

# Excessive Variation of Risk Factor Correlations and Volatilities

by

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# 1 Introduction.

This paper argues that excessive time variation in volatility and correlations is a critical problem in effective risk management. This goes beyond the standard mean-variance portfolio analysis.

A market practitioner uses volatility and correlation estimates for two important purposes besides asset management. First of all, these parameters play a crucial role in *daily* risk management and form the basis of value at risk (VaR) calculations. Second, these estimates are used in calibrating dynamic models for underlying assets during the valuation process of derivative securities. In a typical portfolio management problem where assets are linear in risk factors, *time variation of volatilities and correlations* can easily be taken into account by estimating conditionally time varying variance-covariance matrices.<sup>1</sup> On the other hand, in a risk management and especially valuation perspective the problem of unstable second moments is significantly more serious.

Many assets contain (several) implicit options. These options are sometimes compound, in the sense that they depend on the implicit volatilities as well as covariances of the underlying. If the underlying second moments exhibit strong unpredictable variation, then the pricing of portfolios of assets containing such options will indeed be more complicated.

The objective of this paper is to investigate the linkages in terms of time varying, stochastic volatilities in a sample of asset returns from three major markets and for three types of instruments. We focus on the time series properties of correlations of returns, volatilities of returns, and correlations of volatilities. These parameters play a crucial role in the recommendations of the Basle Committee concerning capital adequacy requirements against market risk. The financial market turmoil of the last few years is another reason that justifies this study. Market participants have realized that the estimated correlations and the volatilities of implied volatilities can vary excessively during turbulent periods.

In risk management, the use of estimated volatilities and correlations is based upon the assumption that the daily estimates of these parameters are “stable,” so that yesterday’s estimates can be used to determine today’s

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<sup>1</sup>Within the asset allocation context the problem reduces to one of time varying estimation and can be handled in a straightforward fashion. The resulting portfolio weights are still optimal locally in a mean square error sense.

VaR. High variation in the underlying correlations, both in terms of sign and in terms of size are ruled out. However, the results provided in this paper suggest that this may not be true at all. Using a Kalman filter approach, this paper investigates the stability of the *daily* second moments for important risk factors and concludes that they are highly unstable both in magnitude and in the sign.

Short-term volatility and correlation estimates are also used in calibrating equity prices and interest rate models. Here the instantaneous volatility and correlation parameters need to satisfy some important conditions. Depending on whether the volatility(correlation) coefficients are *constant*, *deterministically time-varying*, or *stochastic* the pricing and hedging of interest rate derivatives will be different. In case of equity and commodity derivatives depending on the magnitude of the variation the same problem will again be present.

This paper uses a robust set of estimators to provide evidence on an extreme instability in the correlations of most risk factors and their volatilities. In particular, we show that the volatilities of the risk factors not only are unstable, but also behave in a stochastically *non-linear* fashion, where volatility itself becomes highly volatile for some brief periods and then reverts back to an essentially deterministic behavior.

Also, we provide evidence on a surprisingly stable correlation between these stochastically varying volatilities. In fact, although the risk factors appear to display *arbitrary* correlation when measured in terms of their *levels*, they display surprisingly stable correlation in their *volatilities*.

The results reported in this paper may be of interest in a broader sense as well. The financial market turmoil of 1997-99 has made clear how much international financial markets have become integrated. This is in part due to the gradual removal of controls on capital movements in many countries, in part to liberalization of rules regarding international diversification of portfolios of institutional investors, and in part to greater awareness on the part of investors of the benefits of international diversification. One would expect the increased integration to bring about closer connections between asset price movements in different markets and across different asset classes. The disturbance in any one location or market segment is rapidly transmitted elsewhere, as investors rebalance their portfolios in response to new information.

Implicit volatility and correlations estimates as well as asset market link-

ages have generated significant new research, especially after the contagion effect of the 1992 EMS crisis and the contagion of Thai devaluation in 1997. An excellent survey of the role played by volatility dynamics can be found in Rebonato(2000). Bates(1996) deals with jumps and stochastic volatility and discusses issues closely related to our empirical results. Neftci(2000) provides evidence that extreme movements in risk factors do not have a Gaussian behavior. Dumas et. al(1998) is a good source for implicit empirical volatility functions.

At the other extreme, studies that take an asset-allocation perspective are naturally interested in the evolution over time of co-movements in asset prices across markets and instruments. It is not only trend-like evolutions resulting from removal of restrictions on international portfolio-diversification which have been of particular concern, but also more general time-varying patterns of measured correlations. Longin and Solnik (1993) is one recent example of such research. This study looks at the evolution of monthly excess returns on national stock indices for seven major countries over the period 1960-1990. While there is evidence of increased international correlations between markets over the sample period, Longin and Solnik find that correlation and covariance matrices of the excess returns are unstable over time. <sup>2</sup>

The paper by Bertero and Mayer (1989) implies that interdependence between markets varies systematically with the degree of instability in the international system. They find evidence that correlations of returns increase when turbulence in the markets is higher. Similar results are reported in the study by Kupiec (1991) which covers not only national stock markets but also corporate bond yields and foreign exchange. Investigating not only equities but also foreign exchange and government bonds, McNelis (1993) finds significant correlations of volatilities across national markets.

The paper is organized as follows. The next section lays out the definitions and the notation for calculating correlations of data with high observation frequency. The empirical results and the data are presented in Section III of the paper. Section IV discusses some implications of the findings for risk management, for short-term portfolio management and asset allocation decisions, and for asset pricing and the transmission of shocks across markets

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<sup>2</sup>They attempt to account for this instability using a multivariate GARCH(1,1) model for the time-variation. Their results show that this model does help to capture some of the time variation in inter-country correlations, but formal tests imply that the GARCH specification is not entirely satisfactory.

and asset classes.

## 2 Definitions and Notation.

As noted in the introduction, the main purpose of this paper is to investigate the time-series properties of volatilities and correlations of returns across instruments and markets using daily data. In particular we outline a simple statistical model that can be used to answer the following questions:

1. Are volatilities of returns stable over time? When they change, is the change smooth or violent?
2. Are correlation coefficients between returns stable over time, or do they fluctuate between +1 and -1?
3. Are there any systematic relationships between the risk factor volatilities? For example, does the correlation between volatilities increase when the market volatility “jumps”?

We use a simple framework to provide some preliminary answers to these questions. We report a narrow set of results from a very large pool, and we summarize our results only informally using charts rather than more formal measures. Nevertheless, we believe that our findings are suggestive of interesting patterns which have important implications in particular for risk management and valuation of some exotic options such as range structures. In this section we begin by laying out the definition of correlation within a continuous time asset pricing framework.

We assume that  $\{X_t, Z_t\}$  is a 2x1 vector of stochastic processes obeying the following stochastic differential equation:

$$\begin{bmatrix} dX_t \\ dZ_t \end{bmatrix} = \begin{bmatrix} a_t^x \\ a_t^z \end{bmatrix} dt + \begin{bmatrix} \sigma_t^{11} X_t & \sigma_t^{12} X_t \\ \sigma_t^{21} Z_t & \sigma_t^{22} Z_t \end{bmatrix} \begin{bmatrix} dW_{1t} \\ dW_{2t} \end{bmatrix} \quad (1)$$

The  $\{a_t^i, i = x, z\}$  and  $\{\sigma_t^{ij}\}$  are the drift and diffusion parameters. The  $W_{1t}, W_{2t}$  are two *independent* Wiener processes. Note carefully the way we have written the diffusion component. The diffusion component portrays the  $\{\sigma_t^{ij}\}$  as percentage volatilities, although they may potentially depend

on  $X_t, Z_t$  as well. This is important since the markets quote percentage volatility and we would like our tests tailored towards this convention.

In this bivariate framework, the (conditional) volatilities and the correlation between  $X_t, Z_t$  over a finite time interval  $\Delta$  can be defined as follows. The volatility of  $X_t$  over an interval  $[t - \Delta, t]$ , is denoted by  $\sigma_t^x$ :

$$\sigma_t^x = \left[ \frac{1}{\sqrt{\Delta}} E_t^P \left[ \int_{t-\Delta}^t [dX_s - a_s^x ds]^2 \right] \right]^{1/2} \quad (2)$$

$$= \left[ \frac{1}{\sqrt{\Delta}} \int_{t-\Delta}^t [(\sigma_s^{11})^2 + (\sigma_s^{12})^2] ds \right]^{1/2}. \quad (3)$$

where the expectation  $E[\cdot]$  is taken with respect to the information available at time  $t$ , denoted by  $H_t$ . The  $P$  is the real-world probability. The fact that the Wiener processes increments are uncorrelated over time, and uncorrelated with each other is used to derive the last equality.

Similarly for  $\sigma_t^z$ , the volatility of  $Z_t$ , over  $[t - \Delta, t]$  we have:

$$\sigma_t^z = \left[ \frac{1}{\sqrt{\Delta}} E_t^P \left[ \int_{t-\Delta}^t (dZ_s - a_s^z ds)^2 \right] \right]^{1/2} \quad (4)$$

$$= \left[ \frac{1}{\sqrt{\Delta}} \int_{t-\Delta}^t [(\sigma_s^{21})^2 + (\sigma_s^{22})^2] ds \right]^{1/2} \quad (5)$$

The (conditional) covariance,  $\rho_t^{xz}$ , between the increments of  $X_t$  and  $Z_t$  during  $\Delta$ , is defined by:

$$\rho_t^{xz} = E_t^P \left[ \frac{1}{\Delta} \int_{t-\Delta}^t (dX_s - a_s^x ds) \int_{t-\Delta}^t (dZ_s - a_s^z ds) \right] \quad (6)$$

$$= \frac{1}{\Delta} \int_{t-\Delta}^t [\sigma_s^{11} \sigma_s^{21} + \sigma_s^{22} \sigma_s^{12}] ds, \quad (7)$$

where the  $s$  subscript is a reminder of possible dependence of  $\sigma_s^{ij}$  on the  $X_t, Z_t$ .<sup>3</sup>

Finally, from these parameters, time dependent conditional correlation coefficients can be calculated:

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<sup>3</sup>The integrals on the right hand side of equations (3),(5) and (7) are defined pathwise.

$$r_t^{xz} = \frac{\rho_t^{zx}}{\sigma_t^z \sigma_t^x}. \quad (8)$$

The  $r_t^{xz}$  are considered as time-dependent “short-term” correlation coefficients between the risk factors  $X_t, Z_t$ .

As can be seen from the expressions in (3),(5) and (7), the volatilities and covariances during a finite time interval  $\Delta$  are (standard) integrals of expressions depending on  $\{\sigma_s^{ij}, t \leq s \leq t+\Delta\}$ . Further these non-anticipating random variables may depend on the  $X_t$  and  $Z_t$  in complicated ways and may make the time dependent correlations  $r_t^{xz}$  fluctuate between +1 and -1.

The section below describes a Kalman filter framework that can be used to obtain estimates for  $\sigma_t^x, \sigma_t^z, \rho_t^{xz}$  and especially of the  $r_t^{xz}$  for data that are *one day* apart.

### 3 A recursive model

In the following,  $X_t$  represents an interest rate process, while the  $Z_t$  will denote either an exchange rate, a stock price, or a commodity price. Set  $\Delta = 1$  so that  $\{t = 0, 1, 2, \dots\}$  is an index of *days* elapsed. Let  $H_t^x$  and  $H_t^z$ , be the observed histories of the  $\{X_t, Z_t\}$  respectively:

$$H_t^x = \{X_t, X_{t-1}, \dots, X_0\} \quad (9)$$

$$H_t^z = \{Z_t, Z_{t-1}, \dots, Z_0\}. \quad (10)$$

For interest rates, we let  $x_t$  be given by the first differences:

$$x_t = X_t - X_{t-1}. \quad (11)$$

We define  $z_t$  using logarithmic first differences:

$$z_t = \log(Z_t) - \log(Z_{t-1}). \quad (12)$$

We assume that the  $x_t, z_t$  have the conditional means:

$$E^P [x_t | H_{t-1}^x] = \bar{x}_t$$

$$E^P [z_t | H_{t-1}^z] = \bar{z}_t.$$

Our objective is to estimate the parameters  $\{\sigma_t^x, \sigma_t^z, \rho_t^{xz}\}$  defined in (3),(5) and (8) respectively. Note that in discrete time these volatilities and correlations are calculated conditional on histories available at time  $t - 1$  and that they will in general be *time-dependent*. To investigate the dynamics of these parameters, we exploit the Kalman filter framework.

There are two basic steps in the way we proceed.

We first identify the random variables  $|x_t - \bar{x}_t|^2$ ,  $|z_t - \bar{z}_t|^2$  and the  $[(x_t - \bar{x}_t)(z_t - \bar{z}_t)]$  as noisy observations for the unobserved time-varying parameters  $(\sigma_t^x)^2$ ,  $(\sigma_t^z)^2$  and  $\rho_t^{xz}$ , respectively. This is because we can always write, using orthogonal projections of the left-hand side variables on the spaces generated by  $H_{t-1}^x, H_{t-1}^z$ :

$$|x_t - \bar{x}_t|^2 = E^P [x_t - \bar{x}_t | H_{t-1}^x]^2 + \epsilon_t^x \quad (13)$$

$$|z_t - \bar{z}_t|^2 = E^P [z_t - \bar{z}_t | H_{t-1}^z]^2 + \epsilon_t^z \quad (14)$$

$$(x_t - \bar{x}_t)(z_t - \bar{z}_t) = E^P [(x_t - \bar{x}_t)(z_t - \bar{z}_t) | H_{t-1}^x, H_{t-1}^z] + \epsilon_t^{xz} \quad (15)$$

where the  $\{\epsilon_t^x, \epsilon_t^z, \epsilon_t^{xz}\}$  have zero mean and are serially uncorrelated due to orthogonality of projections.<sup>4</sup> We need to specify their variances explicitly. The conditional and the unconditional variances of these error terms are assumed to be the same and are given by:

$$\left[ E^P [\epsilon_t^x]^2 \right]^{1/2} = \omega_\epsilon^x, \quad \left[ E^P [\epsilon_t^z]^2 \right]^{1/2} = \omega_\epsilon^z, \quad \left[ E^P [\epsilon_t^{xz}]^2 \right]^{1/2} = \omega_\epsilon^{xz}. \quad (16)$$

Rewriting the equations in (14)-(16) in terms of the volatility and correlation parameters  $\{\sigma_t^x, \sigma_t^z, \rho_t^{xz}\}$  introduced in the previous section gives the *observation equations* of the Kalman filter:

$$|x_t - \bar{x}_t|^2 = (\sigma_t^x)^2 + \epsilon_t^x \quad (17)$$

$$|z_t - \bar{z}_t|^2 = (\sigma_t^z)^2 + \epsilon_t^z \quad (18)$$

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<sup>4</sup>They may very well be correlated among themselves, both over time and contemporaneously.

$$(x_t - \bar{z}_t)(z_t - \bar{z}_t) = \rho_t^{xz} + \epsilon_t^{xz}. \quad (19)$$

The quantities on the left-hand side of these equations are observed. The terms on the right hand side consist of two sets of time-dependent unobserved variables.

The  $\{\epsilon_t^x, \epsilon_t^z, \epsilon_t^{xz}\}$  are unobserved disturbances to *variances* and *covariances*, respectively, whereas the  $\sigma_t^x, \sigma_t^z$ , and the  $\rho_t^{xz}$  are unobserved time varying  $H_{t-1}^x$  and  $H_{t-1}^z$  measurable variables. Clearly, these equations can be exploited in a Kalman filter framework if a second set of equations describing the (a-piori) evolution of  $\{\sigma_t^i\}$  can be obtained.

To do this, note that thus far, we have not imposed any significant restrictions on the observed price series.<sup>5</sup> In order to complete the Kalman filter framework we now assume that, the parameters  $\sigma_t^x, \sigma_t^z$  and the  $\rho_t^{xz}$  *smoothly* vary overtime and that this evolution can be captured by the system of equations:

$$\sigma_t^x = \alpha_x \sigma_{t-1}^x + v_t^x \quad (20)$$

$$\sigma_t^z = \alpha_z \sigma_{t-1}^z + v_t^z \quad (21)$$

$$\rho_t^{xz} = \alpha_{xz} \rho_{t-1}^{xz} + v_t^{xz}, \quad (22)$$

where the  $\{v_t^x, v_t^z, v_t^{xz}\}$  are zero-mean random variables whose conditional and unconditional second-order moments are denoted by:

$$\left[ E^P[v_t^x]^2 \right]^{1/2} = \omega_v^x \quad \left[ E^P[v_t^z]^2 \right]^{1/2} = \omega_v^z \quad \left[ E^P[v_t^{xz}]^2 \right]^{1/2} = \omega_v^{xz}$$

The parameters  $\{\omega_v^x, \omega_v^z, \omega_v^{xz}\}$  control the tightness of the priors in (20)-(21). In fact, smoothness of time-varying volatility and correlations imply that these parameters are *smaller* than the variances of  $\{\epsilon_t^x, \epsilon_t^z, \epsilon_t^{xz}\}$  denoted by  $\{\omega_\epsilon^i\}$ .

The smoothness constraint also suggests that the  $\alpha_x, \alpha_z, \alpha_{xz}$  are close to +1. The disturbance terms  $\{v_t^x, v_t^z, v_t^{xz}\}$  are further assumed to be serially uncorrelated and uncorrelated among themselves.

Thus, we obtain a framework where Kalman filter becomes the natural estimator. In fact, the general setup to estimate all the parameters can be described by a two equation dynamic system:

$$y_t = \beta \mu_t + \epsilon_t \quad (24)$$

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<sup>5</sup>Apart from the implicit assumption that the respective conditional expectations exist.

$$\mu_t = \alpha\mu_{t-1} + v_t, \quad (25)$$

where  $y_t$  is the noisy observation on the unobserved  $\mu_t$ . The variances of  $\epsilon_t$  and  $v_t$  are denoted respectively by  $\omega_1$  and  $\omega_2$ . The covariance is denoted by  $\omega_{12}$ .

The Kalman filter estimate of the parameter  $\mu_t$ , denoted by  $\hat{\mu}_t$ :

$$\hat{\mu}_t = E[\mu_t | H_{t-1}^y],$$

is then given by the following formula:

$$\hat{\mu}_t = \alpha\hat{\mu}_{t-1} + \left[ \frac{\alpha\beta s_t + \omega_{12}}{\omega_2 + \beta^2 s_t} \right] [y_t - \alpha\hat{\mu}_{t-1}], \quad (26)$$

where the  $s_t$  is the conditional variance-covariance matrix of  $\hat{\mu}_t$  and is given by the recursive equations

$$s_t = \alpha^2 s_{t-1} + \omega_1 \quad (27)$$

$$s_{t+1} = s_t - \frac{(\alpha\beta s_{t-1} + \omega_{12})^2}{(\omega_2 + \beta^2 s_{t-1})}, \quad (28)$$

with  $s_0$  specified a-priori. Here the  $\omega_1, \omega_2$  are the volatilities of the errors in equations(24) and (25) respectively. The  $\omega_{12}$  is the covariance between  $\epsilon_t, v_t$ .

### 3.1 Selection of parameters

The recursive formulae of (26)-(28), are applied to various interest rates, exchange rates and stock prices. Once time-varying estimates for  $\{\sigma_t^x, \sigma_t^z, \rho_t^{xz}\}$  are found, time-varying correlation coefficients  $r_t^{xz}$  are calculated and plotted. This means that for each pair of risk factors, the equations (26)-(28) have to be applied three times.

First, some parameters of the dynamic equations (24)-(25) need to be specified a-priori.

We assume that the  $X_t, Z_t$  are reasonably close to being equilibrium prices so that their conditional means are zero:

$$E^P [x_t | H_{t-1}^x] = 0$$

$$E^P [z_t | H_{t-1}^z] = 0.$$

This is easier to justify for exchange rates than for the interest rates which are known to “follow” mean-reverting stochastic differential equations. However, even with mean-reverting interest rate processes, the *daily* drift will be negligible relative to the diffusion terms. In any case, the instability of volatilities and of correlation coefficients is much larger to be explained by daily drifts.

Next, for the variance of the noisy observation,  $\omega_1$ , we always use:

$$\omega_1 = \frac{1}{T} \sum_{i=1}^T [y_t - \hat{y}]^2, \quad (29)$$

where  $T$  is arbitrarily selected as 150 days. We set the variance of the error in the state equation (25) according to:

$$\omega_2 = \lambda \omega_1. \quad (30)$$

Thus  $\lambda$  is a proportionality constant that determines the relative weights put on the observations and on the priors at time  $t$ . We set  $\lambda$  equal to .2. Thus, we stack the cards *against* erratic movements in the time varying parameters.

The correlation between  $\epsilon_t$  and  $v_t$  in (24)- (25) is set to zero:

$$\omega_{12} = 0. \quad (31)$$

With respect to a-priori dynamics of  $\{\sigma_t^x, \sigma_t^z, \rho_t^{xz}\}$ , we assume that:

$$\alpha_x = \alpha_z = \alpha_{xz} = .9. \quad (32)$$

This assumption is consistent with the conventions used in risk management literature. <sup>6</sup>

Further, the parameter  $\beta$  is naturally selected as:

$$\beta = 1. \quad (33)$$

Finally, the initial point for  $s_t$  was obtained by letting:

$$s_1 = \omega^1 \quad (34)$$

In order to reduce the effects of initial point selection, the Kalman filter estimates for the first 15 days are subsequently dropped. It turns out that the results reported below are not sensitive to *significant* variations in  $\omega^i$ ,  $\alpha$ ,  $\beta$  and in  $\lambda$ .

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<sup>6</sup>For example, in RiskMetrics J.P. Morgan assumes that volatilities change by an exponential time decay factor of .9.

## 3.2 The Results

We report results on daily data on six assets: the DEM/USD, FF/USD exchange rates, the one-month French and German money market rates and the one-year French and German government bond yields. The sample period was January 1, 1990 to October 31, 1995. We work with 1520 daily observations.<sup>7</sup>

We report the results in two stages. First we discuss the empirical results that show an *extreme* variation in the correlation coefficients between risk factors. Next we discuss the correlation and the time variation of risk factor volatilities. Both sets of empirical results provide strong evidence concerning practices of risk management and calibration of dynamic models for pricing purposes.

The first set of results deal with the *instability* of correlation coefficients between risk factors. We estimate time-varying correlation coefficients between risk factors and plot them. Conclusions are drawn from a visual examination of these charts.

Figures I(a)-I(c) display the time varying correlation coefficients between various risk factors. These figures were obtained by first applying the Kalman filter formulas in (26)-(28) to pairwise risk factors designed in the titles. The result is the estimate of the time-varying correlation,  $r_t^{xz}$ , which is plotted. We have the following observations concerning these time varying correlations.

With one exception, the *daily* correlations between risk factors are very *unstable*, varying from +1 to -1 throughout the sample period. This instability contrasts sharply with the very high positive correlation we find between dollar-French frank and dollar-German mark rates as shown in Figure I(a), which in fact provides some evidence that, if there were meaningful correlation between risk factors our procedures would have detected it. In fact, French Frank and German Mark are expected to be highly correlated due to their relation in EMU. Thus, when the Kalman filter is applied to estimate the corresponding  $r_t^{xz}$ , we obtain a correlation that rarely deviates from +1.

<sup>8</sup> Yet, when our procedure is applied to other risk factors we see that the

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<sup>7</sup>In fact, many other series were tried. Interest rates on maturities longer than one year and other risk factors such as equities were used. Data from smaller EC economies such as Spain were also investigated. Results were similar.

<sup>8</sup>There is one major instance of deviation and that is the 1992 EMS foreign exchange

*daily* correlation coefficients are highly unstable. We consider some details.

As expected, time-varying *daily* correlation coefficients for Equity indices fluctuate, but are in general *positive*, as can be seen from I(b). Instead, time-varying *daily* correlation coefficients between Equity indices and interest rates fluctuate within the band  $-0.5$  and  $+0.5$ , and it is difficult to make a statement concerning the *sign* of the daily correlations as well as their size.

Daily correlations between various interest rates belonging to different economies are *on the average* positive, but not throughout the sample period. In fact, we observe intermittent periods where this correlation shift to negative territory. Finally, even the daily correlations between interest rates and currencies are highly *unstable*, contrary to the expectation one may have from economic theory. For these two variables we again have periods of high negative correlation as well as high positive ones.

Further, these switches between high negative and positive correlation seem to occur in a completely arbitrary fashion, to the extent that the use of *daily* estimated correlations for VaR calculations become quite suspect. Indeed, if short-term correlation coefficients display such extreme time-variation, it becomes quite difficult to justify the use of “yesterday’s” estimates in calculating “today’s” VaR.

Thus, our first set of results seem to indicate that, on a short-term basis it is very difficult to use *stable* correlation coefficients for risk management and for pricing securities with historically estimated parameters. Instead, the results indicate that the implicit values for correlation parameters need to be extracted from liquid and up to date prices of properly chosen contingent claims. It is interesting that econometric work based upon monthly data may not show this weakness of historical second moment estimation, since the monthly second moments do appear to be quite stable. But, for pricing and risk management purposes the use of monthly data will be less meaningful.

Our second set of results are given in Figures II(a)-II(b). These results provide estimates of the daily time-varying correlations *between volatilities* of various risk factors. First we begin by providing a sample of time-varying *daily* volatilities of German 10 year rates, French Frank, S&P500 and Nikkei. Deterministic periods interrupted by sudden bursts of volatility are clearly visible in Figure I(a). Other risk factor volatilities, not reported here are essentially similar in nature.

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crisis.

Next, in figure II(b) we provide two examples of volatilities *of* volatilities. The charts plotted in these figures were obtained from the same set of formulas as in the above case. But, in contrast to the results reported in the first set above, we now need to apply a *two-stage* process. At the first stage time-varying volatilities for each risk factor are estimated using Kalman filter and this way a process of volatility estimates are generated. Then, time-varying correlations were estimated using these estimated time-varying volatilities. From these, we also estimate the so-called *volatilities of volatilities*. We provide some selected results. Other risk factors again yield quite similar results.

Figure II(b) displays the volatilities of DEM and YEN volatilities. In order to calculate the data displayed here, we first applied the Kalman Filter in order to obtain estimates of time varying daily volatilities. Then, the Kalman filter was applied for a second time to obtain the volatilities of these first round estimates. Hence the results of Figure II can be interpreted as displaying the volatility of the stochastic component in the DEM and YEN volatilities. The interesting result from these Figures is that most of the time daily volatility of DEM and YEN volatilities are close to zero. Only at some occasional times the volatilities become volatile. This occurs as a spike which then gradually goes back to zero within two to three weeks. We see that the size and the timing of these spikes are very highly correlated across the two currencies.

In fact, Figures III(a)-III(c) provide estimates of the correlation between the volatilities of three risk factors; namely the pairs US90 day rates and German 10 year rates, DEM/USD and Yen/USD exchange rates, CAC40 and Nikkei. We see in all these graphs that the time varying daily correlations are very close to 1, in general.

Hence not only the volatilities seem to vary stochastically, but this variation has a surprising characteristic. The variation in volatility processes is stochastically non-linear. That is to say, long periods of stability are interrupted by sudden spurts of high volatility. This stochastically non-linear behavior is present in all risk factor volatilities. Second, we see that the correlations between the volatilities of risk factors are very high.

## 4 Conclusions

We can summarize our results in terms of three empirical propositions.

**Proposition 1:** The correlations between risk factors are highly unstable even over short periods of time, both in terms of the sign and in absolute value. The exception is the correlations between exchange rates which is close to 1.

**Proposition 2:** Although the correlations and volatilities of risk factors are themselves very unstable, the volatilities of risk factors are highly and positively correlated among themselves. The stability of correlations between volatilities contrasts sharply with the instability of the correlation of the variables themselves.

**Proposition 3:** The time-variation of volatilities is stochastically *non-linear*. Long periods of “deterministic” volatilities are interrupted by sudden bursts of highly volatile periods. This pattern is repetitive.

These results have the following implications.

Concerning risk management, according to the rules that will be proposed by the Basel Committee, banks will not be able to use diversification arguments *across* asset classes when they calculate value at risk for the purpose of capital requirements. This has been criticized by the industry on the grounds that correlations of returns are typically less than 1 and that not allowing for the effects of diversification means that capital requirements will be unnecessarily high. Our results show that correlations can quite suddenly approach +1 even though, on the average, they are much smaller. But since value at risk calculations are supposed to be valid in extreme situations, the assumption that the relevant correlations are equal to 1 is perhaps justified. Our results therefore support the interpretations of Basel Committee.

Concerning portfolio management and asset allocation, in view of the instability of correlations of returns in the short run, one can not rely on standard optimal portfolio models for short-run portfolio management. The asset allocation problem needs to take into consideration the unstable nature of these second order moments.

Concerning measurement of volatilities, ARCH and GARCH models may not be the most appropriate. Models of stochastic process switching may be a more appropriate way of capturing the stochastically non-linear behavior of volatility fluctuations.

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