FitchSolutions

Quantitative Research
Special Report

Fitch Equity Implied Rating and Probability of Default Model

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Summary

Fitch's Equity Implied Ratings and Probability of Default (EIR) model is estimated to provide a view of a firm's credit condition given its current equity price and available financial information.

The Fitch EIR model is a proprietary hybrid probability of default and rating model that incorporates an option-based barrier model with hybrid adjustment of firms' financial and market information. Our barrier-option based PD provides a forward-looking structural default probability. Changes in this structural default probability provide leading information about changes in the credit quality of a debt issuer, and thus help to understand impending rating change and default. The model makes use of a small, but very carefully selected subset of accounting and market variables.

Fitch's proprietary default database contains over 7,900 defaults globally, spanning from the 1960s through 2006. Model calibration for Fitch's EIR model is based on data from 1990 through 2005 for North American firms, and for the period between 2000 through 2005 for global firms using daily price history and annual financial statements.

The Fitch EIR model covers approximately 27,000 entities globally: 13,000 in the US and Canada, plus another 14,000 firms from more than 70 other countries. For all firms, the model provides daily output of estimated default probability (PD) for both one-year and five-year horizons, and the implied agency rating (IR).

Accuracy Ratio (AR) power of the model for the rated universe is 86.4% and 68.8% for 1-year and 5-year horizons, respectively. When compared to established benchmark models, the EIR model outperforms peer models by wide margins. Furthermore, it is shown that our hybrid methodology (whereby certain financial ratios and market information are combined with a pure option-based barrier model) outperforms the pure option-based barrier model's AR by 4% and 6% at the 1- and 5-year horizons, respectively.

In addition to AR, a battery of tests are performed to measure the performance of the model, including k-fold tests, walk-forward tests, and lead-lag analysis. The results of these tests reveal that the model is very powerful in distinguishing good credits from bad credits, maintains robust performance through time and is applicable across different industries, geographic regions, and firms with different sizes.

Introduction

Credit risk models can be classified into two groups, known as structural models and reduced form models.

Structural models were pioneered by Black & Scholes (1973) and Merton (1974). The central concept common to all structural-type models is that a company defaults on its debt if the value of the assets of the company falls below a certain default point. For this reason, these models are also known as "firm-value" models. In these models, the default behavior is modeled in an option theoretical framework, and, as a result, one can apply the same principles used for option pricing to the valuation of risky corporate securities. The use of option pricing theory set forth by Black-Scholes-Merton hence provides a significant improvement over traditional methods for valuing default-prone risky bonds. It also offers not only prices that are more accurate but provides information about how to hedge out the default risk, which was not obtainable from traditional methods. Furthermore, the key assumption of these models makes intuitive sense: default behavior is a result of the value of the firm's assets falling below the value of its debt. In the case of Black-Scholes-Merton or barrier model, the outputs of the model show how the credit risk of corporate debt is a function of the capital structure (leverage) and the asset volatility of the issuer. The structural model framework is a useful tool in the analysis of counterparty risk for banks when establishing credit lines with companies and a useful tool in the risk analysis of portfolios of securities. However, structural models are difficult to calibrate and thus are not suitable for the frequent marking to market of credit contingent securities. Structural models are also computationally burdensome.

The second group of credit models, known as reduced form models, is relatively more recent. These models, most notably the Jarrow-Turnbull (1995) and Duffie-Singleton (1999) models, do not model credit risk through firm valuation, they model directly the likelihood of default or downgrade. Also, it is practical to model not only the current probability of default, but also a "forward curve" of default probability that can be used to price instruments of varying maturities. Modeling a probability in this fashion is tantamount to assuming that it is a surprise or a shock – the default event is a random event, which can suddenly occur at any time.

Clearly, both modeling frameworks have their own set of advantages and disadvantages, making the choice of which to use depend heavily on what the model is intended for. Accordingly, in developing the Fitch EIR model, using a barrier model enhanced by other financial and market information, so called enhanced-structural model is deemed more plausible. Section 3 provides the data coverage of the model. Section 4 briefly discusses barrier model selection, PD calibration, and the Implied Rating mapping. Section 5 provides model testing, while section 6 concludes.

Coverage and Data

Fitch's Equity Implied Rating and Probability of Default (EIR) model covers all traded non-financial firms globally, which includes more than 13,000 North American (US and Canada) firms and over 14,000 Non-North American (International) firms from more than 70 countries. The Fitch EIR model is based on large data sources of equity price information, financial statement information, historical rating information, and a large proprietary global default database. Because data reliability is very important for the development of data-driven models, the quality of the data needs special examination. Especially for the calculation of the default point, liability structure is carefully verified. Because of capital structure particularities of financial institutions, banking, insurance and other financial services sectors are left out of the scope of this model. Firms in these sectors shall be addressed in a later extension of the model.

Model calibration for the Fitch EIR model is based on monthly calculated PDs from 1990 through 2005 for North American firms, and monthly derived PDs for the period between 2000 through 2005 for global firms, using daily price history and annual financial statements. Fitch's proprietary default database contains more than 7,900 defaults globally from 1960s through 2006.

The Fitch EIR model covers approximately 27,000 companies across the globe and provides daily output of estimated default probability (PD) for one-year and five-year horizons, implied rating (IR).

Figure 1: Time Distribution of Fitch Global Default Database

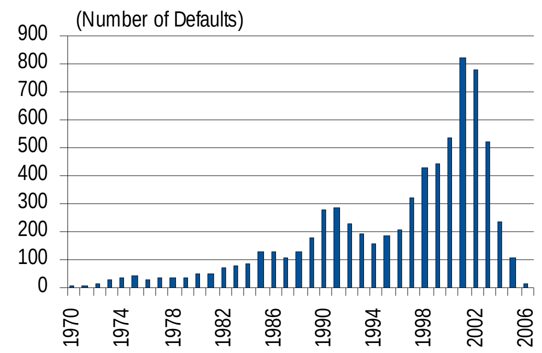


Figure 2: Sector Distribution of Fitch Global Default Database

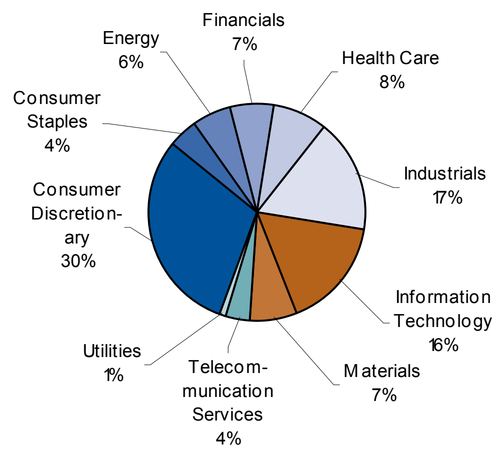
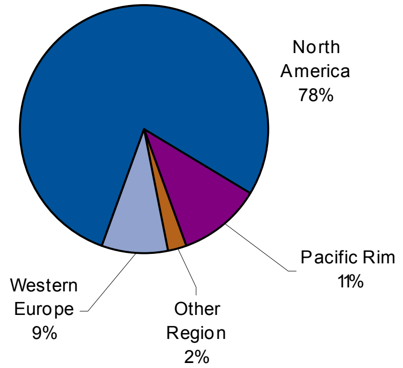


Figure 1 shows the distribution of default events through time of our default database. Starting from 1970, global defaults gradually increase; after a slight drop in the middle of 1990s, defaults jump to the highest level in the early of 2000s and sharply trend down to historically low levels over the past few years.

Figure 2 reports the default distribution by industry. For example, Retail, industrials and IT defaults clearly outnumber the rest of the industries in the dataset.

Figure 3 shows that most of the default events in the database are within North America (NA). Nevertheless, it should be mentioned that any default database will likely fail to capture 100% of defaults – it is possible that this likelihood is higher for smaller firms and Non-US firms.

Figure 3: Fitch Global Default Database by Region



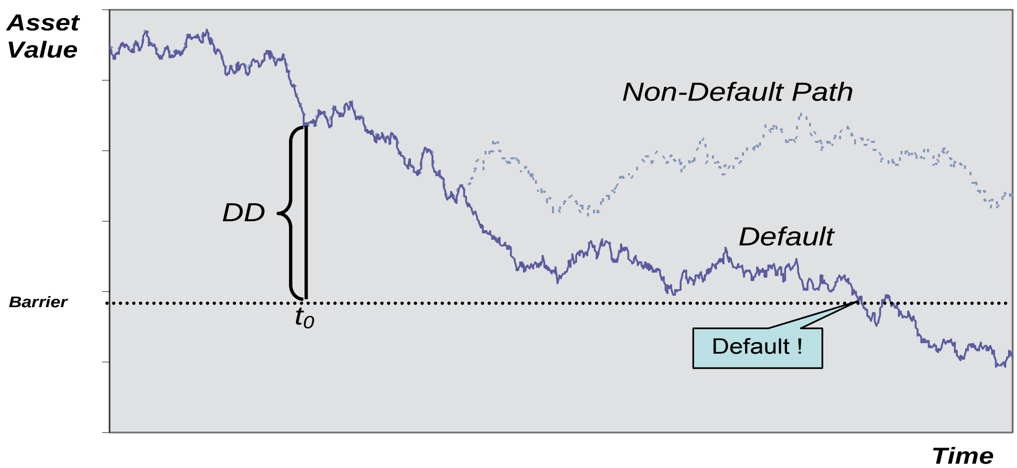
Model Specification

Fitch's EIR model combines a structural approach to credit risk modeling with a statistical mapping approach. Technically, Fitch EIR model is composed of a hybrid default risk model to produce a probability of default and a nonlinear regression model that is utilized in mapping the PDs to implied ratings.

1. Structural PD Model - Barrier Model

All common approaches for Structural default probability models assume the equity value of the firm is the value of a call option on underlining asset value. Accordingly, the firm defaults when its asset value falls below its debt obligation. Following the original work of Merton (1974), most industry standard PD models typically employ the standard European call option framework or a slight variation of it, which assumes that a firm can only default at the risk horizon. However, firms can default whenever their asset value fall below the default point – even if this occurs prior to the maturity of the option. In order to address this fact, previously developed models have introduced ad hoc assumptions, e.g. 'absorbing barrier'. Rather than taking this approach, Fitch's EIR uses a barrier option model as the core-modeling framework for its equity-implied PD model. This approach provides a closed form solution to the problem; it allows firms to default prior to the maturity date, and offers the flexibility of defining a barrier value in addition to the exercise price.

Figure 4: Generic Example of Default Probability Barrier Model



Barrier-based models all assume an exogenous barrier, which when crossed triggers default. Given the firm value process, the probability of crossing a flat barrier is easy to compute. For the down-and-out option, the barrier ("H") is equal to or lower than the default point ("K"), i.e., *H≤K*[1].

Under this assumption, the equity value of the firm (as well as its debt market value), or the corresponding call option value of the firm, is given by.

$$E(t) = A(t)\left\{N(x^{+}) - \left[\frac{H}{A(t)}\right]^{\frac{2r}{\sigma^2}+1} N(y^{+})\right\} - e^{-r(T-t)}K\left\{N(x^{-}) - \left[\frac{H}{A(t)}\right]^{\frac{2r}{\sigma^2}-1} N(y^{-})\right\}$$

$$D(t) = A(t)\left\{N(-x^{+}) + \left[\frac{H}{A(t)}\right]^{\frac{2r}{\sigma^2}+1} N(y^{+})\right\} + e^{-r(T-t)}K\left\{N(x^{-}) - \left[\frac{H}{A(t)}\right]^{\frac{2r}{\sigma^2}-1} N(y^{-})\right\}$$

where

$$x^{\pm} = \frac{\ln A(t) - \ln K + \left(r \pm \tfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$y^{\pm} = \frac{2\ln H - \ln A(t) - \ln K + \left(r \pm \tfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \qquad (1)$$

$E(t)$: Market Value of Equity
$D(t)$: Market Value of Debt
T : Time Horizon
r : Riskless Interest Rate
$A(t)$: Market Value of Firm Asset
H : Barrier of Option
K : Strike Pirce of Option/Default Point of the Firm
$\sigma(t)$: Asset Volatility

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[1] During the model calibration, different values for the barrier (H) have been tested to optimize the model performance. The results reveal that no single universal value satisfies this criterion across different industries and regions; moreover, the performance of the model is reasonably close when compared to the default point. Therefore, in order to avoid complexity, in the current model the barrier is assumed to be equal to the default point, which in turn is defined as short-term debt plus 0.5 long-term debt for 1 year and short-term debt plus long-term debt for 5 years.

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Note that the debt market value consists of two parts, the coupon present value (the second term) and recovery present value (the first term). The coupon present value is the risk neutral survival probability (terms in brackets) times the coupon (face value of strike price, or default point of the firm K) times the risk-free discount factor. The survival probability is the probability of asset value staying above barrier *H* at all times and also above strike *K* at maturity, which is equal to the probability of staying above the barrier at all times (first term in brackets) minus the probability of staying above the barrier but falling below the strike at maturity (second term in brackets). One minus such survival probability is the default probability, which can be written as:

$$PD(t) = N(-x^-) + \left[\frac{H}{A(t)}\right]^{\frac{2r}{\sigma^2}-1} N(y^-) \qquad (2)$$

In equation (1), both asset value and asset volatility are unobservable, where the only observable variables are equity value and equity volatility. In order to derive the asset value and asset volatility and, thus calculate the corresponding PD in equation (2), some researchers use the method of solving two equations for two unknown variables (TETU) (e.g. Vassalou and Xing (2005)). As discussed in Ericsson and Reneby (2005), the TETU approach assumes constant equity volatility, which is inconsistent with a structural model. Moreover, this approach underestimates volatility for those firms with rapidly falling asset value due to changes in leverage. Fitch's EIR uses a double loop converge (DLC) approach to solve the problem. In more specific terms, one can use equity volatility as the initial value and utilize Euler's method to solve for the unobservable asset value given in equation (1)[2]. Once one obtains the asset value, the asset volatility is calculated for the next loop, and the process is repeated until the calculated volatility difference between iterations converges to below a threshold value.

For firms with good credit quality, the structural PD calculated from equation (2) is very small in magnitude and can hardly be used directly[3]. More importantly, it is a risk-neutral measure; therefore, in order to get a PD under real measure, one needs to map this structural PD to historical default rates. Fitch's large proprietary default database (around 7900+ global defaults) enables us to get a reasonable PD mapping from risk neutral measure to real measure.

2. Enhanced PD Mapping – From Structural PD to Final PD

The DD (or structural PD) obtained from the barrier model provides the core part of Fitch EIR PD model. However, the structural (barrier) component does not address certain accounting, market, and credit information. For example, this additional information better describes each firm's assets and borrowing capacity. The result is significantly better discriminatory power of the final PDs, relative to the "structural PDs".

Accordingly, besides the DD from the barrier model, Fitch's EIR PD model also incorporates financial statement information and market information. These variables can be grouped under three general categories:

- Financial Ratios

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[2] It is very easy to get the asset value converged. Normally, less than five iterations can provide an accurate estimation of asset value.

[3] To make it comparable with other models, Fitch also obtains the derived Distance to Default (DD) that is defined as negative normal inversion of the structural PD. The AR power of this DD is compared to the final mapped PD in the following section.

- Market Performance
- Macro Market Variables

Financial Ratios have been found to be useful in predicting a firm's default as early as Ed Altman's original Z-score model published in 1968. Accordingly, Fitch's research into the utility of financial ratios for credit modeling analyzed an exhaustive set of ratios across several different financial measurement categories: Activity, Debt Coverage, Leverage, Liquidity, and Profitability. Out of these categories, Fitch selected 204 ratios based on their univariate accuracy ratio (AR) (power analysis). In general, variable selection for non-linear models is a computationally intensive problem. To select out the minimum set of explanatory variables, a K-fold test was employed to compare the AR power when combined with other ratios[4].Through this exercise, the optimal parsimonious combination of financial ratios that yield the highest AR power was selected. The selected combination of financial ratios include: cash corrected leverage, cash flow to total liability, net income less extraordinary to total asset, equity to sales, and cash to total asset.

Market Performance as a category is incorporated in the structural model to the extent it is captured by the asset value and volatility of a firm. Clearly, market performance affects firm's borrowing capacity and thus affects the firm's refinancing ability, which will ultimately be reflected in its default probability. These can be proxied by measures such as cumulative return and volatility of stock returns. In addition, higher order statistics such as kurtosis of a firm's historical returns can be thought to reflect a given firm's likelihood of rare events[5]. The tests reveal that these factors are significant even in the presence of the DD. Accordingly, both short term (two months) and longer term (one year) market performance measures have been included in Fitch's EIR PD model.

Market Risk has been accepted as a driving factor for the default probability forecasting. In Fitch's EIR PD model, Fitch tested commonly used macro-economic market variables including S&P 500 index return, Russell 2000 small stock index return, volatility index, and the term structure of interest rates. Moreover, Fitch tested some industry-wide or market-wide averages based on the barrier model: these variables are industry average of volatility, industry average of distance to default and the market average of distance to default.

Visually examining each predictive factor is a powerful tool for model construction. First, it shows a variable's power to distinguish defaulting versus non-defaulting firms. Second, it shows any potential non-linearity across the variable's range. Mechanically, the plots show the historical default rate across all the percentiles of each variable. These graphs not only help us select powerful variables, but also provide evidence of the non-liner character of the relationship. More importantly, these graphs have been applied to the non-linear mapping from each explanatory variable. For example, Figure 5 shows the relationship between DD and historical default rates. In other words, the 1-year default rate (tabulated from Fitch's proprietary default database) is plotted at

[4] K-fold analysis uses out-of-sample testing for model selection. First, divide the sample into N groups, then N-1 group firms are used to estimate the model parameters, score the group using the estimated parameters; repeat this process for each group until for all firms; finally calculate the AR power using these scores.

[5] Kurtosis captures the fat-tail of the firm's stock return distribution. Including kurtosis also addresses presence of a jump risk embedded in our barrier models to some extent.

each percentile of an explanatory variable. There is a clear positive relationship between DD and historical default frequency.

Similarly, the same relationship is plotted for each of the explanatory variables used in Fitch's EIR PD model. All these relationships are consistent with one's expectations[6]. Note that the relationships shown in these graphs are only univariate in nature. That is, this relationship is for each specific variable with respect to historical default rates given that all other variables are ignored. The true relationships are much more complex once the influence of other variables is added[7].

The procedure for selecting the minimum set of explanatory variables, Fitch began with a set of fundamental accounting and market-based variables in addition to the output of the barrier model. Using conventional statistical techniques such as linear and non-linear discriminant analysis the most likely candidate variables are determined. In the last step, a nested logistic regression is implemented to combine the variables together based on historical default/non-default observations. In particular, the following exponential logistic function is utilized to get the functional form of probability of default, where X's are transformed PD's (non-linear mapping) for each selected explanatory variable including structural PD from barrier model.

$$PD^*(t,\tau) = \frac{1}{1+\exp(\sum_{i=1}^{n} a_0 + a_i X_i(t,\tau))} \qquad (3)$$

where τ is maturity (1 year and 5 year maturity respectively)

Parameters in equation (3) are estimated through maximum likelihood estimation.

Figure 5: Historical Default Rate and Distance to Default (DD)

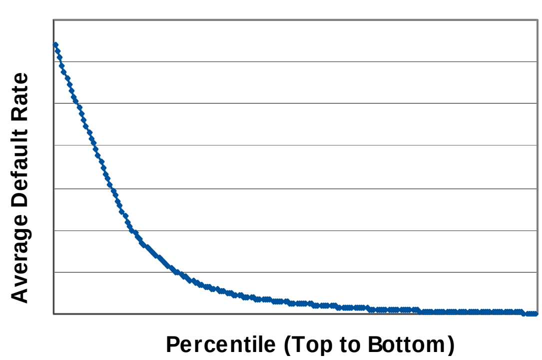


Figure 5 and Figure 6 show the univariate mapping from each explanatory variable to historical default rate. Equation (3) provides a multivariate linkage of these explanatory variables to historical default rates. To get the default probability based on the historical default prediction (default probability under real measure), Fitch uses another empirical mapping from PD* to final PD. The mapping is designed to answer the question of, given the level of PD*, what is the likelihood of firm's default in the coming 1 year (5 years). To conduct

[6] Manual adjustment is applied for some variables to guarantee the monotonic relationship of explanatory variable with the default rate in our final mapping.

[7] To accurately capture the high dimensional and non-linear nature of the default event, a flexible approach is required. To address this need some researchers have even tried artificial neural network techniques to link them together. However, this approach involves overly complex network architectures which can yield unintuitive directional relationships

the mapping, Fitch groups firm/month PD* into different buckets, then track the historical default rate of each bucket using Fitch's proprietary default database. A polynomial mapping function is applied to get the final PD[8].

Figure 6A: Historical Default Rate with Financial Ratios and Market Variables

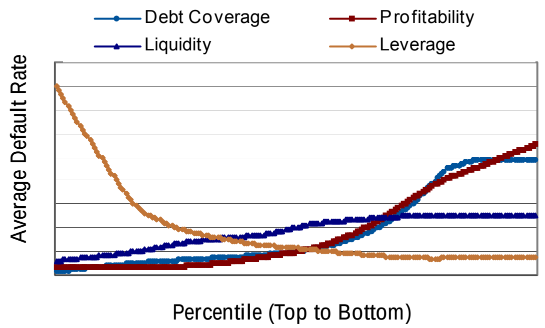
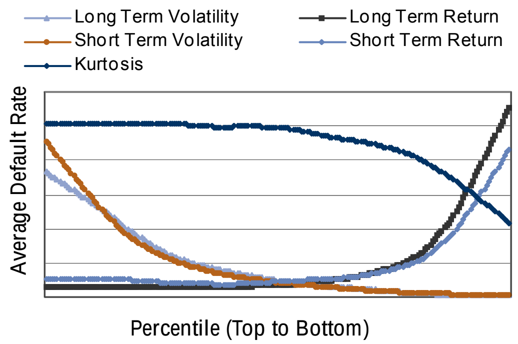


Figure 6B: Historical Default Rate with Financial Ratios and Market Variables



2.1. Global Adjustment

Firms in two different countries, even though they may possess similar accounting-based performances, might have differing default probabilities. This may be due to various reasons; for example, it can be a reflection of differences in government regulation, taxation policies, or differences in accounting policy and standards, and so forth.

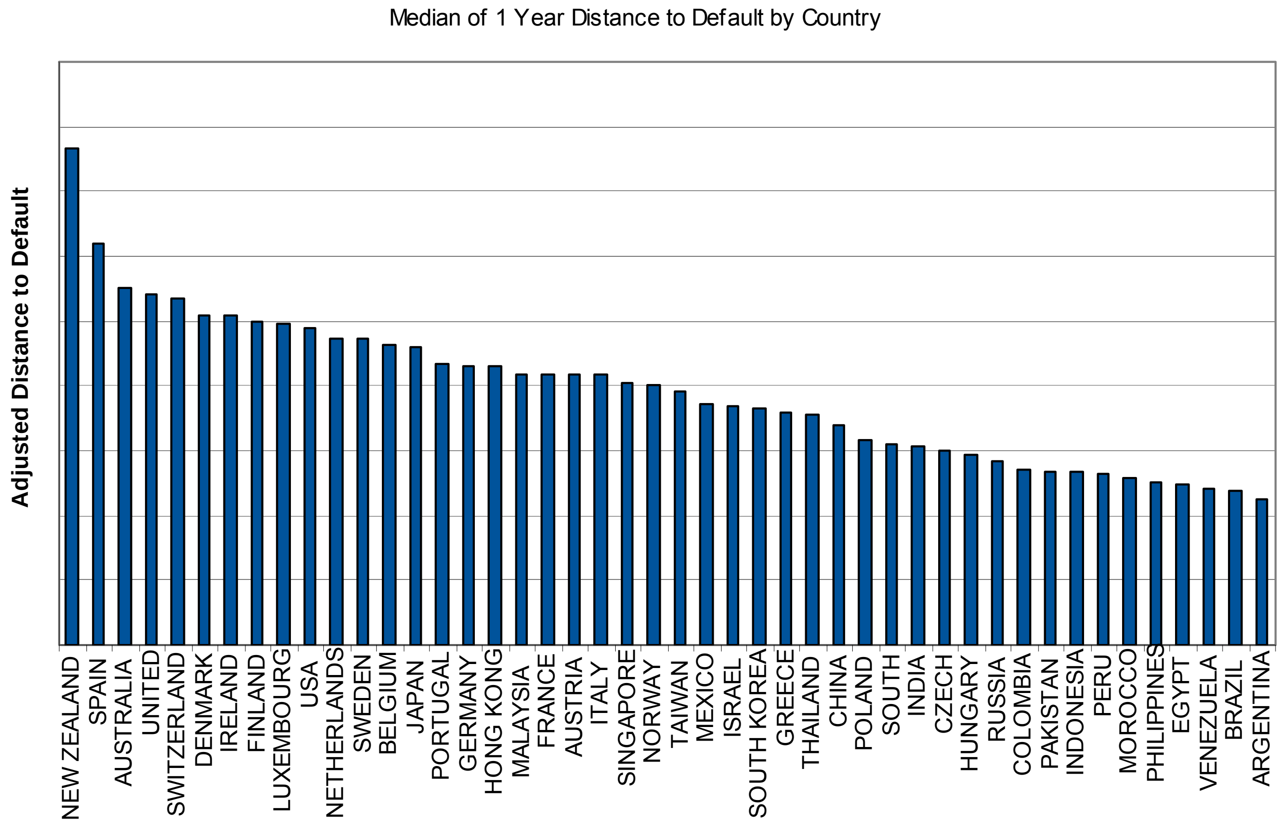
In order to get a good approximation of the default rate for firms from different countries, adjustments are applied to average distance to default by country according to historical default frequency and expert judgment. Following the same structural barrier model framework, Fitch gets the Distance to Default (DD) from structural PD for global firms. This DD is then adjusted by region and country to reflect the average credit quality of each country such that median of each country's DD is in line with its corresponding sovereign rating [9]. Figure 7 shows the median distance to default by country after this adjustment.

Once the country-adjusted DDs are obtained, utilizing the North America based mapping, the DDs are converted to obtain the final PD under the real measure for all global firms.

[8] The mapped PD for 1 and 5 years are bounded between 2bp ~ 3000 bp for 1 year and 4 bp ~ 5500 bp for 5 year. PD for other time horizons can be interpolated from the credit curve from these two points.

[9] The model uses sovereign ratings to distinguish between credit conditions between countries.

Figure 7: Global DD Adjustment by Country



3. Hybrid IR Mapping

As the next step, Fitch maps the estimated default probabilities to implied ratings. It is noted that rating analysts consider several other factors beyond the default probability of a firm in assigning a rating. Some of these factors are: industry information, size and the macroeconomic environment. For instance, large firms normally get credit and thus have higher ratings because of diversification benefits and their stronger ability to access extra resources under financial distress[10]. Besides size, firms from different industries also exhibit different rating characteristics.

Accordingly, two types of mapping approaches have been applied for the IR mapping – dynamic mapping and static mapping. The dynamic mapping process also uses a multivariate mapping process, combining the default probability with size and other key information that are relevant for rating decisions, whereas the static mapping utilizes a univariate mapping from PDs to implied ratings.

[10] The EIR model uses sales as a proxy for the firm size. The model also considers the size of one firm within the same industry (relative size) and average size of the whole industry to test the rating power, but find other size factors' contribution is marginal so the model only includes size in the final IR mapping.

3.1. Dynamic and Static Mapping

The dynamic mapping combines the PD with some other factors, such as macro variables (proxy by market leverage), size factor, and industry information in a Logistic function to reach an implied rating. The final Implied Rating is based on the comparison of a firms' rating scores to the threshold of each rating bucket.

Static mapping, on the other hand is based solely on PD:

$$Score_{dynamic}(t) = a_0 * \log PD(t) + b_1 * Size(t) + b_2 * MarketLeverage(t)$$
$$+ \sum_{i=1}^{n} c_i * IndustryDummy_i$$

$$Score_{static}(t) = d_0 * \log PD(t) \qquad (4)$$

where:

logPD(t)→ log (PD5y(t))

Size→ log (Sales(t))

MarketLeverage(t) → Market Median index of the book leverage of all the firms

Industry Dummy → Equal to 1 if the firm comes from that industry and 0 otherwise.

3.2. Global Adjustment

Dynamic mapping is applied to all firms globally, where firm size, industry information and country level factors along with 5-year PD's are used in the modeling. Based on observed regional differences the IR mapping procedure is separated into three broad groups:

- Region 1 – Pac Rim, EMEA, Latin America
- Region 2 – West European, Australia, New Zealand
- Region 3 – Japan, South Korea

Utilizing a LOGIT model, PD is combined with macro variables, size factor and industry information to reach a dynamic IR mapping. LOGIT model provides coefficient for each explanatory variable as well as the threshold of each rating bucket. IR is based on the comparison of a firm's rating score to the threshold of each rating bucket.

$$Score^k_{i,j}(t) = a_0 * \log PD(t) + b_1 * Size(t) + b_2 * CountryDD_i(t)$$
$$+ b_3 * SectorDD^k_j(t) + \sum_{n=1}^{5} c_n * IndustryDummy_n \qquad (5)$$

where:

logPD (t)→ log (PD5y) at date t

Size(t)→ log (Sales(t))

CountryDDi (t)→ Average of DD at date t for the Country i that the firm is located in

SectorDDjk (t)→ Average of DD at date t for the Sector (industry) j in Region k that the firm is located in

DD → Adjusted DD by different region, and countries

Industry Dummy → Equal to 1 if the firm comes from that industry and 0 otherwise.

3.3. Smoothing

In order to avoid frequent changes in the calculated implied ratings, two smoothing techniques are used to arrive at the final IR:

3.3.1. Moving average smoothing (MA smoothing)

For both dynamic and static mapping, moving average of the past 22 day's unsmoothed IR are used to reflect the final IR mapping.

3.3.2. Three standard deviation smoothing (3SD smoothing)

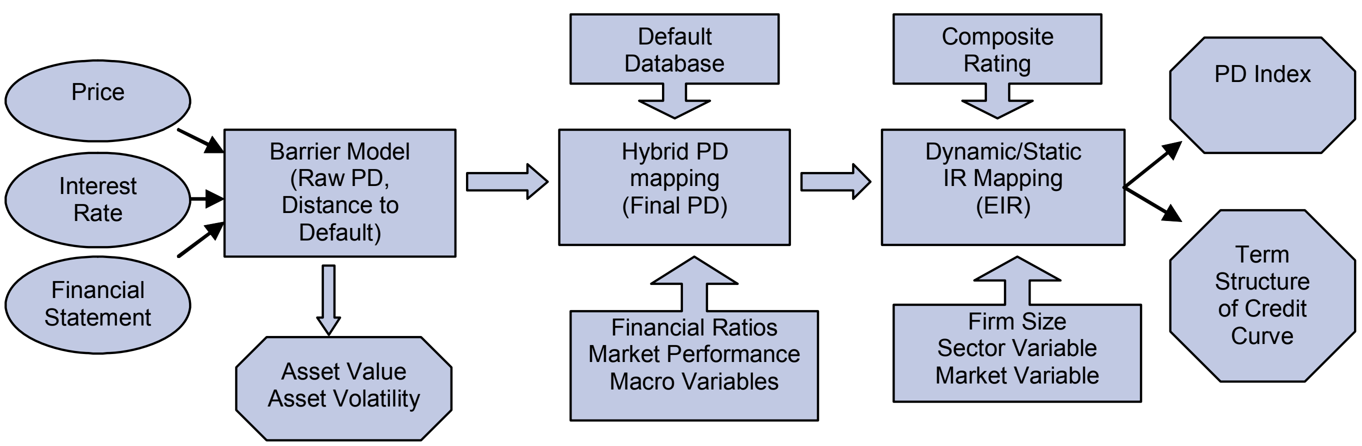
For both dynamic and static mapping, whenever an IR is upgraded or downgraded, the new rating score has to be larger/smaller than three standard deviations of threshold estimation away of each rating bucket threshold to confirm the rating changes. i.e., Fitch upgrade/downgrade IR only when the change of rating score is larger than three standard deviations across the threshold.

3.4. Country Ceiling

For global firms from different countries, country ceiling ratings reflect the risk of capital and exchange controls being imposed by the sovereign authorities that would prevent or materially impede the private sector's ability to convert local currency into foreign currency and transfer to non-resident credits – transfer and convertibility risk. To reflect this countrywide risk into corporate level rating, Fitch imposes proxy country ceilings, defined as sovereign rating plus two notches, into our final implied rating for global firms.

Figure 8 summarizes the modeling steps of Fitch EIR:

Figure 8: Modeling Step of EIR



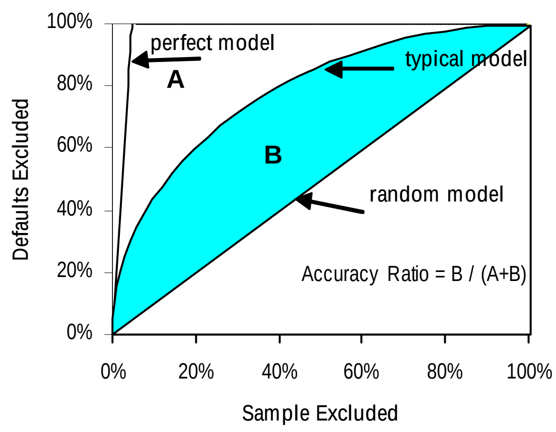
Robustness Check and Model Performance

In the following section, Fitch provides the predictive power (AR) comparison between different PDs, which support the inclusion of other financial variables.

1. PD Model Testing

To test the performance of the EIR PD model, Fitch uses a standard statistic for measuring the ranking power of a model called, Accuracy Ratio (AR). The basic idea of this statistic is to measure the type I and type II errors using one ratio. To understand AR, one should first understand the Cumulative Accuracy Profile (CAP) curve. To obtain the CAP curve, all firms are sorted from riskiest to safest by model generated PD's. For a given fraction of x of total number of firms, the CAP curve is constructed by calculating the percentage of the defaulters whose PD is within x percentile of PD from the total sample. A perfect default probability model would assign high PD's for those defaulters, thus the CAP would increase linearly up to 100% and then stay at 100% as it travels to the right across the remainder of the population. In other words, all defaulters should fall into the highest PD buckets. For a random PD model without any discriminative power, the fraction of x% PD firms will contain x% defaulters, which is the cross-diagonal line in the following graph. The ratio between area of typical model with random model and perfect model is defined as the accuracy ratio (AR). Calculation of AR is explained in Figure 9.

Figure 9: CAP Curve and Accuracy Ratio



This statistic ranges from 0 to 1, the higher the AR, the better the PD model differentiates defaulters from non-defaulters[11].

Table 1: AR Power Analysis for North American — Full Universe

AR	1 Year	5 Year
DD	72.4	50.7
Final PD	76.3	56.2
Z-score(Z4)	54.6	37
Z-score(Z5)	49.5	32.4
Shumway	57.9	41.2

[11] More discussion of AR can be found in Engelmann, Hayden and Tasche (2003)

Altman Z-score

$Z4 = 6.56\,X1 + 3.26X2 + 6.72X3 + 1.05X4$

$Z5 = 1.2\,X1 + 1.4X2 + 3.3X3 + 0.6X4 + X5$

where X1 = working capital / total assets, X2 = Retained Earnings / total assets, X3 = EBIT / total assets, X4 = Mkt Value of Equity / total liability, X5 = Sales / total assets

Shumway Statistics

$S = -6.307\,Y1 + 4.068Y2 - 0.158Y3$

Table 1 shows the AR power using structural form DD and the final hybrid PD for North American companies. For comparison, the performance of Altman Z-score and Shumway statistics are also included as a benchmark. As the table illustrates, the AR for pure structural form DD is already 72.4%, which outperforms the traditional default risk model by 20%. Final mapped PDs enhanced by other financial ratios and market information can further improve the AR by 4% for 1 year and by 6% for 5 year horizons. If we focus on the rated universe, the AR figures are higher[12].

[12] AR power for the rated universe is 86.4% and 68.8% for 1 year and 5 year respectively.

Table 2: AR Power Comparison

	1 Year		5 Year	
AR	**DD (%)**	**Final PD (%)**	**DD (%)**	**Final PD (%)**
By Region				
North American	72.3	76.1	50.7	56.2
Pacific Rim	53.7	60.2	63.7	65.6
Arica, East Euro Middle East	83.1	86.5	84.5	84.9
West Euro	67.5	68.2	57.8	57.9
Latin America	32.5	33.6	21.1	22.5
Global	72.7	73.2	55.5	56.1
By Industry – NA				
Auto	77	79.5	49.7	59.4
Capital Goods	77.4	78.7	56.3	61.9
Commercial Service	63.8	68.8	37.8	48.6
Consumer Durables	73.3	77.3	51	57
Consumer Service	67.5	73.2	47.9	55.7
Energy	79.8	81.5	61.9	65.1
Food	79	80.2	60.4	62.6
Beverage & Tobacco	74.6	81.4	57.7	66.1
Healthcare	73.8	77.1	44.2	54.1
Household Products	59.1	68.1	28.1	48.4
Materials	77	77.3	56.1	60.1
Media	70.2	72.8	45.9	48.9
Pharmaceuticals Biotech	68.7	74	55.6	63.2
Retailing	67.4	70.8	45.4	46.1
Semiconductor	71.7	79.8	69.2	76.3
Software	60.2	69.9	39.1	51.3
Technology Hardware	71.2	75.2	47.5	52.1
Telecommunication	67.8	73.7	40.2	48.5
Transportation	75	78.4	59.2	60.6
Utilities	71.6	78.4	54.9	64
By Size – NA*				
Small	63.9	71.3	38	47.5
Middle	71.1	75.8	46.7	53.7
Large	81.2	82.2	58.6	62.1

* Small firms are defined as CPI-adjusted sales less than 30mm, large firms are CPI-Adjusted sales larger than 300mm, and the rest are middle size firms.

Table 2 compares the AR power by different region and by industry. It should be recalled that AR is a relative measure and not an absolute one. In other words, a lower score in one industry versus another one does not necessarily lead to the conclusion that the model performance in the first industry is inferior to the performance in the latter industry. Rather, it is more informative to compare the performance of two different models on the very same sample, e.g. DD versus Final PD.

Figure 10: AR Curve for Global Rating Universe

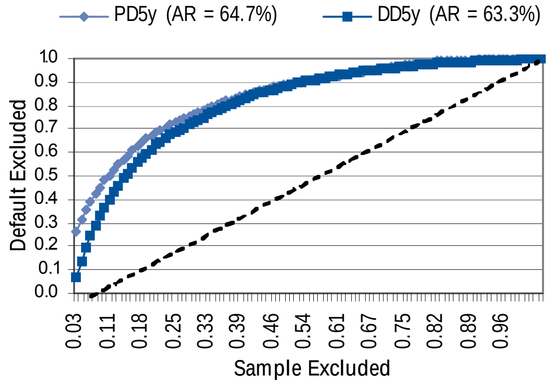


Figure 11: AR Curve for North America

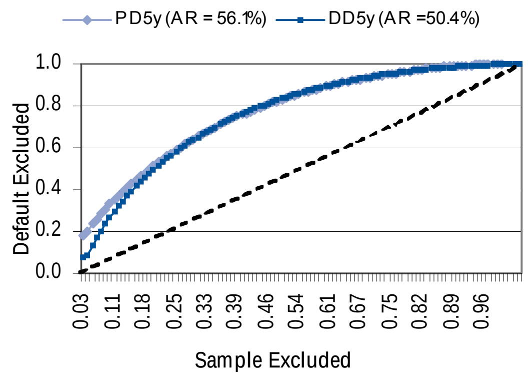


Figure 12: AR Curve for Western Europe

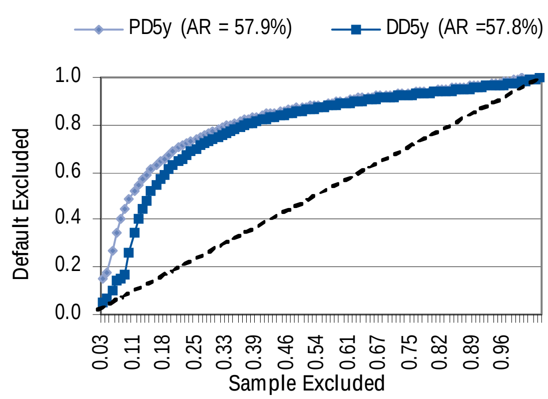


Figure 10 through Figure 13 show the graphs of AR power curves for the full universe of different regions and the global rated universe.

To test the robustness and performance of our model, a walk-forward test is implemented. Walk forward tests reproduce and therefore verify how the model will succeed in practice and examine if the models exhibit any symptoms of "overfitting".

In conducting the walk forward test, 1995 was chosen as the starting year. Consequently, the model is fitted using all of the data before the selected year. Once the model is built, Fitch generates the model output (final PD) for the following year (in this case 1996) and store them in a validation data set. By construction, the predicted model outputs are out-of-sample results. This process is repeated by moving the window up one year and store all the out-of-sample output. Finally, the AR power of this walk-forward analysis is compared to the in-sample ARs. Table 3 reports the AR for the final PD and the walk-forward PD. The two ratios are very close to each other, which supports the robustness of Fitch's Equity Implied PD model for out-of-sample data.

Figure 13: AR Curve for Pacific Rim

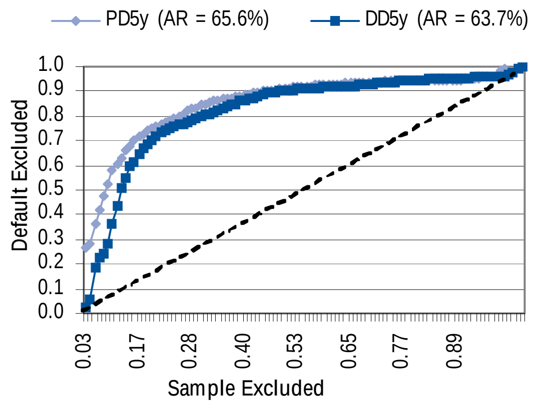


Table 3: Walk-Forward Out-of-Sample AR Power Analysis for North American

(%)

	Final PD		Walk-Forward PD	
AR	1 Year	5 Year	1 Year	5 Year
1996	75.0	48.2	75.1	48.4
1997	73.8	49.2	73.8	49.4
1998	70.2	50.3	69.7	50.2
1999	72.7	55.7	72.4	55.5
2000	68.0	58.4	66.8	58.3
2001	72.1	63.0	71.7	62.8
2002	73.8	64.2	72.8	64.2
2003	74.1	67.4	73.7	67.6
2004	81.1	77.3	80.9	76.8
2005	86.4	87.2	86.2	86.1
2000-2005	76.9	68.6	75.7	69.0

2. Implied Rating Model Testing

AR power analysis is a useful tool to test the performance of Fitch EIR model (both type I and type II error). In this section, Fitch will test the performance of the implied rating model. The most natural summary of implied rating is to see how many issuers exactly match the agency rating, how many are within one notch, two notches, and so on. Fitch calls this a Hit-Miss-Match (HMM) matrix. Table 4 reports the HMM matrix for North American companies. Accordingly, one observes that 19.4% issuers match agency rating perfectly. 51.3% and 73.7% issuers are within one notch and two notches. It is important to note that one ought to not expect a 100% match of Fitch EIR with agency ratings, and anticipate that there must be differences between these two. Clearly a thorough analysis of the mismatched cases would be of interest and would provide lots of value-added insight. To focus at the mismatched cases (off-diagonal terms), a forward analysis is conducted.

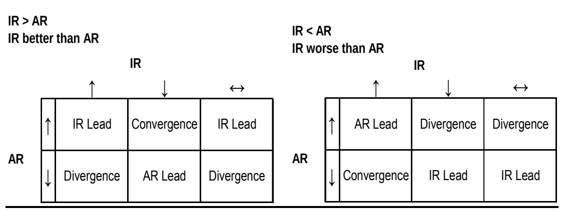
Table 4: Hit-Miss-Match Matrix for Equity Implied Rating (EIR)

Agency Rating	CCC<	B-	B	B+	BB-	BB	BB+	BBB-	BBB	BBB+	A-	A	A+	AA-	AA	AA+	AAA	Total
CCC<	2.07%	0.98%	0.90%	0.66%	-	-	-	-	-	-	-	-	-	-	-	-	-	5.09%
B-	0.93%	0.73%	0.95%	0.63%	-	-	-	-	-	-	-	-	-	-	-	-	-	3.80%
B	1.21%	1.16%	1.75%	1.57%	0.76%	-	-	-	-	-	-	-	-	-	-	-	-	7.46%
B+	0.73%	1.12%	2.24%	2.54%	1.79%	0.80%	0.59%	-	-	-	-	-	-	-	-	-	-	10.76%
BB-	-	-	1.23%	2.38%	2.06%	1.09%	0.78%	0.54%	0.53%	-	-	-	-	-	-	-	-	9.66%
BB	-	-	-	1.07%	1.30%	1.04%	0.81%	0.60%	0.59%	-	-	-	-	-	-	-	-	6.66%
BB+	-	-	-	0.51%	0.82%	0.82%	0.83%	0.72%	0.63%	-	-	-	-	-	-	-	-	5.30%
BBB-	-	-	-	-	0.83%	0.90%	0.92%	1.05%	1.40%	0.89%	0.49%	-	-	-	-	-	-	7.65%
BBB	-	-	-	-	0.86%	0.91%	1.18%	1.43%	1.74%	1.32%	0.95%	0.74%	-	-	-	-	-	9.97%
BBB+	-	-	-	-	-	0.53%	0.74%	0.92%	1.61%	1.36%	0.86%	0.81%	-	-	-	-	-	7.78%
A-	-	-	-	-	-	-	-	0.62%	1.27%	1.54%	1.12%	0.93%	0.38%	-	-	-	-	6.91%
A	-	-	-	-	-	-	-	-	1.24%	1.67%	1.55%	1.62%	0.76%	0.35%	-	-	-	8.39%
A+	-	-	-	-	-	-	-	-	-	0.69%	0.84%	1.32%	0.68%	0.28%	0.08%	-	-	4.55%
AA-	-	-	-	-	-	-	-	-	-	-	-	0.68%	0.49%	0.33%	0.16%	0.02%	-	2.56%
AA	-	-	-	-	-	-	-	-	-	-	-	-	0.41%	0.39%	0.34%	0.09%	0.05%	1.84%
AA+	-	-	-	-	-	-	-	-	-	-	-	-	-	0.06%	0.03%	0.01%	0.01%	0.45%
AAA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.37%	0.12%	0.10%	1.17%
Total	5.31%	4.75%	7.99%	10.48%	9.66%	7.09%	6.92%	7.32%	10.08%	8.97%	6.93%	7.30%	3.61%	1.99%	1.19%	0.25%	0.17%	

*- indicates values are less than 0.5%.

In a market with perfect information, agency rating and equity implied rating should be consistent with each other. Any inconsistency would suggest that the market and rating agencies have different opinions about the future performance of individual firms. To test this, whenever a change in agency rating is observed, Fitch looks at the history of the equity implied rating to determine whether the model anticipated the agency rating change. If the implied rating (IR) moves downward prior to an agency rating downgrade, the IR "leads" the agency rating migration.

Figure 14: Classification of Lead-Lag Analysis between IR and Agency Rating



Similarly, if IR moves up followed by agency rating upgrade, the IR "leads" the agency rating migration. Contrarily, if IR upgrade (downgrade) is after agency rating upgrade (downgrade), the agency rating leads implied rating. If their changes are in different directions, the IR and agency rating either "diverge" (agency rating moves away from IR) or "converge" (both agency rating and IR move to each other). Based on the current relationship between IR and agency rating, the changes of IR and agency rating are summarized in the following graph, and call this as Lead-Lag analysis.

Table 5: Lead-Lag Analysis for Equity Implied Rating

(%)

	IR Lead	AR Lead	Converge	Diverge
1 Month	74	4	4	18
3 Month	63	9	13	14
6 Month	55	15	17	12
9 Month	52	19	18	11
12 Month	49	22	20	10

Table 5 reports the lead-lag analysis for various time intervals prior to agency rating changes. It is clear from the table that agency rating movements are anticipated by the equity implied rating. 74% of those off-diagonal issuers IR leads agency rating changes, while 4% of agency rating lead IR, indicating that the EIR model can accurately forecast agency rating changes for most cases.

The general trend is that EIR leading behavior becomes increasingly more prominent while approaching the announcement date. This indicates that rating agencies and markets often respond to the same information, however, the market may be able to “price” this information into firm and security valuations faster than this information appear in the published rating. Interestingly, as time goes by, the market appears to polarize around IR leading and divergence.

Convergence, which occurs when the market and rating agencies appear to influence each other, subsides as we approach the announcement date. The implication is that, either ratings follow the market (IR lead), or they do not (diverge). In addition, the convergence declines as we get closer to the ratings change suggests that agencies, although influence by the market, are not over-sensitive to its signals and yield to market pressure only gradually[13].

Overall, the analysis shows that the Fitch EIR model can predict the future movements of agency rating. The leading effect is highly significant. The results for various intervals show that EIR can provide early warnings before an agency rating change.

3. Rating Migration

The EIR model can also provide a rating migration matrix. Table 6 and Table 7 compare the 1 year rating migration for equity implied rating and agency rating. Within one year, only 42.5% of IR will stay the same while this number is 69.4% for agency ratings. This is consistent with more volatile market changes and stability of agency’s ratings.

Table 6: 1 Year Equity Implied Rating Migration Matrix

From\To	CCC<	B-	B	B+	BB-	BB	BB+	BBB-	BBB	BBB+	A-	A	A+	AA-	AA	AA+	AAA	Total
CCC<	80.78%	10.91%	5.62%	1.94%	-	-	-	-	-	-	-	-	-	-	-	-	-	216942
B-	35.21%	25.85%	24.75%	10.47%	2.71%	-	-	-	-	-	-	-	-	-	-	-	-	68958
B	15.07%	19.48%	30.48%	23.71%	7.81%	2.00%	-	-	-	-	-	-	-	-	-	-	-	86626
B+	5.01%	8.28%	21.81%	32.48%	20.32%	7.04%	3.06%	1.23%	-	-	-	-	-	-	-	-	-	89165
BB-	1.56%	3.16%	9.71%	24.54%	28.71%	15.58%	9.46%	4.60%	2.13%	-	-	-	-	-	-	-	-	69399
BB	-	1.41%	4.54%	13.96%	24.09%	20.06%	16.57%	11.12%	6.12%	1.24%	-	-	-	-	-	-	-	43327
BB+	-	-	2.57%	7.96%	16.05%	17.87%	19.64%	17.62%	12.88%	3.30%	-	-	-	-	-	-	-	37716
BBB-	-	-	1.29%	4.26%	9.40%	13.26%	17.83%	20.75%	22.15%	8.12%	2.14%	-	-	-	-	-	-	35512
BBB	-	-	-	1.71%	4.74%	7.81%	12.49%	18.15%	27.97%	17.98%	6.77%	1.56%	-	-	-	-	-	41208
BBB+	-	-	-	-	1.75%	2.93%	5.54%	10.84%	25.42%	28.11%	17.46%	6.58%	-	-	-	-	-	30321
A-	-	-	-	-	-	1.28%	2.19%	5.31%	15.93%	25.45%	27.23%	19.44%	1.96%	-	-	-	-	22299
A	-	-	-	-	-	-	-	1.87%	6.58%	15.81%	25.36%	36.79%	10.98%	1.21%	-	-	-	20571
A+	-	-	-	-	-	-	-	-	1.20%	3.94%	11.24%	39.36%	33.04%	8.98%	1.17%	-	-	9412
AA-	-	-	-	-	-	-	-	-	-	1.18%	1.57%	14.49%	41.35%	30.71%	9.72%	-	-	4588
AA	-	-	-	-	-	-	-	-	-	-	-	3.13%	15.60%	39.07%	36.39%	4.66%	-	2616
AA+	-	-	-	-	-	-	-	-	-	-	-	-	2.87%	11.04%	56.73%	20.97%	7.73%	453
AAA	-	-	-	-	-	-	-	-	-	-	-	-	-	1.62%	30.46%	30.46%	37.20%	371
Total	218578	68952	85501	89497	70350	44173	38715	35910	41797	30414	21871	19287	8353	3638	1907	347	194	779484
		Diagonal	42.49%															
		1 Notch	76.73%															
		2 Notch	91.96%															

[13] Fitch would admit that outlook and watches from rating agencies can also reflect the market perception of firms’ credit quality in advance, even though this might not be as soon as price changes

Table 7: 1 Year Agency Rating Migration Matrix

From\To	CCC<	B-	B	B+	BB-	BB	BB+	BBB-	BBB	BBB+	A-	A	A+	AA-	AA	AA+	AAA	Total
CCC<	81.40%	9.28%	6.52%	1.92%	-	-	-	-	-	-	-	-	-	-	-	-	-	5795
B-	21.40%	56.58%	15.35%	4.96%	1.38%	-	-	-	-	-	-	-	-	-	-	-	-	4860
B	9.29%	10.52%	62.85%	13.28%	2.92%	-	-	-	-	-	-	-	-	-	-	-	-	9969
B+	3.93%	4.44%	12.13%	63.82%	11.06%	3.42%	-	-	-	-	-	-	-	-	-	-	-	15374
BB-	1.19%	1.31%	4.60%	13.21%	64.54%	11.10%	3.07%	-	-	-	-	-	-	-	-	-	-	13735
BB	-	1.18%	2.20%	6.03%	13.27%	59.29%	12.84%	3.77%	-	-	-	-	-	-	-	-	-	9567
BB+	-	-	1.17%	2.74%	5.96%	11.29%	58.27%	15.69%	3.45%	-	-	-	-	-	-	-	-	7750
BBB-	-	-	-	1.03%	1.87%	4.84%	8.01%	68.21%	13.18%	1.60%	-	-	-	-	-	-	-	11005
BBB	-	-	-	-	-	1.12%	3.35%	10.30%	73.43%	9.18%	1.18%	-	-	-	-	-	-	14618
BBB+	-	-	-	-	-	-	-	3.09%	13.84%	71.50%	8.31%	-	-	-	-	-	-	11751
A-	-	-	-	-	-	-	-	-	4.40%	12.76%	71.37%	9.51%	-	-	-	-	-	10443
A	-	-	-	-	-	-	-	-	1.02%	3.16%	9.71%	79.58%	5.18%	-	-	-	-	13036
A+	-	-	-	-	-	-	-	-	-	-	3.62%	15.21%	76.23%	3.34%	-	-	-	7128
AA-	-	-	-	-	-	-	-	-	-	-	1.06%	3.78%	14.01%	78.09%	2.77%	-	-	4075
AA	-	-	-	-	-	-	-	-	-	-	-	-	2.19%	12.69%	82.55%	1.45%	-	2963
AA+	-	-	-	-	-	-	-	-	-	-	-	-	1.78%	5.07%	9.59%	78.22%	4.93%	730
AAA	-	-	-	-	-	-	-	-	-	-	-	-	-	1.13%	3.12%	2.96%	92.15%	1860
Total	7605	5389	10304	14235	13034	9527	7874	11235	14817	11812	10208	12803	6807	3891	2699	669	1750	144659
		Diagonal	69.41%															
		1 Notch	90.72%															
		2 Notch	97.03%					-										

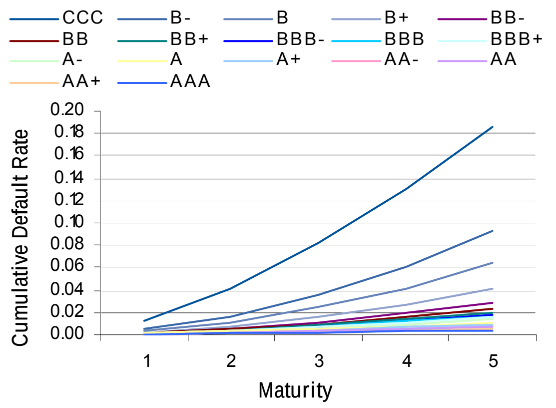
* - indicates values are less than 1.0%.

4. Term Structure of PD and PD Index

The term structure of default probability is useful for examining the changing credit structure of individual firms, industries, and the whole economy. Thus, a term structure of default probability contains information about the business cycle. Typically, for investment-grade firms, the term structure of unconditional default probability should be upward sloping because the probability of default increases with time. However, the term structure of default probability is inverted when the short-term default probability is greater than the forward default probability, $\left(FPD(t,T_1,T_2) = \dfrac{PD(t,T_2) - PD(t,T_1)}{1 - PD(t,T_1)} \right)$.

Figure 15: Term Structure of Credit Curve

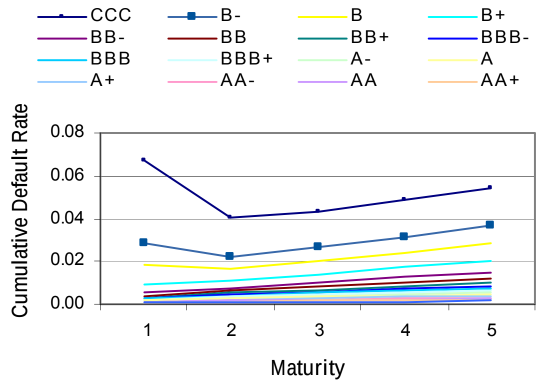
Panel A



Common for sub-investment grades, this occurs whenever the firm has a high probability of default in the short term. But if it can survive through the next year and payoff its short term obligations, then the firm's default probability might decline. In this situation, the Forward default probability would be less than the short-term default probability. Fitch's EIR model provides a term structure of default probabilities. Fitch uses a Weibull distribution ( ) to fit the term structure for each firm. At any given date for any firm, Fitch estimates the parameters of the Weibull distribution using 1 year PD and 5 year PD; then interpolate and extrapolate the term structure of PD for all other maturities. Figure 15 shows the term structure of default probability for different

Figure 15: Term Structure of Credit Curve

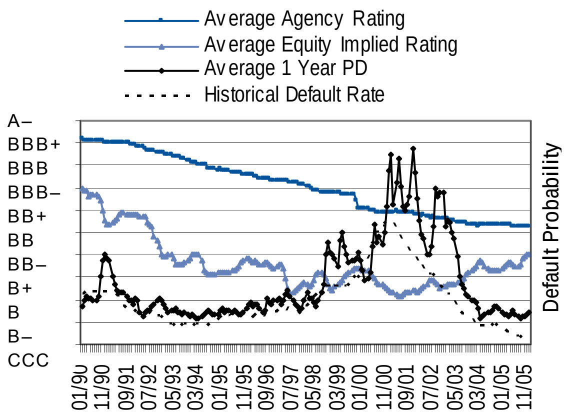
Panel B



implied ratings up to 5 years. The first one shows the average cumulative default probability by implied ratings and the second graph shows the average forward default probability by implied ratings.

Figure 16 draws the graph for average agency rating and equity implied rating over time. From 1990 through 2005, the average agency rating has been downgraded from BBB+ to BB while the equity implied rating also shows a corresponding pattern of downgrades. Compared to the rated universe, our sample firms cover more small firms, which can explain why on average the equity implied rating is about 2 to 4 notches below agency rating. Moreover, the equity implied rating index also exhibits higher volatility. The PD index for the same universe reflects the higher volatility of equity markets, and the change of PD index is consistent with the change of average implied rating index. The Fitch EIR is a hybrid mapping model, meaning that besides PD, Fitch also incorporates other factors into the final mapping, thus, the PD index is even more volatile than the IR index, especially for the period between 1999 through 2003.

Figure 16: PD Index and Average IR for North American



To test the level of final PD, Fitch also compares the average PD with historical default rates by each month. As indicated in Figure 16, the hybrid PD predictions are consistent with the observed historical default rates. After experiencing a high-risk bubble-burst period in the first few years of the 21st century, the average default rate gradually drops. Since 2004, we observe that the default rate remains at the lowest level since 1990.

Figure 17: Panel A and B provides the PD indices for different industries. Even though all industries exhibit similar patterns through credit cycles, there still exists differences among industry groups. Traditional industries such as utility and materials have much lower default risk and less volatility compared to the high technology and telecommunication sectors.

Figure 17: PD Index by Industry

Panel A—1 Year PD Index by Industry

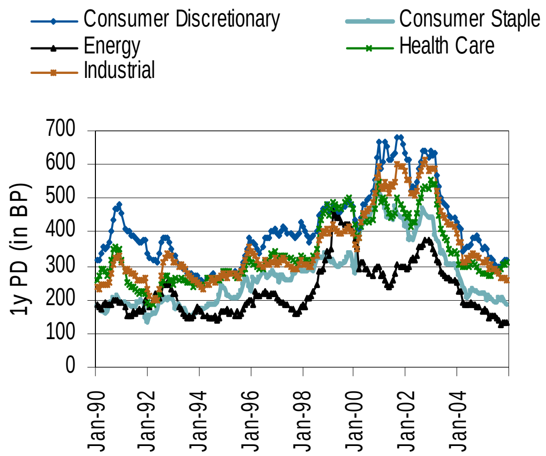
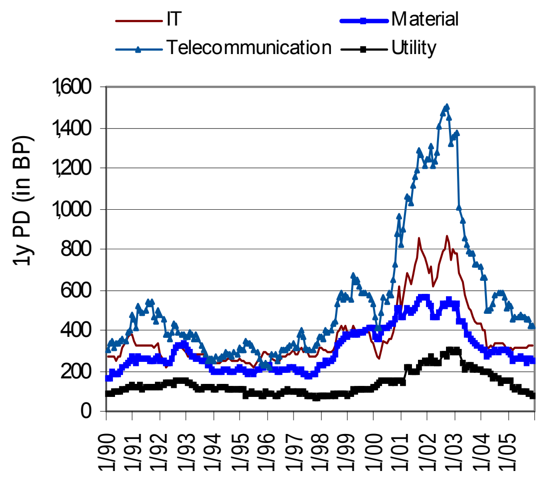


Figure 17: PD Index by Industry

Panel B — 1 Year PD Index by Industry



Conclusion and Future Research

The Fitch EIR model is a proprietary hybrid probability of default and rating model that incorporates an option-based barrier model with hybrid adjustment of a firm's financial performance and market information. The barrier-option based PD provides a forward-looking structural default probability provide leading information about changes in the credit quality of a debt issuer, and thus help to understand impending rating changes and default. The model makes use of a small, but very carefully selected subset of accounting and market variables.

Fitch also carefully adjusts the distance to default by region and country to reflect the average credit risk for each country. The use of disciplined non-linear modeling techniques allows this model to capture the complex relationships between different explanatory variables and default rate extracted from Fitch's extensive default database of over 7900 defaulters globally. Fitch applied a multi-period logistic regression and proprietary mapping to get the final equity implied rating. Results show Fitch EIR model is consistently more effective than alternative models at predicting defaults and can be used as an early warning system for credit risk assessment and portfolio risk management.

Because of the particular structure of financial institutions, the current Fitch EIR model specifically excludes the financial, banking and insurance industries; firms in these sectors will be considered in a later extension of the model.

FitchSolutions

Appendix

Industry Classification

North America

Fitch uses the following industry classification in the Fitch EIR model for North American. The classified 20 industries are:

```
Automobiles & Components                      → ind1
Capital Goods                                 → ind2
Commercial Services & Supplies                → ind3
Consumer Durables & Apparel                   → ind4
Consumer Services                             → ind5
Energy                                        → ind6
Food & Staples Retailing                      → ind7
Food Beverage & Tobacco                       → ind8
Health Care Equipment & Services              → ind9
Household & Personal Products                 → ind10
Materials                                     → ind11
Media                                         → ind12
Pharmaceuticals Biotechnology & Life Sci      → ind13
Retailing                                     → ind14
Semiconductors & Semiconductor Equipment      → ind15
Software & Services                           → ind16
Technology Hardware & Equipment               → ind17
Telecommunication Services                    → ind18
Transportation                                → ind19
Utilities                                     → ind20
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Global

Industry classification for global firms is from WorldScope, which includes 19 sectors and 121 industries. Fitch group them into the following 6 general industries:

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Hi-Tech – TMT                                 → ind1
Consumer Cyclical                             → ind2
Consumer NonCyclical                          → ind3
Materials, Energy & Transportation            → ind4
Industrial Goods & Service                    → ind5
Utilities                                     → ind6
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