \middle| X \right) \approx W(X) \qquad \text{.......A.1.2}$$

where Θ and L are the expected and actual values of percentage default losses in the portfolio, respectively. Equation A.1.2 may be construed as under,

Expected value of the fraction of obligors defaulting in the large homogeneous portfolio (Θ), over a given time horizon, can be approximated by the corresponding individual conditional default probabilities, denoted as W(X), of the obligors.

Note that for any given value of Θ, one can back out the implied value of X upon which the conditional expectation in A.1.2 is based

$$X = W^{-1}(\theta) \qquad \text{.......A.1.3}$$

Suppose now that the actual realized value of X turns out to be larger than one used in equation A.1.3. From equation 11 of Section 3, this translates into higher realized asset return R_i. Hence, other things being equal, the actual percentage loss L will be smaller than the expected percentage loss Θ. Mathematically, this can be summed up as,

$$X \geq W^{-1}(\theta) <=> L \leq \theta \qquad \text{........A.1.4}$$

Thus, we can make the following statement,

$$\Pr\left[X \geq W^{-1}(\theta)\right] = \Pr\left[L \leq \theta\right] \qquad \text{........A.1.5}$$

Given the standard normal distribution assumption for X and relying on the symmetric nature of the Normal PDF,

$$\Pr\left[X \geq W^{-1}(\theta)\right] = \Pr\left[X \leq -W^{-1}(\theta)\right] = N(-W^{-1}(\theta)) \qquad \text{.........A.1.6}$$

Thus we arrive at the result,

$$\Pr\left[L \leq \theta\right] = N(-W^{-1}(\theta)) \qquad \text{.........A.1.7}$$

From equation A.1.2 and equation 15 of Section 3,

$$\theta = W(X) = N\left(\frac{C - \sqrt{\beta}X}{\sqrt{1-\beta}}\right) \qquad \text{.........A.1.8}$$

and we can write,

$$W^{-1}(\theta) = X = \frac{C - N^{-1}(\theta)\sqrt{1-\beta}}{\sqrt{\beta}} \qquad \text{..........A.1.9}$$

From equation A.1.7 and equation equations 5 of Section 3,

$$\Pr[L \leq \theta] = N\left(-\frac{N^{-1}(W) - N^{-1}(\theta)\sqrt{1-\beta}}{\sqrt{\beta}}\right) \qquad \text{........A.1.10}$$

Note that W is unconditional PD, as against W(X), which is conditional PD. Given the 99.9% confidence level under Basel II IRB Approach, equation A.1.10 becomes,

$$\Pr\left[L \leq \theta\right] = N\left(-\frac{N^{-1}(PD) - N^{-1}(\theta)\sqrt{1-R}}{\sqrt{R}}\right) = 0.999 \qquad \text{..........A.1.11}$$

$$N^{-1}(0.999) = \left(-\frac{N^{-1}(PD) - N^{-1}(\theta)\sqrt{1-R}}{\sqrt{R}}\right) \qquad \text{.........A.1.12}$$

Thus,

$$N^{-1}(\theta) = \left(\frac{N^{-1}(PD) + N^{-1}(0.999)\sqrt{R}}{\sqrt{1-R}}\right) \qquad \text{.........A.1.13}$$

Finally,

$$\theta = N\left(\frac{N^{-1}(PD) + N^{-1}(0.999)\sqrt{R}}{\sqrt{1-R}}\right) \qquad \text{……….A.1.14}$$

Thus, Θ is a non-linear function in PD and asset correlation R#. It represents the expected value of percentage *total credit loss* associated with a hypothetical, infinitely-granular portfolio of exposures with 100% LGD. Together with equation A.1.2, Θ approximates the conditional default probability W(X), given the parameter values for unconditional (average) default probability PD and asset correlation R for any obligor.

Regulatory Capital (RC) requirement may be written as K% of total credit exposures (EAD).Also, RC should suffice to cover Θ (i.e., EL+UL), adjusted for "downturn" LGD.

$$\frac{RC}{EAD} = K = \theta \times LGD \qquad \text{...……A.1.15}$$

Thus, capital charges are calibrated for both Expected and Unexpected Loss (EL+UL). However, in the January 2004 notification, BCBS announced its intention to move to a UL-only risk weighting construct. According to this amendment, capital would only be needed for absorbing UL. The Committee stipulated EL to be adjusted directly against loan loss provisions and not capital; any shortfall between EL and actual provisions should be deducted equally from Tier 1 and Tier 2 capital and any excess to be eligible for inclusion in Tier 2 capital (subject to a cap). Hence, the modified capital requirement formula may be written as,

$$K = \theta \times LGD - EL \qquad \text{...……A.1.16}$$
$$= \theta \times LGD - PD \times LGD \qquad \text{..…….A.1.17}$$

Maturity Adjustment (MatAdj) is subsequently incorporated in the above formula to account for effective maturity of the exposure. It highlights the fact that BCBS has calibrated Θ based on 2.5-year average maturity assumption. The adjustment has been made a decreasing function of firm-wise PD. For effective maturity of one year, MatAdj function yields the value 1.

$$K = (\theta \times LGD - PD \times LGD) \times MatAdj \qquad \text{..…….A.1.18}$$

Since Basel II stipulates Regulatory Capital (RC) to be 8% of total risk weighted assets (RWA),

$$RC = 8\% \times RWA = K \times EAD \qquad \text{………A.1.19}$$
$$\therefore RWA = 12.5 \times K \times EAD \qquad \text{..…….A .1.20}$$

……………………………………………………………………………………………

Asset correlation function in IRB approach also comprises of 'firm-size adjustment' for SMEs.

Plugging equation A.1.18 in equation A.1.20,

$$RWA = 12.5 \times (\theta \times LGD - PD \times LGD) \times MatAdj \times EAD \qquad \text{.........A.1.21}$$

It is worth noting that RWA is a linear function of LGD & non-linear concave function of PD. Hence, If LGD falls by half, the risk weight also falls by half. However, If PD falls by half, risk weight also falls—but it falls by less than half. This anomaly could possibly incentivize banks to make low-LGD loans through over-collateralization, with reduced regard for default risk. This practice, known as *Lending On Collateral*, gives primary consideration to collateral and LGD, rather than to the borrower's ability to repay (Jon Frye, 2001). Furthermore, the risk weight function overlooks PD-LGD correlation.

Appendix 2: **<u>Internal Analytical Model for estimating Implied Capital Multiplier</u>**

Mathematically, we may write the expression for portfolio Expected Loss as,

$$EL_P = \sum_{i=1}^{N} EL_i = \sum_{i=1}^{N} \left(EAD_i \times PD_i \times LGD_i \right) \qquad \text{.......A.2.1}$$

However, portfolio Unexpected Loss is not equal to the linear sum of the individual unexpected losses of the credit exposures. As a result of the diversification effects through default correlations,

$$UL_P = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} UL_i UL_j} \leq \sum_{i=1}^{N} UL_i \qquad \text{......A.2.2}$$

where, assuming constant LGD across all exposures,

$$UL_i = EAD_i \times LGD_i \times \sqrt{PD_i (1 - PD_i)} \qquad \text{.......A.2.3}$$

Matrix form representation for UL_P is,

$$UL_P = \sqrt{\mathbf{UL_i}'_{(1\times N)} \times \boldsymbol{\rho}_{\mathbf{ij}(N\times N)} \times \mathbf{UL}_{\mathbf{i}(N\times 1)}} \qquad \text{.......A.2.4}$$

where UL_i is the vector of individual unexpected losses and ρ_{ij} symbolizes the default correlation matrix.

Economic Capital (EC) to be allocated for the credit portfolio is a multiple of ULp. This multiple is termed as Implied Capital Multiplier ($CM_{Implied}$)

$$CM_{Implied} = \frac{EC}{UL_P} = \frac{VaR_{99.9\%} - EL_P}{UL_P} \qquad \text{.......A.2.5}$$