Loss Given Default Implied by Cross-sectional No Arbitrage[*]

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Abstract

I develop various frameworks for the separation of loss given default and default intensity present in securities with credit risk. They include spot and forward credit default swaps, digital default swaps and bonds. Cross-sectional no-arbitrage restriction between different securities extracts the pure measure of default intensity and loss given default not contaminated by the other. Using spot and forward CDS premium data of 10 emerging market sovereigns, I find that 75% level of loss given default prevails in the sovereign CDS markets across countries over time. Positive correlation between loss given default and default intensity is only found in Brazil and Venezuela during the period of political turmoil in each country. This result is puzzling considering diverse fundamentals across countries and time variation of the marginal rate of substitution. Loss given default below (above) the 75% generates negative (positive) pricing errors in forward CDS and the magnitude of them is economically significant. This persistent negative (positive) pricing errors with mis-specified loss given default higher (lower) than the true one are consistent with the model developed.

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1 Introduction

For the recent several years, credit derivatives markets have grown explosively. A recent report by the British Bankers' Association (2006) estimates that by the end of 2006, the size of market would be USD 20 trillion, which is far beyond its own prediction of USD 8.2 trillions made in 2004. It expects that at the end of 2008 the global credit derivatives market will have expanded to USD 33 trillion and continue to grow.[1] In addition to the fast growth of the credit derivatives markets, the composition of obligors also is shifting. AAA-BBB rating classes represent 59% in 2006 falling from 65% in 2004 and it is expected to continue to fall to 52% by end of 2008. In contrast, under investment grade classes have expanded and expected to reach nearly half of the market.[2] The development of credit derivatives in Emerging Markets parallels that of the "global" credit derivatives market.[3]

Two main components of default risk are (risk neutral) default intensity (λ) and (risk neutral) loss given default(L). A number of studies have focused on modeling default intensity, but research on loss given default are rare.[4] The difficulties in disentangling default intensity and loss given default have been well known since Duffie and Singleton (1999). In order to identify default intensity, both academics and practitioners often assume loss given default as a constant. Longstaff et al. (2005) pre-specify loss given default as 50% in their study on credit default swaps and bonds for investment grade corporates. For sovereigns, Adler and Song (2007) set loss given default as 75% in their study on the dynamics of emerging markets sovereign CDSs and bonds. Zhang (2004) and Pan and Singleton (2006) also take loss given default as a constant in their work on Sovereign CDS, but their studies are different from

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[1] Single name credit default swap (CDS) represents a substantial portion of the market. It represents 51% of the total market in year 2004 and 33% in year 2006 respectively. Index trades, the second largest product, represent 30% in year 2006 growing from 9% in year 2004. Synthetic CDOs (collateralized debt obligation) come in the third place at 16% of the market for both year.

[2] In contrast, the BB-B classes have expanded from 13% to 23% and are expected to grow to 27% by end 2008 according to the report. Under B classes represent the remaining portion of the market.

[3] CDSs are the most basic product for Emerging Markets as well. They are based on standard ISDA contract documentation, and enjoy an active broker market with dealers quoting two-way pricing for standard contract sizes (see Dages et al. (2005) for more details). More interestingly and importantly, there are quotes for CDS premiums from 1-10 years for the EM sovereigns, whereas the quotes are heavily concentrated on the 5 year contract for the corporate both in the U.S. and in the Emerging Markets. For more details, see Packer and Suthiphongchai (2003) and Pan and Singleton (2006).

[4] Altman and Kishore (1996) and Acharya, Bharath, and Srinivasan (2004), for example, provide analysis on actual recoveries of defaulted securities. Note that this study is on the actual loss given default, not the risk neutral one. For a survey paper on the recovery risk, see Das (2005).

the previous ones in that they estimate it rather than pre-specify it. However, the separation of default intensity and loss given default relying only on CDS may be difficult as shown in Duffie (1999).

Despite of difficulties of separation of default intensity and loss given default, it is crucial in many pricing circumstances. Even the basic CDS requires the separation when traders do mark-to-market of premium payment leg of a contract. We need to calculate PV01, the present value of a one-basis-point annuity with the maturity of the credit default swap that terminates following a credit event, for the mark-to-market purpose. And PV01 is a function of default intensity, not a function of loss given default. CDS swaption and digital default swap (DDS) also require a separate measure of default intensity in their pricing. Furthermore, as the market for credit risk continues to develop, there will be more trading of contingent credit securities that depend on default intensity and loss given default separately, rather than in combination.

In this paper, I suggest frameworks for the separation by imposing cross-sectional no-arbitrage restrictions on different securities. I develop, first, pricing models for various credit instruments including spot and forward credit default swap (CDS), digital default swap (DDS) and defaultable bonds. Pricing functions of those securities are derived in term of default intensity and loss given default of underlying reference entity. Then, I impose cross-sectional no-arbitrage restrictions between them. Due to cross default provisions and absolute priority rules, each credit instrument with the same level of seniority is exposed to the same level of risk out of default intensity and loss given default of a certain reference entity. It makes possible for default intensity or loss given default in a pricing model for a certain type of security to be replaced with (observable) prices of other securities. Once either default intensity or loss given default is identified, the remaining component can be sequentially estimated.

Forward CDS premiums, implied by no-arbitrage, are derived in terms of default intensity and other CDS premiums. Loss given default is canceled out with a cross-sectional no-arbitrage restriction between spot and forward CDSs, and only default intensity remains. One of the merits of this framework is that the separation does not require any assumption on the process specification of loss given default. It can be a constant or time varying.

Furthermore, it can be correlated or uncorrelated to other model parameters such as default intensity. The fact that spot and forward CDS contracts are with constant maturities is also convenient since we do not need to do maturity match of them.

Stand alone digital default swap (DDS) premiums reveals a pure measure of default intensity, since loss given default in DDS is a contractually fixed number. Protection buyer pays premiums until the maturity or the time of default. In exchange, the protection seller pays a pre-specified loss amount to the protection buyer in the event of default. Differences between DDS and CDS are, first, the amount of payment from the protection seller is pre-specified. In addition to it, DDS contract is usually cash settled, while CDS contract, in many cases, requires physically delivery of reference obligation when default occurs. Therefore, DDS transfer of default event risk while usual CDS transfer default loss risk among counterparties. A cross-sectional no-arbitrage restriction between CDS and DDS, interestingly, leads to a measure of expected loss given default.

Bond price can also be used in extracting a pure measure of default intensity with cross-sectional restrictions with CDS. When bond is floating rate note and at par, it does not add additional information over CDS. However, when it is not at par, the parity relation between bond yield spread and CDS premiums does not hold as pointed out in Adler and Song (2007).

I find that loss given defaults around 75% prevails in the sovereign CDS markets based on the cross-sectional restriction between spot and forward CDSs. 75% level of loss given default persistently generates the smallest pricing error in the pricing of forward CDS premiums for all sovereigns in the sample; 10 emerging market sovereigns are in the sample and sample periods are from 1999 to 2005.[5] This finding is in contrast with the result found in Pan and Singleton (2006). Estimates in Pan and Singleton (2006) are 24%, 23% and 83% for Turkey, Mexico, and Korea. Loss given default around 25% causes the pricing error of about 100-200bp in forward CDS premiums for Turkey while loss given default of 75% leads to only about 10bp pricing error.[6] For Mexico, loss given default around 25% causes the pricing error of about 20bp in forward CDS premiums for Turkey while loss given default of 75% leads to

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[5] Countries include Bulgaria, Brazil, Colombia, Korea, Mexico, Malaysia, Philippines, Poland, Turkey, and Venezuela.

[6] Bid-ask spreads for 5 year CDS for corresponding periods are about 25-50bp.

only about 2bp pricing error.[7] The difference in pricing error depending on loss given default is less than bid ask spread for Korea.

The remainder of this paper is organized as follows. Section 2 presents pricing models for stand-alone spot credit default swaps, forward credit default swaps, digital defaults swaps, and defaultable bond. Section 3 investigates the loss given default in the actual and risk neutral probability space in a simple three state economy. This section also illustrates how cross sectional restrictions work using CDS and bond. Section 4 provides the framework for the separation of default intensity and loss given default by imposing cross-sectional no-arbitrage restrictions. No-arbitrage restrictions are imposed between spot CDS and forward CDS, resulting in the pure measure of default intensity. The pure measure of loss given default is obtained by imposing the restriction between spot CDS and digital default swaps. No arbitrage restriction between CDSs and bonds also lead to the separation of default intensity. Section 5 documents the empirical findings on loss given default in the CDS market. Section 6 summarizes the results and offers concluding remarks.

2 Pricing Models

In this section, I derive pricing formulae for credit default swaps, forward credit default swaps, digital default swaps, and defaultable bonds. The stand-alone pricing formula for each security is derived in terms of default intensity and loss given default. Digital default swap is an exception in that it provides default intensity not contaminated by the loss given defaults. It is because loss given default in digital default swaps is contractually pre-fixed. Other than that, the separate estimation of default intensity and loss given default is not feasible and they should be jointly estimated. However the accuracy of the joint estimation is questionable (see Longstaff et al. (2005), Duffie (1999) and Pan and Singleton (2006) for more details). Though the difficulties of identification of default intensity and loss given default in stand-alone pricing model are well known, stand-alone models form bases for the separation when they are used in combinations via cross-sectional no-arbitrage restrictions.

I, first, set-up the common notations to be used in this paper. A probability space

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[7] Bid-ask spreads for 5 year CDS for corresponding periods are about 10bp.

$(\Omega, \mathcal{F}, \mathbb{Q})$ is well defined, where the filtration $\mathcal{F} = \{\mathcal{F}_t | 0 \leq t \leq T\}$ satisfies $\mathcal{F}_T = \mathcal{F}$ and it is complete, increasing and right continuous where $\mathbb{Q}$ is the equivalent martingale measure. Suppose also a locally risk-free short rate process r. Let $\chi(\tau) = 1_{\xi \geq \tau}$ be a default indicator function of a reference entity, where ξ is the stopping time that characterizes the time of default by the reference entity. An risk neutral default intensity process $\lambda(\tau)$ for a stopping time ξ is characterized by the property that the following is the martingale,

$$\chi(\tau) - \int_0^\tau \left(1 - \chi(\mu)\right)\lambda(\mu) d\mu$$

L denote the risk-neutral fractional loss of face value on a reference obligation in the event of a default.

2.1 Spot Credit Default Swap

In this section, I derive a pricing model for CDSs.[8] Suppose that two parties make a spot CDS contract at time t with maturity of τ_c. A buyer of protection periodically pays premiums, s_{t,τ_c}, to a seller. The payment is made M_c times per unit time until any one of the following events happens: the underlying reference entity defaults on its reference obligation or the maturity of the CDS contract comes. The payment begins at $t + \frac{\tau_c}{M_c}$.

The seller of protection receives the premium payment and its present value at t is

$$\frac{s_{t,\tau_c}}{M_c} \sum_{j=1}^{M_c \cdot (\tau_c - t)} E^{\mathbb{Q}} \left[\frac{B(t)}{B(t + \frac{j}{M_c})} \left(1 - \chi\left(t + \frac{j}{M_c}\right)\right) \middle| \mathcal{F}_t \right]$$

where $B(\tau) = e^{-\int_t^\tau r(s)ds}$. Then the value of the 'premium leg' is

$$\frac{s_{t,\tau_c}}{M_c} \sum_{j=1}^{M_c \cdot (\tau_c - t)} E^{\mathbb{Q}} \left[e^{-\int_t^{t + \frac{j}{M_c}} r(s) + \lambda(s) ds} \middle| \mathcal{F}_t \right]$$

The buyer of protection will receive a unit face value of the reference obligation in exchange

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[8] My model is different from the previous model by Longstaff et al. (2005) and Zhang (2004) in that it explicitly reflect the discrete premium payment in actual contracts. Their models assume a continuous payment of premiums. My model is also different from the model proposed by Pan and Singleton (2006) in that it does not assume loss given default as a constant in the risk neutral space.

of the physical delivery of the obligation when a credit event happens. The payoff process, D(t), follows

$$dD(t) = (1 - \chi(t))\lambda(t)L(t)dt + dM_{D}(t)$$

where $M_D(t)$ s a martingale with respect to $\mathbb{Q}$. Then the present value of the protection payment is

$$E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu \middle| \mathcal{F}_t\right]$$

Since the net present value of a spot CDS at its initiation be zero, the spot CDS premium can be obtained by equating the value of the two legs

$$\frac{s_{t,\tau_c}}{M_c} = \frac{E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu \middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t\right]} \tag{1}$$

2.2 Forward Credit Default Swap

In this section, I develop a pricing model for forward CDSs.[9] A forward CDS contract is an obligation to buy or sell a CDS on a specified reference entity for a specified spread at a specified future time. Suppose that two parties make a forward CDS contract at time t with maturity of τ_c. A buyer of protection will begin to pay premiums, s_{τ_f,τ_c}, to a seller at a certain pre-set future time (expiry of a forward contract), which is denoted as τ_f. The payment is made M_c times per unit time until any one of the following events happens: the underlying reference entity defaults on its reference obligation or the maturity of the forward CDS contract comes.

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[9] The previous model developed by Hull and White (2003) assumes a constant loss given default and independence between the short rate and default probability. I extend the model with more flexible structure. The new model does not require either a constant loss given default or the independence between the short rate and default probability. The flexible features of the model allow the models to be valid regardless of various model restrictions made in the previous literature.

The seller of protection receives the premium payment and its present value at t is

$$\frac{s_{\tau_f,\tau_c}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[\frac{B(t)}{B(\tau_f+\frac{j}{M_c})}\left(1-\chi\left(\tau_f+\frac{j}{M_c}\right)\right)\bigg|\mathcal{F}_t\right]$$

where $B(\tau)=e^{-\int_t^{\tau} r(s)ds}$. Then the value of the 'premium leg' is

$$\frac{s_{\tau_f,\tau_c}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\bigg|\mathcal{F}_t\right]$$

The buyer of protection will receive a unit face value of the reference obligation in exchange of the physical delivery of the obligation when a credit event happens. The payoff process, D(t), follows

$$dD(t)=(1-\chi(t))\lambda(t)L(t)dt+dM_D(t)$$

where $M_D(t)$ s a martingale with respect to $\mathbb{Q}$. Then the present value of the protection payment is

$$E^{\mathbb{Q}}\left[\int_{\tau_f}^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\bigg|\mathcal{F}_t\right]$$

Since the net present value of a forward CDS at its initiation is zero, the forward CDS premium can be obtained by equating the values of the two legs

$$\frac{s_{\tau_f,\tau_c}}{M_c}=\frac{E^{\mathbb{Q}}\left[\int_{\tau_f}^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\bigg|\mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\bigg|\mathcal{F}_t\right]} \tag{2}$$

Pricing formula for a forward CDS is quite similar to the one for a spot CDS; They are virtually identical except the beginning point of the premium payment. With $\tau_f=t$, the equation (2) is identical to the equation (1).

2.3 Digital Default Swap

In this section, I develop a pricing model for digital default swaps.[10] Digital default swaps contract is an obligation that a protection seller pays a pre-specified dollar amount to a protection buyer in the event of default. Spot and forward credit default swaps and digital default swaps are designed to protect against different types of risk. The credit default swaps transfer the risk of loss of the obligation holder in the time of default. Therefore pricing function contains both default intensity and loss given default. However, digital default swaps transfer only the risk of a default event. Regardless of the realization of the loss, pre-specified amount is paid, and as a result the pricing function contains only default intensity.

Suppose that two parties make a digital default swaps contract at time t with the maturity of τ_c. A buyer of protection will begin to pay premiums, s^D_{t,τ_c}, to a seller at time $t + \frac{\tau_c}{M_c}$. The payment is made M_c times per unit time until any one of the following events happens: the underlying reference entity defaults on its reference obligation or the maturity of the CDS contract comes. In the event of default, the seller of a protection pay the pre-specified $\mathcal{L}$ to the buyer.

The seller of protection receives the premium payment and its present value at t is

$$\frac{s^D_{t,\tau_c}}{M_c} \sum_{j=1}^{M_c\cdot(\tau_c - t)} E^{\mathbb{Q}} \left[\frac{B(t)}{B(t+\frac{j}{M_c})} \left(1 - \chi\left(t + \frac{j}{M_c}\right)\right) \middle| \mathcal{F}_t \right]$$

where $B(\tau) = e^{-\int_t^\tau r(s)ds}$. Then the value of the 'premium leg' is

$$\frac{s^D_{t,\tau_c}}{M_c} \sum_{j=1}^{M_c\cdot(\tau_c - t)} E^{\mathbb{Q}} \left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t \right]$$

The buyer of protection will receive the pre-specified dollar amount $\mathcal{L}$ when a credit event

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[10] Berd and Kapoor (2003) also derived the pricing formula for digital default swaps. Their model is different from mine with two respects. Their model is derived under the actual probability space while mine is under the risk neutral probability space. For the consistency with other pricing model in this paper and future use in the next section, a model under risk neutral measure is necessary. More importantly, my model is a model for the absolute pricing. Their model is derived using the hedge ratio in the relative pricing set up. Their pricing function is expressed with hedging instrument.

happens. The payoff process, D(t), follows

$$dD(t) = \mathcal{L} \cdot (1 - \chi(t))\lambda(t)dt + dM_{_D}(t)$$

where $M_D(t)$ s a martingale with respect to $\mathbb{Q}$. Then the present value of the protection payment is

$$\mathcal{L} \cdot E^{\mathbb{Q}} \left[\int_t^{\tau_c} \lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t \right]$$

Since the net present value of a digital default swaps at its initiation is zero, the digital default swap premium can be obtained by equating the value of the two legs

$$\frac{s^D_{t,\tau_c}}{M_c} = \mathcal{L} \cdot \frac{E^{\mathbb{Q}} \left[\int_t^{\tau_c} \lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t \right]}{\sum_{j=1}^{M_c \cdot (\tau_c - t)} E^{\mathbb{Q}} \left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t \right]} \tag{3}$$

Note that pricing formula DDS premiums is similar to that of spot CDS. The only difference is that loss given default in the numerator of the equation (1) is pulled out as a constant $\mathcal{L}$ in the equation (3).

2.4 Defaultable Bond

Let P_t denote a bond price at time t with the maturity of τ_b. A bond holder receives a periodic coupon payment until maturity conditional on no default at each coupon payment time. Coupon is paid M_b times a year and C denotes an annualized coupon rates for the bond. If there is no default until its maturity, the investor receives the principal (normalized as one). In the event of default, the bond holder only receive a fractional recovery of face value, $1 - L$ at the time of default.

The value of coupon payment until default is

$$\frac{C}{M_b} \sum_{j=1}^{M_b \cdot (\tau_b - t)} E^{\mathbb{Q}} \left[e^{-\int_t^{t+\frac{j}{M_b}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t \right]$$

The value of principal conditional on no default until maturity is

$$E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_b} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]$$

Finally, the value of recovery at default is

$$E^{\mathbb{Q}}\left[\int_t^{\tau_b}\big(1-L(\mu)\big)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right]$$

Then, the price of the bond is derived by summing up the value of coupon, principals and recovery.

$$\begin{aligned} P_t &= \frac{C}{M_b}\sum_{j=1}^{M_b\cdot(\tau_b-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_b}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]+E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_b} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] \\ &\quad +E^{\mathbb{Q}}\left[\int_t^{\tau_b}\big(1-L(\mu)\big)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \end{aligned} \tag{4}$$

3 Loss Given Default

Although loss given default is critical to the pricing of credit-related securities as shown in the previous section, convention within both academic analysis and industry practice is to treat it as a constant (e.g., Zhang (2004) and Pan and Singleton (2006)) and it is often pre-specified based on a historical average (e.g., Longstaff et al. (2005) and Adler and Song (2007)). It is set to lie in the $50-60\%$ range for U.S. corporates, and about 75% for sovereigns (see Das and Hanouna (2006) and Pan and Singleton (2006) for more details). However, historical averages of loss given default are measured in the actual probability space. Loss given default in the actual probability space cannot proxy loss given default in the risk neutral space, unless there is no risk premiums on recovery risk.

In this section, I contrast loss given default in the actual and risk neutral probability space and show that expected loss given default in the risk neutral space is generally bigger than that in the actual probability space. I also discuss how the historical average could have been used as a proxy for the risk neutral one.

3.1 LGD in actual and risk neutral probability space: illustration

In this section, I assume a three-state economy for the exposition of the relation between loss given default in the actual and risk neutral probability space. By construction, the payoff space is complete and the unique state price is defined. However, the result holds when the asset span is not complete, which allowing many combination of state prices, as long as there is no-arbitrage.

Suppose a three-state economy. State one represents no default of an obligor. State two represents the case of default of the obligor and a big loss (L_B). Finally the last state is for the case of default of the obligor with a small loss (L_S). The marginal rate of substitution (the pricing kernel) is respectively denoted as m_1, m_2, and m_3 for each state with probability of $\mathbb{P}_1^P$, $\mathbb{P}_2^P$, and $\mathbb{P}_3^P$. Let $\mathbb{P}_D^P$ denote the probability of default in the actual probability space. Let $\mathbb{P}_{D\cdot B}^P$ denote the joint probability of default with a big loss and $\mathbb{P}_{D\cdot S}^P$ denote the joint probability of default with a small loss in actual probability space. For state prices denoted q_1, q_2, and q_3, following equations hold.

$$\begin{aligned} q_1 &= \mathbb{P}_1^P \cdot m_1 = (1-\mathbb{P}_D^P)\cdot m_1 \\ q_2 &= \mathbb{P}_2^P \cdot m_2 = \mathbb{P}_{D\cdot B}^P \cdot m_2 \\ q_3 &= \mathbb{P}_3^P \cdot m_3 = \mathbb{P}_{D\cdot S}^P \cdot m_3 \end{aligned}$$

Risk neutral probability for each state is defined as

$$\begin{aligned} \mathbb{P}_1^Q &= \frac{q_1}{q_1+q_2+q_3} = (1-\mathbb{P}_D^Q) \\ \mathbb{P}_2^Q &= \frac{q_2}{q_1+q_2+q_3} = \mathbb{P}_{D\cdot B}^Q \\ \mathbb{P}_3^Q &= \frac{q_3}{q_1+q_2+q_3} = \mathbb{P}_{D\cdot S}^Q \end{aligned}$$

where $\mathbb{P}_D^Q$, and $\mathbb{P}_{D\cdot B}^Q$ ($\mathbb{P}_{D\cdot S}^Q$) respectively denotes the probability of default and the joint probability of default with a big (small) loss in the risk neutral probability space. Then

$$\begin{aligned}
\mathbb{P}^Q_{D} &= \frac{q_2+q_3}{q_1+q_2+q_3} \\
&= \frac{\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2+\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3}{(1-\mathbb{P}^P_{D})\cdot m_1+\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2+\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}^Q_{D\cdot B} &= \frac{q_2}{q_1+q_2+q_3} \\
&= \frac{\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2}{(1-\mathbb{P}^P_{D})\cdot m_1+\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2+\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3} \\
\mathbb{P}^Q_{D\cdot S} &= \frac{q_3}{q_1+q_2+q_3} \\
&= \frac{\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3}{(1-\mathbb{P}^P_{D})\cdot m_1+\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2+\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3}
\end{aligned}$$

The conditional probability of big loss given default in the risk neutral probability space is

$$\mathbb{P}^Q_{B|D} = \frac{\mathbb{P}^Q_{D\cdot B}}{\mathbb{P}^Q_{D}} = \frac{\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2}{\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2+\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3} = \frac{1}{1+\frac{\mathbb{P}^P_{D\cdot S}}{\mathbb{P}^P_{D\cdot B}}\cdot\frac{m_3}{m_2}} = \frac{1}{1+\frac{\mathbb{P}^P_{S|D}}{\mathbb{P}^P_{B|D}}\cdot\frac{m_3}{m_2}}$$

$\mathbb{P}^Q_{B|D}$ depends not only on the conditional probability of loss given default in the actual probability space, but also on the ratio of marginal rate of substitution of small and big loss states.

For the expectation of loss given default L in the risk neutral probability space,

$$\begin{aligned}
E^{\mathbb{Q}}[\,L\,] &= L_B\cdot\mathbb{P}^Q_{B|D}+L_S\cdot\mathbb{P}^Q_{S|D} \\
&= (L_B-L_S)\cdot\left(\frac{\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2}{\mathbb{P}^P_{D}\cdot\mathbb{P}^P_{B|D}\cdot m_2+\mathbb{P}^P_{D}\cdot\left(1-\mathbb{P}^P_{B|D}\right)\cdot m_3}\right)+L_S \qquad (5)
\end{aligned}$$

For the expectation of loss given default L in the actual probability space,

$$E^{\mathbb{P}}[\,L\,] = L_B\cdot\mathbb{P}^P_{B|D}+L_S\cdot\mathbb{P}^P_{S|D} = (L_B-L_S)\cdot\mathbb{P}^P_{B|D}+L_S \qquad (6)$$

Then the difference between the expectation of loss given default in the risk neutral and actual probability space is

$$
\begin{aligned}
E^{\mathbb{Q}}[\,L\,] - E^{\mathbb{P}}[\,L\,] &= (L_B - L_S) \cdot \left(\frac{\mathbb{P}^P_D \cdot \mathbb{P}^P_{B|D} \cdot m_2}{\mathbb{P}^P_D \cdot \mathbb{P}^P_{B|D} \cdot m_2 + \mathbb{P}^P_D \cdot \left(1 - \mathbb{P}^P_{B|D}\right) \cdot m_3} \right) - (L_B - L_S) \cdot \mathbb{P}^P_{B|D} \\
&= (L_B - L_S) \cdot \left(\frac{(m_2 - m_3) \cdot \left(1 - \mathbb{P}^P_{B|D}\right) \cdot \mathbb{P}^P_{B|D}}{\mathbb{P}^P_{B|D} \cdot m_2 + \left(1 - \mathbb{P}^P_{B|D}\right) \cdot m_3} \right) \qquad (7)
\end{aligned}
$$

It is notable from equation (7), $E^{\mathbb{Q}}[\,L\,] = E^{\mathbb{P}}[\,L\,]$ if and only if $L_B = L_S$ or $m_2 = m_3$. Significant cross-sectional variations in loss given default in the actual probability space found in Altman et al. (2005) implies that $L_B \neq L_S$.[11] In addition, loss given default on recent sovereign defaults also exhibits significant variations according to Moody's (2006).[12] Marginal rate of substitution (MRS) for the big loss state and small loss state differs unless there is no loss at the aggregate level. Higher MRS for the big loss state implies that $E^{\mathbb{Q}}[\,L\,] > E^{\mathbb{P}}[\,L\,]$.

3.2 LGD in actual and risk neutral probability space

Suppose that an insurance buyer faces the following maximization problem and solves it, where his utility function is denoted by $U(\cdot)$ and $U'(\cdot) > 0$, $U''(\cdot) < 0$. c_i denotes the consumption at time i, e_i denotes the endowment at time i and P, ξ and X respectfully

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[11] Altman et al. (2005) find default probability and loss given default are positively correlated at the aggregate level. They attribute the correlated relation in U.S. corporates to a business-cycle.

[12] Loss given default is as following with defaulted year in parenthesis: Dominican Republic (2005) 8%; Ukraine (2000) 31%; Moldova (2001) 34%; Uruguay (2003) 34%; Grenada (2004) 35%; Pakistan (1998) 35%; Ecuador (1999) 56%; Argentina (2001) 67%; Ivory Coast (2000) 82%, Russia (1998) 82%. It is the average, issuer weighted, trading price on a sovereign's bonds thirty days after its initial missed interest payment, or in cases in which the initial default event was the distressed exchange itself, it reports the average price shortly before the distressed exchange. Interestingly, at the announcement of exchange offers, which often occurred months after the first default event, the loss were substantially lower except Ukraine and Argentina: Dominican Republic 5%; Ukraine 40%; Moldova N/A; Uruguay 15%; Grenada N/A; Pakistan 35%; Ecuador (1999) 40%; Argentina 70%; Ivory Coast N/A, Russia 50%. For valuing CDS contracts, it is the loss in value on the underlying bonds within a month when an actual physical delivery occurs between the insurer and the insured. Pan and Singleton (2006) quotes traders that recovery depends on the size of the country (and the size and distribution of its external debt).

denote a price vector, number of holdings of each security and payoff.

$$\begin{aligned} \max \quad & U(c_0) + e^{-\beta} E^{\mathbb{P}} \left[U(c_1) \right] \\ & s.t. \\ & c_0 \leq e_0 - P'\xi \\ & c_1 \leq e_1 + \xi X \end{aligned}$$

Assume that ξ^* is the solution of the problem.

In addition, suppose that an insurance buyer faces a risk of loss rate L in the event of default of the principal claim F, where L is a random variable with probability density function $f(L)^{\mathbb{P}}$ under the actual probability space. An insurer provides $I(L)$ to the insurance buyer if loss L occurs. The insurance buyer maximizes the expected value of his utility of wealth. Let $P_{_I}$ be the price of the insurance policy and $P_D^{\mathbb{P}}$ be the probability of default under the actual probability space.

The insurance buyer's problems can be rewritten as

$$\begin{aligned} \max \quad & U(c_0) + e^{-\beta} E^{\mathbb{P}} \left[U(c_1) \right] \\ & s.t. \\ & c_0 \leq e_0 - P'\xi - \varphi \cdot P_{_I} \\ & c_1 \leq e_1 + \xi X - \varphi \cdot P_{_I}(1+r) + \varphi \cdot I(L)_{\{D\}} \end{aligned}$$

The solution will be ξ^* and $\varphi^* = 0$ and the following first order condition holds.

$$-U'(e_0 - P'\xi^*)P_{_I} + e^{-\beta} \left((1 - P_D^{\mathbb{P}}) U' \left(e_1 + \xi^* X_{\{\mathrm{ND}\}} \right) (-P_{_I}(1+r)) + P_{_D}^{\mathbb{P}} \int_0^1 U' \left(e_1 + \xi^* X_{\{\mathrm{D}\}} \right) (I(L) - P_{_I}(1+r)) \, f^{\mathbb{P}}(L) dL \right) = 0$$

Then

$$P_{_I} = \frac{P_{_D}^{\mathbb{P}} \cdot \int_0^1 U' \left(e_1 + \xi^* X_{\{\mathrm{D}\}} \right) \cdot I(L) \cdot f^{\mathbb{P}}(L) dL}{e^{\beta} U' \left(e_0 - P'\xi^* \right) + (1+r) \left[(1 - P_{_D}^{\mathbb{P}}) \cdot U' \left(e_1 + \xi^* X_{\{\mathrm{ND}\}} \right) + P_{_D}^{\mathbb{P}} \int_0^1 U' \left(e_1 + \xi^* X_{\{\mathrm{D}\}} \right) \cdot f^{\mathbb{P}}(L) dL \right]} \tag{8}$$

We need to rearrange the equation (8) such that

$$P_{_I} = P^{\mathbb{Q}}_{_D} \cdot \int_0^1 I(L) f^{\mathbb{Q}}(L) dL \tag{9}$$

Let's define $P^{\mathbb{Q}}_{_D}$ as

$$P^{\mathbb{Q}}_{_D} = \frac{P^{\mathbb{P}}_{_D} \cdot \int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L) dL}{e^{\beta} U'\Big(e_0 - P'\xi^*\Big) + (1+r)\Big[(1 - P^{\mathbb{P}}_{_D}) \cdot U'\Big(e_1 + \xi^* X_{\{\mathrm{ND}\}}\Big) + P^{\mathbb{P}}_{_D} \int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L) dL\Big]}$$

Then

$$f^{\mathbb{Q}}(L) = \frac{U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)}{\int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L) dL}$$

It follows that

$$\begin{aligned} E^{\mathbb{Q}}[L] &= \int_0^1 L \cdot f^{\mathbb{Q}}(L) dL \\ &= \int_0^1 L \cdot \frac{U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)}{\int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L) dL} dL \end{aligned}$$

Then the difference in the expectation of loss rate in the risk neutral and actual probability space is

$$E^{\mathbb{Q}}[L] - E^{\mathbb{P}}[L] = \int_0^1 L \cdot \frac{U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)dL}{\int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)dL} dL - \int_0^1 L \cdot f^{\mathbb{P}}(L)dL$$

$$= \frac{\int_0^1 L \cdot U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)dL - \int_0^1 L \cdot f^{\mathbb{P}}(L)dL \cdot \int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)dL}{\int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)dL}$$

$$= \frac{\mathrm{COV}^{\mathbb{P}}\Big[L, U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big)\Big]}{\int_0^1 U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big) \cdot f^{\mathbb{P}}(L)dL} > 0$$

The last relation hold because $U'\Big(e_1 + \xi^* X_{\{\mathrm{D}\}}\Big)$ increases as L increases; higher L implies less payoff at default states. Since we assume $U''() < 0$, marginal utility is bigger when payoff is less.

3.3 Identification of Loss Given Default

In spite of the shortfall of the historical average as a proxy, it has been used mainly because of the econometrical infeasibility of the separation of two components; when contracts are priced under the fractional recovery of market value convention (RMV) introduced by Duffie and Singleton (1999), the product of default intensity and loss given default determines prices. Arbitrary choice of loss given default is compensated by the corresponding adjustment of default intensity. In this case, random choice of a fixed number for loss given default does not affect the statistical fit of the model being tested.

CDS is priced with the framework of fractional recovery of face value (RFV). And default intensity and loss given default can be identified *in principle*. However, at practical level, several sets of loss given default and default intensity provide equally good fits for observed CDS premiums.[13] Pan and Singleton (2006) jointly estimates loss given default and default intensity using CDS data for sovereigns. They take loss given default as a constant and estimate it using the (quasi) maximum likelihood estimation method. Estimates for loss

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[13] see Duffie (1999) for details.

given default in their study are 23%, 24% and 83% for Mexico, Turkey and Korea. However, it should be noted the maximum likelihood with unrestricted loss given default is about the same with the case where they impose the 75% restriction. Likelihood are 32.030 (32.126), 27.213 (27.700), and 36.626 (36.626) for Mexico, Turkey and Korea in restricted (unrestricted) model. It illustrates the difficulties of identification of loss given default and default intensity by solely using CDS data.

However, a certain set of $\{L, \lambda\}$ providing good fits for CDS may not work well for other securities, e.g. 'classic' bullet bonds. Figure 1 illustrates this. For the simplicity, the short rate is set as a constant, 5%. Bond pays semi-annual coupon with coupon rate, 8%. Default intensity, λ, is set with a range of $[\ 0.01, 0.2\ ]$ and loss given default, L is set with $[\ 0.1, 0.9\ ]$. With this set-up, the range of CDS premiums generated is within $[\ 0.0, 0.2\ ]$ (figure 1(a)). It should be note that many combination of $\{L, \lambda\}$ fit a given CDS premium. The intersection between the plane, '$premium = f(L, \lambda)$', where f is the graph such that '$f : (L, \lambda) \rightarrow premiums$' and the other plane '$premium = a\ constant$' is generally a line, not a point. All points on the line are combinations of $\{L, \lambda\}$ that perfectly fit a given CDS premium.

As with CDS, many combination of $\{L, \lambda\}$ fit a given bond price. The intersection between the plane, '$Price = g(L, \lambda)$', where g is the graph such that '$g : (L, \lambda) \rightarrow\ Price$' and the other plane '$Price = a\ constant$' is generally a line, not a point. All points on the line are combinations of $\{L, \lambda\}$ that perfectly fit a given bond price. With the same range of default intensity and loss given default, bond price is in $[\ 0.5, 1.2\ ]$ (figure 1(b)).

[Insert Figure 1 here]

However, combination that matches both CDS premiums and bond price are not as many as those which match only one of them. In this example, cross-sectional restriction between CDS and bond not only explores the additional price information in both securities, but also significantly improves the identification of loss given default and default intensity. This fact is illustrated in figure 2; Given a pair of observed bond price and CDS premium, the L is uniquely identified. It is also notable that when CDS premiums are low, bond prices does not vary a lot as L varies. However, as CDS premiums get high, bond prices significantly varies.

[Insert Figure 2 here]

From above, I show that cross-sectional restrictions among different securities improve the identification. Usually, for a certain reference entity, there are several types of securities traded with exposure to the credit risk of that entity. With cross-section of these securities, we can improve the identification of loss given default and default intensity.

4 Separation of Default Intensity and Loss Given Default

In this section, I develop frameworks for the separate identification of default intensity and loss given default. The separate identification comes from the cross-sectional no-arbitrage restriction between securities with exposure to the credit risk of the common reference entity. Spot CDS, forward CDS, digital default swaps (DDS) and defaultable bonds are among the most common single name securities with credit exposure. Imposing the restrictions between stand-alone pricing functions of each security, which are derived in terms of default intensity and loss given default in the previous section, I derive new pricing formulae for forward CDS, DDS and defaultable bonds in terms of spot CDS premiums and one of the two components, either default intensity or loss given default. These pricing methods are different from those in the previous section. While the pricing functions in the previous section are 'absolute' pricing in the sense that they are derived in terms of default intensity and loss given default of the reference entity, the pricing functions in this section are 'relative' pricing in that they are expressed with other securities' price.[14]

4.1 Implied Forward CDS Premiums: No Arbitrage Restriction between Spot CDS and Forward CDS

In this section, I impose a cross-sectional no-arbitrage restriction between spot CDS and forward CDS and develop a new pricing framework for forward CDS. I call the new pricing equation for the forward CDS premiums 'implied forward CDS premiums', since the price is

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[14] Prior studies such as Madan and Unal (1998), Unal, Madan, and Guntay (2001) and Bakshi et al. (2001) use multiple debt securities for the separation. Madan and Unal (1998) requires the existence of two debt securities with different seniorities. Bakshi et al. (2001) needs large cross-section of bonds in their estimation.

implied by the no-arbitrage restriction. The new pricing formula has two distinctive features. First, it allows the separation between default intensity and loss given default. The difficulties of the separation are well documented in the previous literature (e.g. Duffie and Singleton (1999), Longstaff et al. (2005), Pan and Singleton (2006) and Le (2007)). In equation (2), the forward CDS premium is derived in terms of the short interest rate, default intensity, and loss given default. However, the implied forward CDS premiums are derived in terms of other observable spot CDS premiums and default intensity. In the derivation of the implied CDS premiums, loss given default is canceled out leading to the separate identification of default intensity. In addition, the cross-sectional no-arbitrage restriction between spot and forward CDS does not require any assumption on the process specification of default intensity and loss given default. Previous studies(e.g. Duffie (1999), Zhang (2004), Longstaff et al. (2005), Pan and Singleton (2006)) assume the independence among the short rate, default intensity and loss given default. Furthermore, they assume that loss given default is constant, not time varying.

Suppose a forward CDS contract, with expiry τ_f, to buy and sell a CDS with time to maturity $(\tau_c - \tau_f)$. Suppose also two spot CDS contracts, at time t, with time to maturity $(\tau_f - t)$ and $(\tau_c - t)$. From equation (1), two spot CDS premiums with time to maturity $(\tau_f - t)$ and $(\tau_c - t)$ are respectively priced as below.

$$\frac{s_{t,\tau_f}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] = E^{\mathbb{Q}}\left[\int_t^{\tau_f} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \quad (10)$$

$$\frac{s_{t,\tau_c}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] = E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \quad (11)$$

From equation (2), the following equation holds for the forward CDS.

$$\frac{s_{\tau_f,\tau_c}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] = E^{\mathbb{Q}}\left[\int_{\tau_f}^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \quad (12)$$

When I add equation (10) to equation (12), it leads to the equation(11). It is notable that the equality should always hold regardless of the process specification of default intensity and loss given default, for the right hand side of summation.

$$
\begin{aligned}
& E^{\mathbb{Q}}\left[\int_t^{\tau_f} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] + E^{\mathbb{Q}}\left[\int_{\tau_f}^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \\
= \; & E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right]
\end{aligned}
$$

For the left hand side,

$$
\begin{aligned}
& \frac{s_{t,\tau_f}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{Q}\left[e^{-\int_t^{t+\frac{j}{M}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] + \frac{s_{\tau_f,\tau_c}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{Q}\left[e^{-\int_t^{\tau_f+\frac{j}{M}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] \\
= \; & \frac{s_{t,\tau_c}}{M_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]
\end{aligned}
$$

Then the forward CDS premiums, s_{t_f,τ_c}, is

$$
s_{\tau_f,\tau_c} = \frac{s_{t,\tau_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] - s_{t,\tau_f}\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]} \tag{13}
$$

It should be noted that, in equation (13) for the forward CDS premium, s_{τ_f,τ_c}, loss given default is not present. With the implied forward premiums, default intensity, not contaminated by loss given default, is derived in terms of other observable spot and forward CDS premiums. More importantly, the equation (13) holds regardless of the process specification of the default arrival intensities and loss given default. Unlike the separation through the ratio of CDS premiums with different maturity, proposed by Pan and Singleton (2006), the assumption that $L(\mu)$ is a constant, is not necessary any more.

4.2 Implied DDS Premiums: No Arbitrage Restriction between CDS and DDS

In this section, I impose a cross-sectional no-arbitrage restriction between CDS and digital default swaps (DDS) and develop a new pricing framework for DDS. I call the new pricing equation for DDS premiums 'implied DDS premiums', since the price is implied by the no-arbitrage restriction. The implied DDS premiums have a remarkable feature that they result in the separation where loss given default remains with the absence of default intensity. We can directly get a measure of expected loss given default by comparing the CDS and DDS premiums. It is notable that in the equation (3), DDS premiums are derived in terms of default intensity, not loss given default. However, implied DDS premiums derived in this section are expressed in term of loss given default, not default intensity.

Suppose that two parties make a digital default swaps contract with maturity τ_c at time t, with premiums s^D_{t,τ_c}. The payment is made M_c times per unit time until any one of the following events happens: the underlying reference entity defaults on its reference obligation or the maturity of the CDS contract comes. In the event of default, the seller of a protection pay the pre-specified amount, $\mathcal{L}$, to the protection buyer. Then from the equation (3),

$$\frac{s^D_{t,\tau_c}}{M_c} = \mathcal{L} \cdot \frac{E^{\mathbb{Q}}\left[\int_t^{\tau_c} \lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c - t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t\right]} \tag{14}$$

From the equation (1), a spot CDS contract with the same maturity with premiums, s_{t,τ_c}, is prices as below

$$\frac{s_{t,\tau_c}}{M_c} = \frac{E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c - t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t\right]} \tag{15}$$

From equations (14) and (15),

$$s^D_{t,\tau_c} = \mathcal{L} \cdot \frac{s_{t,\tau_c} \cdot E^{\mathbb{Q}}\left[\int_t^{\tau_c} \lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t\right]}{E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t\right]} \tag{16}$$

When we assume that $L(\mu)$ is not correlated with $\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}$,

$$s^D_{t,\tau_c} = \frac{\mathcal{L}}{E^{\mathbb{Q}}\left[L(\mu) \middle| \mathcal{F}_t\right]} \cdot s_{t,\tau_c} \tag{17}$$

Note that the implied DDS premiums are derived in terms of risk neutral expectation of loss given default. Extension to the forward DDS premium pricing is straightforward; replace subscript t in equation (17) with τ_f, the expiry of the forward contract.

4.3 Implied Bond Price:

No Arbitrage Restriction between CDS and Bond

In this section, I impose a cross-sectional no-arbitrage restriction between CDS and bond, and develop a new pricing framework for bond. I call the new pricing equation for bond price 'implied bond price', since the price is implied by the no-arbitrage restriction. The implied bond price is derived in term of default intensity, not loss given default.

Suppose a bond with an annualized coupon C, number of payment M_b and maturity τ_b. From the equation (4), bond price P_t is as below

$$\begin{aligned} P_t = \quad & \frac{C}{M_b} \sum_{j=1}^{M_b \cdot (\tau_b - t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_b}} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t\right] + E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_b} r(s)+\lambda(s)ds} \middle| \mathcal{F}_t\right] \\ & + E^{\mathbb{Q}}\left[\int_t^{\tau_b} \big(1 - L(\mu)\big)\lambda(\mu) e^{-\int_t^{\mu} r(s)+\lambda(s)ds} d\mu \middle| \mathcal{F}_t\right] \end{aligned} \tag{18}$$

Suppose a spot CDS contract with premiums s_{t,τ_c}, number of payment M_c and maturity τ_c.

From the equation (1),

$$\frac{s_{t,\tau_c}}{M_c} = \frac{E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]} \tag{19}$$

When $M_b = M_c$ and $\tau_b = \tau_c$,

$$\begin{aligned} P_t = \quad & \frac{C - s_{t,\tau_c}}{M_b} \sum_{j=1}^{M_b\cdot(\tau_b-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_b}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] + E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_b} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] \\ & + E^{\mathbb{Q}}\left[\int_t^{\tau_b} \lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \end{aligned} \tag{20}$$

With cross sectional no-arbitrage restriction between CDS and bond, defaultable bond price is derived in term of default intensity that is not contaminated by loss given default. The equation (20) holds regardless of the process specification of the default intensity and loss given default.

When we assume that $L(\mu)$ is not correlated with $\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}$, same maturities of CDS and bond are not necessary. In this case bond price is as below

$$\begin{aligned} P_t \;=\; & \frac{C}{M_b} \sum_{j=1}^{M_b\cdot(\tau_b-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_b}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] + E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_b} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] + E^{\mathbb{Q}}\left[\int_t^{\tau_b} \lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right] \\ & - \frac{s_{t,\tau_c}}{M_c} \sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right] \cdot \frac{E^{\mathbb{Q}}\left[\int_t^{\tau_b} \lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right]}{E^{\mathbb{Q}}\left[\int_t^{\tau_c} \lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu\middle|\mathcal{F}_t\right]} \end{aligned} \tag{21}$$

5 Estimation of Loss Given Default

5.1 Loss Given Default in Spot and Forward CDS of Sovereigns

In this section, I provide the empirical estimate of loss given default prevailing in the CDS market. Unlike the previous studies, I impose a cross-sectional restriction between *spot* and *forward* CDS premiums. The cross sectional restriction between spot CDSs and forward CDSs is crucial, since it cancel out loss given default in the pricing model. For forward CDS contract, the premiums are function of the short rate, default intensity and loss given default as in the equation (22).

$$\frac{s_{\tau_f,\tau_c}}{M_c} = \frac{E^{\mathbb{Q}}\left[\int_{\tau_f}^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu \middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle| \mathcal{F}_t\right]} \tag{22}$$

However, with cross-sectional restriction, forward CDS premiums are derived in terms of observable spot CDS premiums and default intensity, as in the equation (23).

$$s_{\tau_f,\tau_c} = \frac{s_{t,\tau_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle| \mathcal{F}_t\right] - s_{t,\tau_f}\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle| \mathcal{F}_t\right]} \tag{23}$$

5.1.1 Data

Daily data from 1999 to 2005 for spot and forward CDS premiums are supplied by J.P. Morgan Securities, one of the leading players in the CDS market. Countries in the sample are Bulgaria, Brazil, Colombia, Korea, Mexico, Malaysia, Philippines, Poland, Turkey, and Venezuela. These CDS contracts are standard ISDA contracts for physical settlement for Emerging Market (EM) Sovereigns. The notional value of contract (lot size) is between five to ten million USD for a large market like Brazil, while it is typically between two to five million for small markets. The prices hold at 'close of business.' For riskless rates, I collect data for the constant maturity rate for six-month, one-year, two-year, three-year, five-year, seven-year, and ten-year rates from the Federal Reserve.

5.1.2 Summary Statistics

Table 1 provides the basic statistic for spot CDS premiums with one, three, five, seven and ten year maturities. Noticeable pattern in spot CDS premiums is that mean, median, minimum of CDS premiums increase in maturity for all countries. However, maximum of CDS premiums with short maturities are often higher than ones with long maturities, which results from the inverted CDS premiums curve during the high credit risk period. Pan and Singleton (2006) document the positive slope of spread curve as a prominent feature of the CDS data. Spread curves for Mexico and Korea never show inversion in their study even though they find inversion for Turkey (Figure 3). In my sample, curves for Korea and Mexico are also inverted during the Long Term Capital Management and Russian Default crisis (Figure 4 and 5).

[Insert Table 1 here]

[Insert Figure 3 here]

[Insert Figure 4 here]

[Insert Figure 5 here]

Table 2 provides the basic statistic for forward CDS premiums with one, three, five, and seven year expiry. They all have the same maturity of ten years. Similar patterns to spot CDS premiums appear in forward CDS premiums; mean, median, minimum of CDS premiums increase in maturity for all countries. But the inversion occurs for maximum of CDS premiums, particularly with high level of CDS premiums.

One interesting movement occurs for Brazil during 2002. Spot CDS premiums reached the highest levels in the sample; one year premiums are 4,645bp and ten year premiums are 3,315bp. Around the highest premium period, the level of premiums of one year expiry and ten year maturity forward CDS also peaks. However, other forward premiums with longer expiry decreased rather than increased (Figure 6). This pattern implies that market believe the near term default is very likely, but conditional on no default in near term, the default likelihood is not high for longer term and it would even get lower.

[Insert Table 2 here]

[Insert Figure 6 here]

The bid-ask spreads for spot CDS with 5 year maturity range between 10 and 110 for Brazil, 10 and 60 for Korea, 10 and 90 for Mexico, 10 and 15 for Malaysia, 6 and 60 for

Panama, 30 and 60 for Turkey, and 2 and 120 for Venezuela. The bid-ask spreads for contracts with other maturities are comparable in magnitude to those of the five-year contracts.

[Insert Table 3 here]

5.1.3 Estimation of Loss Given Default

CDS premiums with maturity τ_c is

$$\begin{aligned} \frac{s_{t,\tau_c}}{M_c} &= \frac{E^{\mathbb{Q}}\left[\int_t^{\tau_c} L(\mu)\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu \middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]} \\ &= \mathbb{L}\cdot\frac{E^{\mathbb{Q}}\left[\int_t^{\tau_c} \lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}d\mu \middle| \mathcal{F}_t\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]} \end{aligned} \tag{24}$$

when $L(\mu)$ is assumed not to be correlated with $\lambda(\mu)e^{-\int_t^{\mu} r(s)+\lambda(s)ds}$. $\mathbb{L}$ denotes $E^{\mathbb{Q}}\left[\ L \middle| \mathcal{F}_t\right]$. Using 10 spot CDS contract with one to ten year maturity, I bootstrap $E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t\right]$, $j = \{1, 2, 3, \dots, M_c(\tau_c - t)\}$ on each day. $\mathbb{L} \in (0, 1)$ and it is varied by 0.01 (equivalent to 1%). I define the pricing error in forward CDS premiums with expiry τ_f and maturity τ_c given loss given default $\mathbb{L}$ as

$$\begin{aligned} &\epsilon(t,\tau_f,\tau_c;\mathbb{L}) \\ = \quad & s_{\tau_f,\tau_c} - \frac{s_{t,\tau_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right] - s_{t,\tau_f}\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]} \end{aligned} \tag{25}$$

Pricing error, $\epsilon(t, \tau_f, \tau_c; \mathbb{L})$, is obtained on the daily basis. As shown in the Figure 7, loss given default of 75% prevails in CDS markets for Mexico. Similar patterns are observed for other countries.

[Insert Figure 7 here]

$E^{\mathbb{Q}}\left[L \middle| \mathcal{F}_t \right]$ is a function of loss given default in the actual probability space and the marginal rate of substitution. $E^{\mathbb{Q}}\left[L \middle| \mathcal{F}_t \right]$ increases either when $E^{\mathbb{P}}\left[L \middle| \mathcal{F}_t \right]$ increases or when marginal rate of substitution at big loss state gets bigger than the one at small loss state. Default probability and loss given default are positively correlated at the aggregate level in actual probability space as found in Altman et al. (2005). This positive correlation can be attributed the correlated relation in U.S. corporates to a business-cycle.[15] Recent severe credit deteriorations of emerging market sovereigns, however, come from idiosyncratic political circumstances rather than a business cycle. Spike in CDS premiums in Brazil corresponds to the period when Luiz Inacio Lula da Silva, known as Lula, won presidential elections and began to lead the first left-wing government in 40 years. Venezuelan crisis also corresponds to the period of political turmoil in the country.[16] Political instability may lead to Sovereign default. Furthermore it may adversely affect the negotiation process after the default, leading to the high loss. I find the high loss given default accompanying high default intensity in Brazil and Venezuela. Other than these two countries, I do not find significant time variation in loss given default during the sample periods.

[Insert Figure 8 here]

Based on the finding that loss given default are not time-varying in most cases, I calculate $RMSE(\mathbb{L})$, defined as below, to find out the prevailing loss given default and pricing error associated with it.

$$RMSE(\mathbb{L}) = \sqrt{\frac{1}{\sum_t 1} \cdot \sum_t \epsilon^2(t, \tau_f, \tau_c; \mathbb{L})} \tag{26}$$

Table 4 reports the $RMSE$ for various $\mathbb{L}$ with various maturities. Some noticeable patterns are observed. First of all, loss given default around 75% provides the smallest pricing error for

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[15] In recession, defaults are correlated aggregate level of default is high. Clustered defaults lead to distress asset sales with high loss given default. Chichilnisky and Wu (2006) show how individual risk event can be propagated and magnified into a major widespread default. They show that in an open set of economies, individual default leads to a widespread default no matter how large the economy is. The propagation of default may cause the devaluation of the assets, leading to the positive correlation between loss given default and default intensity.

[16] Armed forces head announced Chavez has resigned and Chavez was taken into military custody in April, 2002. A few days later, Chavez returned to office. Opposition party demanded that Chavez resign.

most cases. Furthermore, pricing errors exhibits '**V**' shaped pattern: they initially decrease in loss given default, reach the bottom with $\mathbb{L}$ around 75%, and increase thereafter. Pricing errors get larger with lower level of loss given default. 25% level of loss given default generates the pricing error amounting to several multiples of bid-ask spreads of corresponding spot CDS. Considering diverse economic fundamentals across countries in the sample, the uniform 75% result is surprising.

[Insert Table 4 here]

Interestingly, 25% of loss given default persistently generates the negative pricing error as shown in figure 7.[17] It is noticeable that 50% of loss given default also leads to the negative pricing error with smaller magnitude. Pricing errors turn into the positive numbers as the loss given default increases. 75% loss given default persistently generates the small pricing error over time. The reason that loss given default below 75% generates the negative pricing errors is that the corresponding default probability fitting spot CDS premiums is higher than the ones markets believe. From equation (25),

$$\begin{aligned}
&\epsilon(t,\tau_f,\tau_c;\mathbb{L}) \\
=\;& s_{\tau_f,\tau_c} - \frac{s_{t,\tau_c}\sum_{j=1}^{M_c\cdot(\tau_c-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right] - s_{t,\tau_f}\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]} \\
=\;& s_{\tau_f,\tau_c} - s_{t,\tau_c} - \left(s_{t,\tau_c} - s_{t,\tau_f}\right)\cdot\frac{\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]}{\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]} \qquad (27)
\end{aligned}$$

In the equation (27), $(s_{\tau_f,\tau_c} - s_{t,\tau_c}) > 0$ with upward sloping CDS term structure. So does $(s_{t,\tau_c} - s_{t,\tau_f}) > 0$ as well. Negative pricing errors are induced by the large value of $\sum_{j=1}^{M_c\cdot(\tau_f-t)} E^{\mathbb{Q}}\left[e^{-\int_t^{t+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]\Big/\sum_{j=1}^{M_c\cdot(\tau_c-\tau_f)} E^{\mathbb{Q}}\left[e^{-\int_t^{\tau_f+\frac{j}{M_c}} r(s)+\lambda(s)ds}\middle|\mathcal{F}_t,\mathbb{L}\right]$ (to be denoted as $A(t,\tau_f,\tau_c,r,\lambda;\mathcal{F}_t,\mathbb{L})$ hereafter) during the normal time with upward sloping CDS term structure. Lower level of loss given default is associated with higher level of default intensity when it is fitted to the CDS premiums, which leads to the higher value of $A(t,\tau_f,\tau_c,r,\lambda;\mathcal{F}_t,\mathbb{L})$. Therefore, negative pricing errors come as a result with lower level of

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[17] Same pattern are observed for all other countries in the sample.

loss given default than one prevailing in the CDS market. Same reasoning applies to the positive pricing errors associated with loss given default higher than 75%.

Another feature of the result is that different level of loss given default does not induce the pricing error beyond its transaction cost for countries with good credit quality. In my sample, Korea, Malaysia and Panama exhibit relatively flat and small pricing errors comparing to the other countries. These three countries are those with best credit quality in the sample.[18] With loss given default varying from 25% to 75%, the differences in the pricing errors are only about a few basis points, which could be negligible considering the transaction cost. As already shown in the equation (27), $A(t, \tau_f, \tau_c, r, \lambda; \mathcal{F}_t, \mathbb{L})$ is the main determinant of the magnitude of pricing errors in forward CDSs. Difference between the values of $A(t, \tau_f, \tau_c, r, \lambda; \mathcal{F}_t, \mathbb{L})$ with true λ prevailing in the market and mis-estimated λ is small with low level of default intensity, which leads to small pricing errors.

6 Conclusion

Two major components of credit risk, default intensity and loss given default, can be separately identified with cross-sectional no-arbitrage restrictions. I develop various frameworks for the separation by imposing cross sectional no-arbitrage restrictions between credit instruments, including spot and forward credit default swaps, digital default swaps and bonds. These frameworks allow the pure measure of default intensity not contaminated by the loss given default. Particularly, the restriction between spot and forward CDS provides the separation of default intensity independent of process specification of loss given default; it allows time varying loss given default and various correlation structures between loss given default and default intensity. Using spot and forward CDS premiums of 10 emerging market sovereigns, I find that 75% level of loss given default prevails in the sovereign CDS markets across countries over time. Positive correlation between loss given default and default intensity is found in Brazil and Venezuela during the period of political turmoils in each country. Loss given default below 75% generates negative pricing errors in forward CDS and the magnitude of

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[18] They have the smallest CDS premiums during the sample period where both spot and forward CDS premiums are available.

them is economically significant. These persistent negative pricing errors with mis-specified loss given default higher than the true one are consistent with the model developed. Assessing the loss given default with other securities such as bonds remains for further research.

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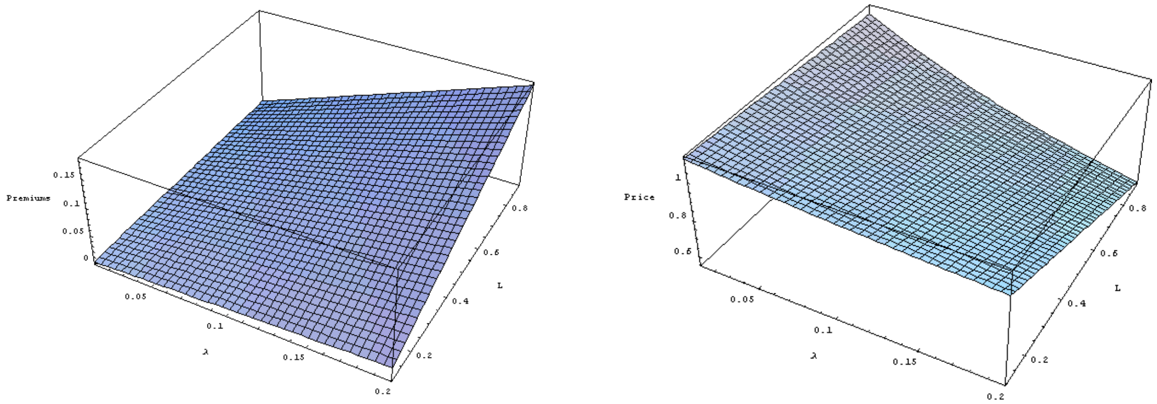
Figure 1: CDS Premiums and Bond Prices



(a) CDS Premiums

(b) Bond Prices

Figure 2: Identification of L^Q

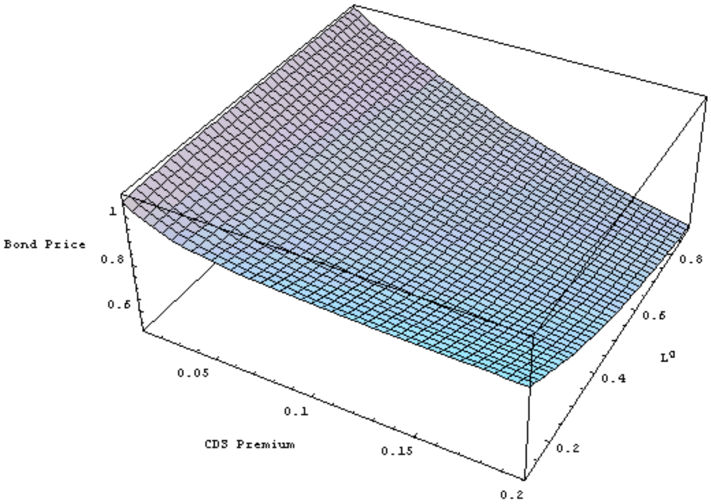
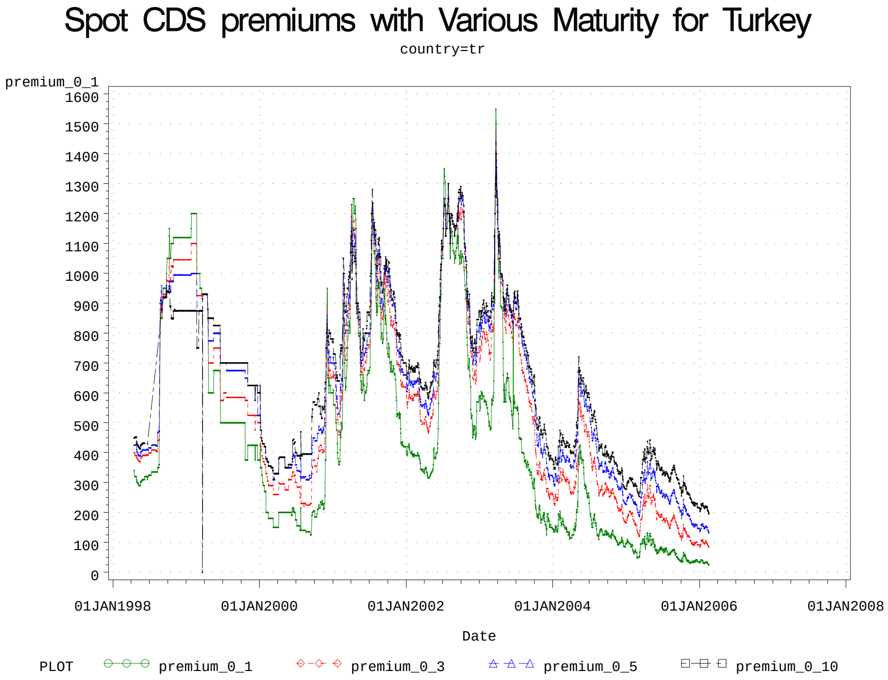
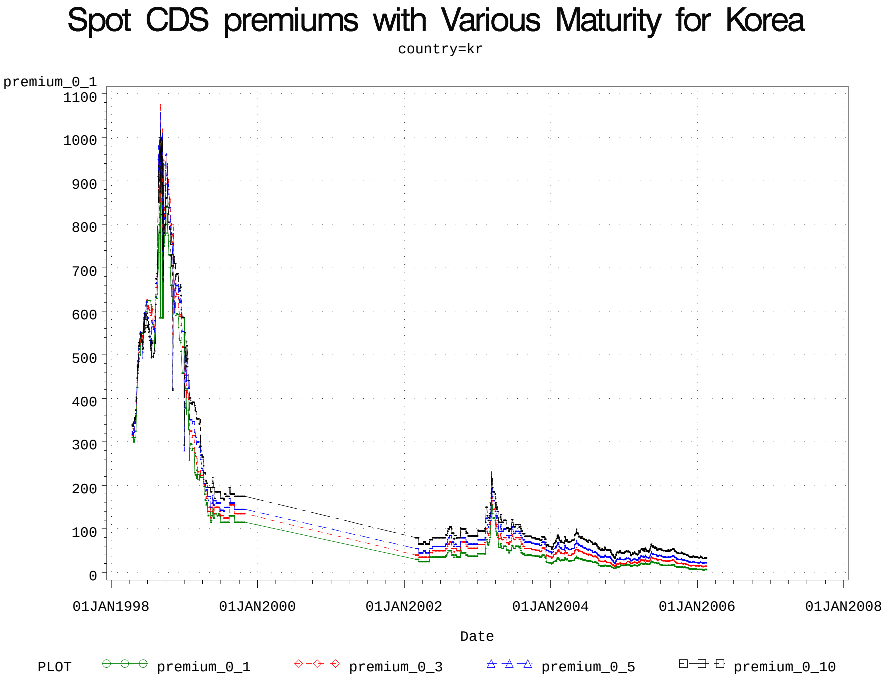


Figure 3: Spot CDS Premiums



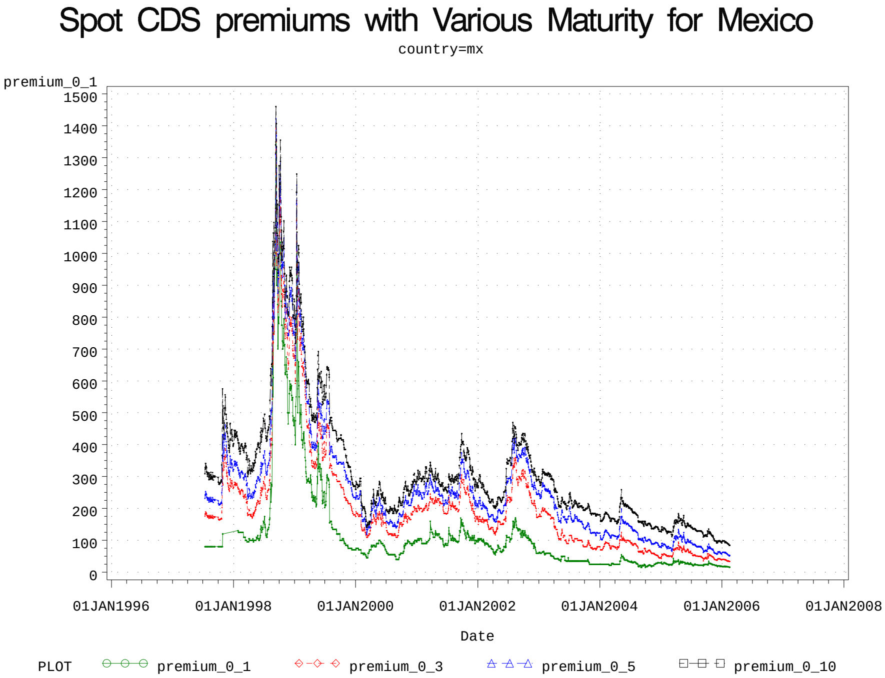
(a) Spot CDS

Figure 4: Spot CDS Premiums



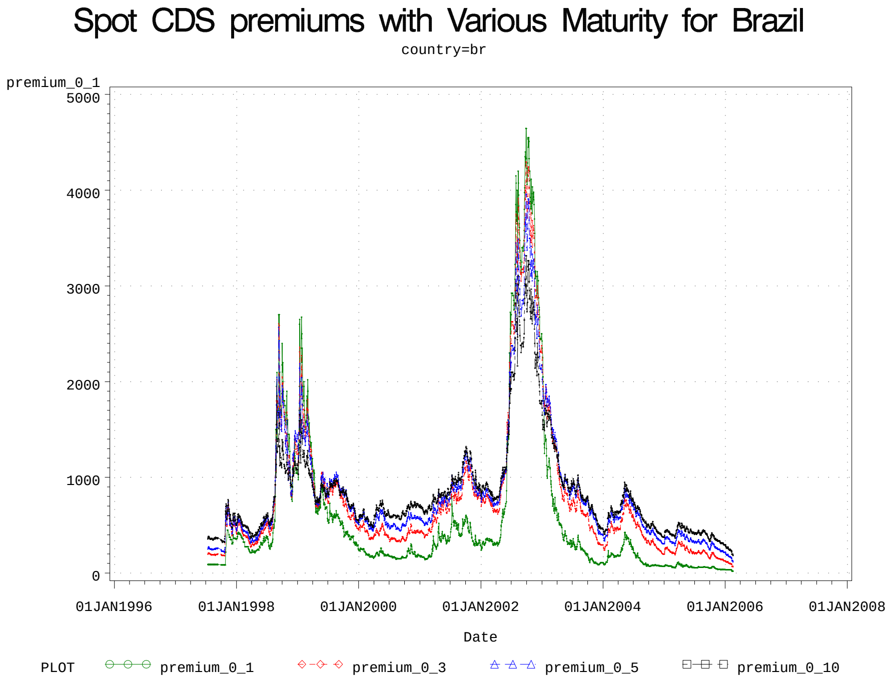
(a) Spot CDS

Figure 5: Spot CDS Premiums

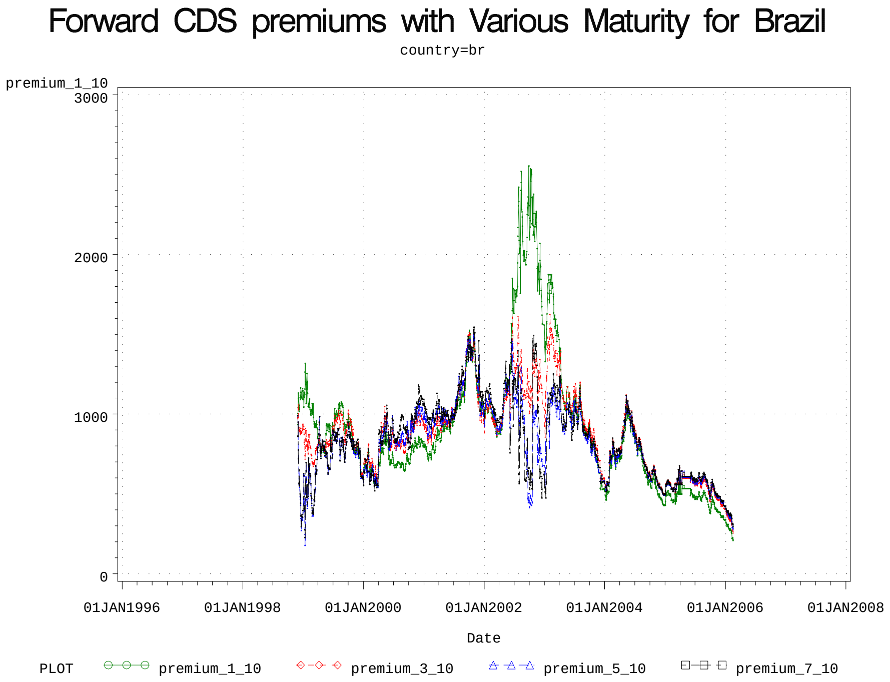


(a) Spot CDS

Figure 6: Spot and Forward CDS Premiums



(a) Spot CDS



(b) Forward CDS

Figure 7: Forward CDS Pricing Error: Mexico

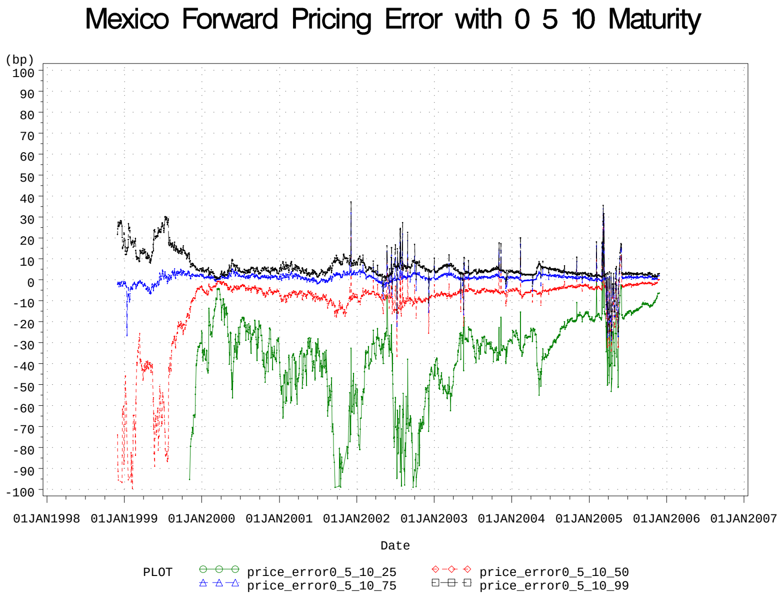
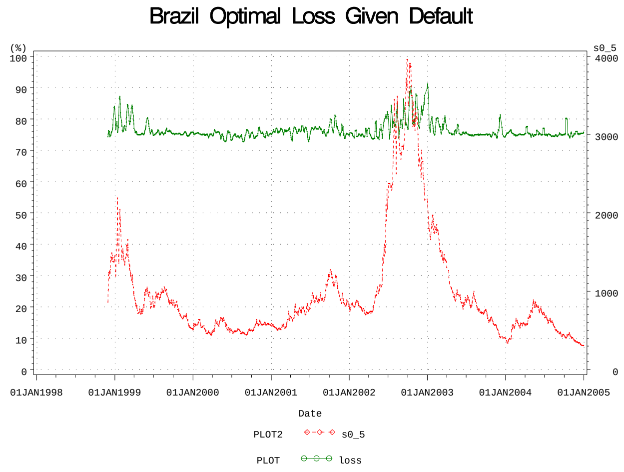
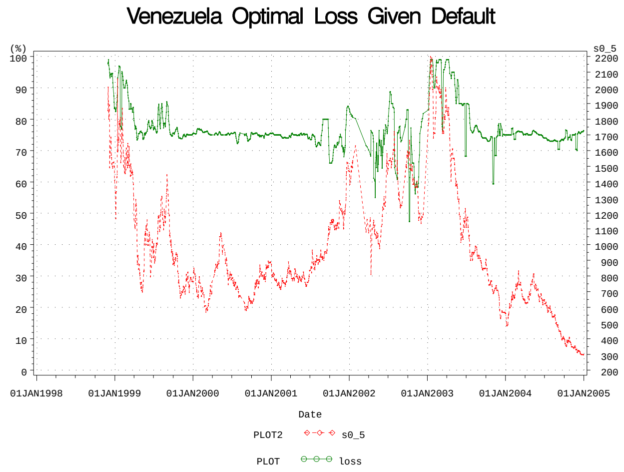


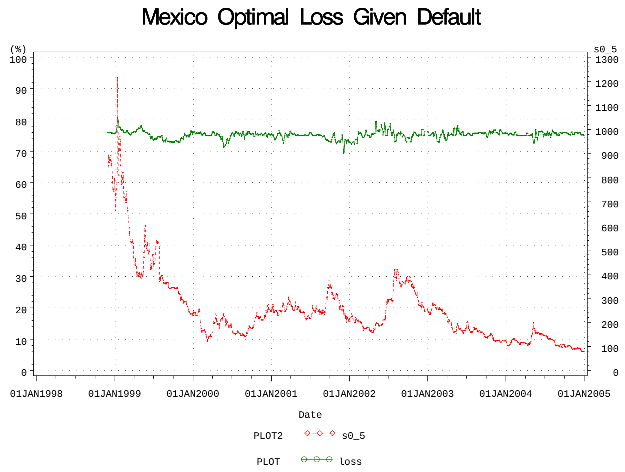
Figure 8: Optimal Loss Given Default



(a) Brazil



(b) Venezuela



(c) Mexico

Table 1: Basic Statistics: Spot CDS Premiums

This table provides the basic statistics of spot CDS premiums.

country	Maturity	beg. date	end. date	obs. Num.	mean	median	std	min	max
BG	1	7/6/1998	2/17/2006	1,903	182.6	95.0	231.4	11.0	2,000.0
	3	7/6/1998	2/17/2006	1,903	273.2	230.0	234.5	16.0	1,901.0
	5	7/6/1998	2/17/2006	1,903	335.8	320.0	246.8	22.0	1,782.0
	7	7/6/1998	2/17/2006	1,903	368.0	360.0	253.6	28.0	1,768.0
	10	7/6/1998	2/17/2006	1,903	393.8	390.0	259.3	35.0	1,743.0
BR	1	7/14/1997	2/17/2006	2,117	584.4	285.0	866.5	20.0	4,645.0
	3	7/14/1997	2/17/2006	2,126	779.7	525.0	748.2	66.0	4,295.0
	5	7/14/1997	2/17/2006	2,126	836.7	634.2	653.4	120.0	3,960.0
	7	7/14/1997	2/17/2006	2,126	834.1	668.9	576.6	151.0	3,610.0
	10	7/14/1997	2/17/2006	2,126	835.6	707.3	516.8	183.0	3,315.0
CO	1	8/25/1998	2/17/2006	1,868	294.7	295.0	208.2	30.0	1,100.0
	3	8/25/1998	2/17/2006	1,868	494.6	520.0	239.1	59.0	1,375.0
	5	8/25/1998	2/17/2006	1,868	573.2	580.0	219.9	110.0	1,380.0
	7	8/25/1998	2/17/2006	1,868	602.8	610.0	209.8	147.0	1,380.0
	10	8/25/1998	2/17/2006	1,868	624.9	635.0	203.2	173.0	1,380.0
KR	1	4/16/1998	2/17/2006	1,376	125.6	37.1	198.2	6.0	1,000.0
	3	4/16/1998	2/17/2006	1,376	144.6	52.1	213.5	13.0	1,076.0
	5	4/16/1998	2/17/2006	1,376	156.0	65.0	210.4	21.0	1,056.0
	7	4/16/1998	2/17/2006	1,376	163.0	73.2	206.5	25.0	1,036.0
	10	4/16/1998	2/17/2006	1,376	170.8	82.0	201.3	32.0	1,016.0
MX	1	7/14/1997	2/17/2006	2,070	116.4	80.0	161.9	15.0	1,350.0
	3	7/14/1997	2/17/2006	2,126	209.2	172.1	186.5	34.0	1,409.0
	5	7/14/1997	2/17/2006	2,126	260.1	221.8	197.1	52.0	1,423.0
	7	7/14/1997	2/17/2006	2,126	290.0	250.0	201.0	68.0	1,444.0
	10	7/14/1997	2/17/2006	2,126	315.2	270.0	203.9	85.0	1,461.0
MY	1	6/15/1998	2/17/2006	1,105	165.6	17.5	320.0	6.6	1,700.0
	3	6/15/1998	2/17/2006	1,105	171.6	34.5	291.0	13.8	1,576.0
	5	6/15/1998	2/17/2006	1,105	181.2	49.5	276.1	20.0	1,501.0
	7	6/15/1998	2/17/2006	1,105	189.0	59.0	269.3	25.5	1,451.0
	10	6/15/1998	2/17/2006	1,105	198.1	71.5	262.3	32.1	1,376.0
PH	1	6/15/1998	2/17/2006	1,618	204.0	189.0	109.8	42.0	800.0
	3	6/15/1998	2/17/2006	1,618	342.9	345.0	108.3	118.0	850.0
	5	6/15/1998	2/17/2006	1,618	437.1	440.0	109.9	196.0	899.0
	7	6/15/1998	2/17/2006	1,618	478.0	478.8	112.8	235.0	930.0
	10	6/15/1998	2/17/2006	1,618	521.6	525.0	121.4	235.0	990.0
PL	1	9/1/1998	2/17/2006	1,862	22.8	19.0	22.6	4.0	160.0
	3	7/14/1997	2/17/2006	2,126	42.8	34.0	33.7	7.0	252.0
	5	7/14/1997	2/17/2006	2,126	57.7	47.0	42.4	10.0	293.0
	7	7/14/1997	2/17/2006	2,126	68.3	55.0	47.6	13.5	325.0
	10	7/14/1997	2/17/2006	2,126	78.6	64.0	52.0	17.0	339.0
TR	1	4/16/1998	2/17/2006	1,958	450.3	375.0	347.6	24.0	1,550.0
	3	4/16/1998	2/17/2006	1,958	558.1	525.0	315.7	84.0	1,475.0
	5	4/16/1998	2/17/2006	1,958	610.2	610.0	291.6	132.0	1,440.0
	7	4/16/1998	2/17/2006	1,958	630.6	625.0	279.1	-	1,410.0
	10	4/16/1998	2/17/2006	1,900	646.3	650.0	271.0	-	1,400.0
VE	1	7/14/1997	2/17/2006	2,029	733.4	485.0	815.3	23.0	8,000.0
	3	7/14/1997	2/17/2006	2,126	834.7	720.0	589.1	67.0	4,854.0
	5	7/14/1997	2/17/2006	2,126	871.7	775.0	517.3	130.0	3,896.0
	7	7/14/1997	2/17/2006	2,126	887.4	795.0	489.8	167.0	3,653.0
	10	7/14/1997	2/17/2006	2,126	894.9	805.0	464.5	200.0	3,420.0

Acronyms for each country are as follows. BG: Bulgaria, BR: Brazil, CO: Colombia, KR: Korea, MX:Mexico, MY: Malaysia, PH: Philippines, PL:Poland, TR: Turkey, VE: Venezuela

Table 2: Basic Statistics: Forward CDS Premiums

This table provides the basic statistics of forward CDS premiums.

country	begin	end	beg. date	end. date	obs. Num.	mean	median	std	min	max
BG	1	10	11/30/1998	2/17/2006	1,772	415.7	428.0	262.8	38.0	998.0
	3	10	11/30/1998	2/17/2006	1,772	475.0	477.0	300.0	46.0	1,248.0
	5	10	11/30/1998	2/17/2006	1,772	494.8	481.0	311.6	52.0	1,200.0
	7	10	11/30/1998	2/17/2006	1,772	508.3	475.5	325.2	57.0	2,014.0
BR	1	10	11/30/1998	2/17/2006	1,772	912.1	829.5	426.3	208.0	2,555.0
	3	10	11/30/1998	2/17/2006	1,772	875.8	880.5	255.5	255.0	1,624.0
	5	10	11/30/1998	2/17/2006	1,772	813.7	821.0	234.7	178.0	1,500.0
	7	10	11/30/1998	2/17/2006	1,772	832.2	838.0	247.4	227.0	1,546.0
CO	1	10	11/30/1998	2/17/2006	1,772	683.1	692.0	212.0	195.0	1,514.0
	3	10	11/30/1998	2/17/2006	1,772	722.1	730.0	182.9	241.0	1,418.0
	5	10	11/30/1998	2/17/2006	1,772	718.7	729.0	177.9	265.0	1,380.0
	7	10	11/30/1998	2/17/2006	1,772	722.4	714.0	188.8	264.0	1,380.0
KR	1	10	10/10/2001	2/17/2006	1,052	83.1	82.0	33.1	35.0	224.0
	3	10	10/10/2001	2/17/2006	1,052	93.9	92.6	35.5	42.0	240.0
	5	10	10/10/2001	2/17/2006	1,052	100.5	98.0	36.6	45.0	244.0
	7	10	10/10/2001	2/17/2006	1,052	109.4	105.0	39.0	49.0	250.0
MX	1	10	11/30/1998	2/17/2006	1,772	311.1	262.4	179.3	96.0	1,312.0
	3	10	11/30/1998	2/17/2006	1,772	346.7	295.2	187.4	115.0	1,348.0
	5	10	11/30/1998	2/17/2006	1,772	362.3	307.0	189.6	130.0	1,352.0
	7	10	11/30/1998	2/17/2006	1,772	369.9	315.0	194.3	137.0	1,349.0
MY	1	10	10/10/2001	2/17/2006	1,052	104.7	78.0	57.4	36.0	225.0
	3	10	10/10/2001	2/17/2006	1,052	120.8	92.0	64.6	42.0	247.0
	5	10	10/10/2001	2/17/2006	1,052	129.6	102.5	64.7	48.0	271.0
	7	10	10/10/2001	2/17/2006	1,052	141.6	116.0	71.1	53.0	312.0
PH	1	10	1/11/2001	2/17/2006	1,239	603.5	598.0	98.2	315.0	857.0
	3	10	1/11/2001	2/17/2006	1,239	690.7	678.9	112.3	379.0	988.0
	5	10	1/11/2001	2/17/2006	1,239	710.7	694.0	127.1	396.0	1,110.0
	7	10	1/11/2001	2/17/2006	1,239	760.7	729.0	156.2	389.0	1,296.0
PL	1	10	11/30/1998	2/17/2006	1,772	69.8	66.0	35.5	19.0	220.0
	3	10	11/30/1998	2/17/2006	1,772	80.9	75.0	41.2	22.0	242.0
	5	10	11/30/1998	2/17/2006	1,772	88.2	81.0	44.1	26.0	243.0
	7	10	11/30/1998	2/17/2006	1,772	92.7	84.0	46.0	28.0	240.0
TR	1	10	10/2/2000	2/17/2006	1,308	699.3	697.5	292.1	222.0	1,357.0
	3	10	5/20/2003	2/17/2006	503	466.2	447.0	124.0	264.0	1,022.4
	5	10	5/20/2003	2/17/2006	503	473.2	457.0	112.9	288.0	983.9
	7	10	5/20/2003	2/17/2006	503	468.3	457.0	100.1	311.0	960.0
VE	1	10	11/30/1998	2/17/2006	1,772	954.2	914.0	391.1	227.0	2,071.0
	3	10	11/30/1998	2/17/2006	1,772	960.5	898.5	366.6	282.0	2,248.0
	5	10	11/30/1998	2/17/2006	1,772	956.2	884.2	398.9	304.0	2,750.0
	7	10	11/30/1998	2/17/2006	1,772	940.4	875.5	406.8	317.0	2,856.0

Acronyms for each country are as follows. BG: Bulgaria, BR: Brazil, CO: Colombia, KR: Korea, MX:Mexico, MY: Malaysia, PH: Philippines, PL:Poland, TR: Turkey, VE: Venezuela

Table 3: Basic Statistics: Bid Ask Spreads in Spot CDS

This table provides the basic statistics of bid ask spreads for spot CDS .

country	Maturity	beg. date	end. date	obs. Num.	mean	median	std	min	max
BG	1	7/6/1998	2/17/2006	1903	20	20	0	20	20
BG	3	7/6/1998	2/17/2006	1903	20	20	0	20	20
BG	5	7/6/1998	2/17/2006	1903	20	20	0	20	20
BG	7	7/6/1998	2/17/2006	1903	20	20	0	20	20
BG	10	7/6/1998	2/17/2006	1903	20	20	0	20	20
BR	1	7/14/1997	2/17/2006	2117	56	60	16	10	100
BR	3	7/14/1997	2/17/2006	2126	50	50	13	10	90
BR	5	7/14/1997	2/17/2006	2126	51	50	16	10	110
BR	7	7/14/1997	2/17/2006	2126	51	50	16	10	120
BR	10	7/14/1997	2/17/2006	2126	44	40	17	10	120
CO	1	8/25/1998	2/17/2006	1868	14	14	0	14	14
CO	3	8/25/1998	2/17/2006	1868	14	14	0	14	14
CO	5	8/25/1998	2/17/2006	1868	14	14	0	14	14
CO	7	8/25/1998	2/17/2006	1868	30	30	0	30	30
CO	10	8/25/1998	2/17/2006	1868	30	30	0	30	30
KR	1	4/16/1998	2/17/2006	1376	51	60	19	10	60
KR	3	4/16/1998	2/17/2006	1376	55	60	14	10	60
KR	5	4/16/1998	2/17/2006	1376	55	60	14	10	60
KR	7	4/16/1998	2/17/2006	1376	56	60	12	20	60
KR	10	4/16/1998	2/17/2006	1376	56	60	12	20	60
MX	1	7/14/1997	2/17/2006	2070	46	50	15	10	80
MX	3	7/14/1997	2/17/2006	2126	50	50	14	10	90
MX	5	7/14/1997	2/17/2006	2126	50	50	14	10	90
MX	7	7/14/1997	2/17/2006	2126	51	50	16	10	110
MX	10	7/14/1997	2/17/2006	2126	44	40	18	10	120
MY	1	6/15/1998	2/17/2006	1105	10	10	0	10	10
MY	3	6/15/1998	2/17/2006	1105	15	15	1	10	15
MY	5	6/15/1998	2/17/2006	1105	15	15	1	10	15
MY	7	6/15/1998	2/17/2006	1105	24	25	4	10	25
MY	10	6/15/1998	2/17/2006	1105	29	30	5	10	30
PL	1	9/1/1998	2/17/2006	1862	17	20	5	6	20
PL	3	7/14/1997	2/17/2006	2126	27	20	16	6	60
PL	5	7/14/1997	2/17/2006	2126	27	20	16	6	60
PL	7	7/14/1997	2/17/2006	2126	27	20	16	6	60
PL	10	7/14/1997	2/17/2006	2126	28	20	16	6	60
TR	1	4/16/1998	2/17/2006	1958	66	70	12	20	70
TR	3	4/16/1998	2/17/2006	1958	57	60	9	30	60
TR	5	4/16/1998	2/17/2006	1958	57	60	9	30	60
TR	7	4/16/1998	2/17/2006	1958	57	60	9	30	60
TR	10	4/16/1998	2/17/2006	1900	57	60	9	30	60
VE	1	7/14/1997	2/17/2006	2029	65	70	13	30	100
VE	3	7/14/1997	2/17/2006	2126	52	50	11	30	100
VE	5	7/14/1997	2/17/2006	2126	52	50	15	20	120
VE	7	7/14/1997	2/17/2006	2126	54	50	14	40	130
VE	10	7/14/1997	2/17/2006	2126	54	50	14	40	130

Acronyms for each country are as follows. BG: Bulgaria, BR: Brazil, CO: Colombia, KR: Korea, MX:Mexico, MY: Malaysia, PH: Philippines, PL:Poland, TR: Turkey, VE: Venezuela

Table 4: RMS of Pricing Error

This table provides the root mean square (RMS) of forward CDS pricing error.

A: Spot CDS with 1 and 10 year maturity, Forward CDS with 1 year expiry and 10 year maturity

	LGD											
Country	10	20	25	30	40	50	60	70	75	80	90	99
bg	183.0	68.7	47.1	33.1	17.8	9.8	5.2	2.8	2.6	2.9	4.0	5.0
br	508.3	241.6	175.5	133.7	80.9	47.3	26.7	17.2	17.7	20.1	27.3	33.8
co	441.4	136.7	87.6	61.1	34.1	21.5	15.5	13.2	12.9	12.9	13.5	14.3
kr	5.0	4.2	4.2	4.1	4.1	4.0	4.0	4.0	4.0	4.0	4.0	4.0
mx	80.6	29.2	20.5	15.1	8.9	5.7	4.1	3.5	3.4	3.5	3.9	4.3
my	5.1	2.9	2.7	2.5	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4
ph	229.5	86.3	60.0	44.1	28.0	21.5	19.0	18.3	18.2	18.3	18.7	19.1
pl	4.4	3.9	3.9	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8
tr	131.1	60.2	45.0	35.3	22.3	15.5	12.1	10.8	10.6	10.6	10.9	11.3
ve	286.4	109.5	78.4	56.9	32.7	20.3	14.3	12.4	12.5	13.0	14.5	16.1

B: Spot CDS with 3 and 10 year maturity, Forward CDS with 3 year expiry and 10 year maturity

	LGD											
Country	10	20	25	30	40	50	60	70	75	80	90	99
bg	543.3	348.0	197.6	123.0	57.6	29.0	13.2	4.1	3.4	5.5	10.2	13.7
br	643.4	503.9	328.8	227.3	136.5	90.1	69.8	56.5	58.2	62.4	74.8	87.8
co	741.1	329.3	172.8	108.7	53.8	30.1	18.4	13.6	13.0	13.2	14.8	16.5
kr	9.7	5.6	5.1	4.9	4.7	4.6	4.6	4.5	4.5	4.5	4.5	4.5
mx	252.3	78.9	51.5	36.3	19.9	11.4	6.6	4.3	4.2	4.6	5.9	7.2
my	17.4	6.1	4.6	3.8	3.1	2.8	2.8	2.8	2.8	2.8	2.9	2.9
ph	699.5	394.8	228.3	146.4	72.7	41.1	26.1	20.5	20.0	20.5	22.6	25.0
pl	9.4	5.8	5.3	5.0	4.9	4.8	4.8	4.9	4.9	4.9	4.9	4.9
tr	319.9	104.9	72.7	53.0	31.9	22.1	17.2	15.3	15.0	15.0	15.3	15.8
ve	525.2	246.5	146.4	96.4	52.3	37.0	30.0	27.8	27.9	28.6	30.6	32.7

C: Spot CDS with 5 and 10 year maturity, Forward CDS with 5 year expiry and 10 year maturity

	LGD											
Country	10	20	25	30	40	50	60	70	75	80	90	99
bg	577.7	525.5	448.1	241.3	98.1	47.8	22.9	10.9	9.5	10.8	15.7	19.9
br	671.5	613.4	517.2	360.0	198.6	133.3	106.9	99.7	96.4	97.6	102.4	108.8
co	702.3	541.0	261.7	143.8	64.2	34.2	20.2	14.5	13.8	14.1	15.8	17.8
kr	14.4	6.5	5.6	5.2	4.8	4.6	4.6	4.5	4.5	4.5	4.5	4.5
mx	326.8	135.9	80.0	54.3	30.2	17.3	9.4	5.1	4.5	5.1	7.1	8.9
my	28.7	8.5	6.2	4.9	3.7	3.3	3.2	3.2	3.2	3.2	3.3	3.3
ph	721.8	657.6	473.9	274.1	115.2	60.3	35.7	26.3	25.3	25.8	28.7	32.0
pl	15.4	8.0	6.8	6.3	5.9	5.9	5.9	5.9	5.9	6.0	6.0	6.0
tr	339.9	145.6	91.1	64.7	37.2	24.7	18.3	15.7	15.2	15.1	15.5	16.1
ve	499.8	363.4	215.5	133.5	69.9	59.1	61.8	64.8	65.8	65.3	64.9	64.6

D: Spot CDS with 7 and 10 year maturity, Forward CDS with 7 year expiry and 10 year maturity

	LGD											
Country	10	20	25	30	40	50	60	70	75	80	90	99
bg	598.6	564.0	534.7	443.3	155.3	69.7	27.5	6.5	5.8	10.8	20.0	26.4
br	659.2	625.2	584.9	494.5	233.1	147.7	112.2	103.4	101.5	105.6	109.5	115.9
co	693.5	655.1	451.7	238.9	92.4	45.5	23.6	13.6	12.3	12.6	15.7	18.8
kr	20.9	8.5	7.4	6.9	6.6	6.5	6.5	6.6	6.6	6.6	6.6	6.7
mx	360.4	189.3	116.7	72.5	40.3	23.8	12.9	6.8	5.7	6.3	8.7	11.1
my	56.8	13.1	8.9	6.6	4.5	3.7	3.6	3.6	3.6	3.7	3.8	3.9
ph	776.4	772.1	682.6	512.4	210.0	96.4	51.0	34.6	33.3	34.7	40.4	46.1
pl	20.8	10.1	8.0	7.1	6.5	6.5	6.5	6.6	6.6	6.6	6.7	6.8
tr	311.6	158.8	98.6	69.8	39.2	25.8	19.0	16.2	15.7	15.5	15.8	16.3
ve	466.4	418.5	274.1	170.6	91.6	80.7	85.2	90.7	93.2	95.4	98.4	100.7

Acronyms for each country are as follows. BG: Bulgaria, BR: Brazil,CO: Colombia,KR: Korea, MX:Mexico, MY: Malaysia, PH: Philippines, PL:Poland, TR: Turkey,VE: Venezuela