Economic Benefit of Powerful Credit Scoring *

Andreas Blöchlinger‡

Credit Suisse

Markus Leippold‡

Swiss Banking Institute, University of Zurich

First Version: November, 2004
This Version: July 20, 2005

*The content of this paper reflects the personal view of the authors, in particular, it does not necessarily represent the opinion of Credit Suisse. The authors thank the seminar participants at the University of Zurich, Marc Paolella, Roger M. Stein, and two anonymous referees for their thoughtful comments. Markus Leippold acknowledges the financial support of the Swiss National Science Foundation (NCCR FINRISK).

‡Correspondence Information: Andreas Blöchlinger, Credit Suisse, Bleicherweg 33, CH-8070 Zurich, Switzerland, tel: +41 (44) 333 45 18, mailto:andreas.bloechlinger@credit-suisse.com

‡Correspondence Information: Markus Leippold, Swiss Banking Institute, University of Zurich, Switzerland, tel: +41 (44) 634 39 62, mailto:leippold@isb.unizh.ch
Economic Benefit of Powerful Credit Scoring

ABSTRACT

We study the economic benefits from using credit scoring models. We contribute to the literature by relating the discriminatory power of a credit scoring model to the optimal credit decision. Given the Receiver Operating Characteristic (ROC) curve, we derive a) the profit-maximizing cutoff and b) the pricing curve. Using these two concepts and a mixture thereof, we study a stylized loan market model with banks differing in the quality of their credit scoring model. Even for small quality differences, the variation in profitability among lenders is large and economically significant. We end our analysis by quantifying the impact on profits when information leaks from a competitor’s scoring model into the market.

JEL Classification Codes: D40, G21, H81

Key Words: Bank loan pricing; Credit scoring; Discriminatory power; Receiver Operating Characteristic (ROC).
In this paper, we investigate the economic benefit of credit scoring models. Ordinal performance measures such as, e.g., the Receiver Operating Characteristic (ROC) curve, are widely used to assess the discriminatory power of credit scoring and rating models. However, performance statistics and common lending practice seem to be two separate worlds. We show how to reconcile ordinal power measures with metrics like profit and loss. In addition, we present a simple loan market model where banks with different credit scoring models compete for loans. By calibrating the model, we find that higher discriminatory power translates into significant profit improvement.

For banking institutions, loans are often the primary source of credit risk. Traditional lending practice has been to grant loans that have a positive net present value (NPV) and to deny those that do not. Recently, the use of statistical models has increased significantly. To assess the risk of these loans, banks use credit scoring models and credit ratings to estimate default risk on a single obligor basis.

Loans to small and medium sized companies, mostly unrated firms, are an important portion of most banking institutions’ portfolios. Since the individual amount of exposure to such firms is often relatively small, it is uneconomical to devote extensive resources to the credit analysis. The credit scoring model should optimize both the likelihood of a bad obligor being accepted and the likelihood of a good obligor being rejected. Similarly, in the case of a pricing-based lending, a credit scoring model with low discriminatory power can lead to underpricing of bad and overpricing of good loans. For a recent survey on the use of credit scoring models, we refer to Thomas (2000) and Thomas, Edelmann, and Crook (2002).

In evaluating the performance of credit scoring models, it is common practice to use ordinal measures such as, e.g., the Receiver Operating Characteristic (ROC) curve and its associated discriminatory power statistics. However, it is not a priori clear how discriminatory power is linked to credit decision making and credit risk pricing. Establishing such a link is essential for the profitability of the bank’s credit business. If in a market with several suppliers of loans in which, by means of a higher default prediction accuracy, one bank has better knowledge of the quality of loans than its competitors, the information advantage may translate into better profitability figures.
In this paper, we show how lenders can incorporate the scoring model and its ROC-based performance measure into traditional lending practices, based on NPV considerations. By relating the discriminatory power of a credit scoring model to the optimal credit decision, we derive a) the profit-maximizing cutoff and b) the pricing curve. In addition, to analyze the economic impact of discriminatory power, we study a stylized loan market with banks that differ in the quality of their credit scoring model. Already for small differences in the discriminatory power of the credit scoring models, we find that profitability varies substantially among lenders. More powerful credit scoring models lead to economically significant differences in credit portfolio performance. Finally, we study the market impact of a model improvement by one bank. Such an improvement will have a negative impact on the profit of the other competitors. We also study the situation in which information leaks and the competitors obtain perfect knowledge of the improved model. We show that the information leak offsets a large part of the profit increase.

To some extent, our work is motivated by the contribution of Stein and Jordão (2003). In their paper, they provide empirical evidence of the economic impact of differences in discriminatory power between various default models. Their simulation is based on historical data of middle-market financial statement and loan performance data. Stein and Jordão (2003) claim that, due in part to the non-parametric nature of power curves, an analytical exploration of their economic benefits is inherently difficult. By deriving the profit-maximizing cutoff and the pricing curve, we are able to resolve this difficulty. In a similar way and independently of our study, the recent contribution of Stein (2005) also derives a link between the power curve and the pricing of loans. However, our derivation is more direct and allows a clear-cut analysis of (stylized) loan market models in several dimensions, such as revenues, profits and losses, and market shares. Further, we model a market regime by mixing cutoff and pricing regime that closely matches a real credit environment. Therefore, our model offers a flexible framework that could be calibrated to market data. Such a calibration exercise would not only give valuable insights in the current loan market structure, but could also help regulators and policy makers to control for the level of market concentration.
The paper is organized as follows. Section I explains the Receiver Operating Characteristic (ROC) concept to assess the discriminatory power of credit scoring models. Section II describes how profit-optimal credit decisions can be deduced from ROC statistics. In Section III, we present a stylized loan market model under different market regimes. For each regime, we calculate different economic figures, like market share and profit. Section IV concludes.

I. Discriminatory power

Credit scoring models can err in two ways. First, the model may indicate low risk when, in fact, the risk is high. This error, typically referred to as $\alpha$-error, corresponds to the assignment of high credit quality to obligors who nevertheless default or come close to defaulting. The cost of the bank is the loss of credit amount and/or interest. Second, the model may indicate high risk when, in fact, the risk is low. This error, usually referred to as $\beta$-error, relates to low-rated firms that should, in fact, be rated higher (see, Table 1). Potential losses resulting from this second type of error include the loss of return and fees as well as a drop in market share when loans are either turned down or lost through non-competitive pricing. Table 1 gives an overview of the various costs occurring from $\alpha$- and $\beta$-errors.

[ Table 1 about here ]

There exist several methods to measure the statistical performance of credit scoring models. One of the most applied methods is the Receiver Operating Characteristic (ROC). The ROC analysis is a technique originally used in medicine, engineering, and psychology to assess the performance of diagnostic systems and signal recovery techniques (see, e.g., Egan (1975)).

The ROC curve is a two-dimensional measure of classification performance and visualizes the information from the Kolmogorov-Smirnov statistics if the ROC is concave.$^1$ It is constructed by calculating the $\alpha$- and $\beta$-errors for every possible cutoff level $t$. The two sets of errors correspond to the coordinates of the Receiver Operating Characteristic (ROC) curve.

---

$^1$The maximum distance between the ROC curve and the diagonal equals a constant times the Kolmogorov-Smirnov statistic, but only if the ROC is concave. If the ROC curve is not concave, there is no such general correspondence. We thank one of the referees for pointing this out to us.
(See also Sobehart and Keenan (2001) as well as Engelmann, Hayden, and Tasche (2003) for a discussion of measuring discriminatory power of credit scoring models).

The ordinate of the ROC curve is scaled as the hit rate, i.e., one minus the $\alpha$-error, under the null hypothesis that high scores translate into high default probabilities

$$1 - \alpha(t) = \mathbb{P}\{S > t|Y = 1\} = \mathbb{P}\{S_D > t\},$$

where $S$ is the credit score and $S_D$ is the conditional credit score of defaulters. The abscissa is scaled as the false alarm rate ($\beta$-error)

$$\beta(t) = \mathbb{P}\{S > t|Y = 0\} = \mathbb{P}\{S_{ND} > t\},$$

where $S_{ND}$ is the conditional credit score of non-defaulters. The construction of the ROC curve is illustrated in Figure 1, where we show possible distributions of rating scores for defaulting and non-defaulting obligors. For a perfect rating model, the left distribution and the right distribution would be separate. For a real credit portfolio, perfect discrimination is not possible. Both distributions will overlap. In Figure 1, the dark area under the population of defaulters represents the $\alpha$-error. The shaded area under the population of non-defaulters represents the proportion of false alarms ($\beta$-error) generated by the model in response to the particular cutoff score $t$.

[Figure 1 about here]

Figure 2 plots the ROC curves for four different models: Model I, Model II, as well as a random and a perfect rating model. In Figure 2, the diagonal line corresponds to random forecasts. When the curve bows away from the diagonal line to the upper left corner, this indicates an improvement of the model’s performance. Thus, the ROC of a powerful rating model is steep at the left end and flat near the point $(0, 1)$ that represents the perfect model in which the two distributions of defaulters and non-defaulters are separate. Similarly, the larger
the area below the ROC curve, the better the model. This area is usually called the AUROC. A nice interpretation of the AUROC is given by, e.g., Bamber (1975) and Hanley and McNeil (1982).

Formally, under the null hypothesis that high score values indicate low creditworthiness, AUROC is defined as

\[
\text{AUROC} = - \int_{-\infty}^{\infty} \mathbb{P}\{S_D > y\} d\mathbb{P}\{S_{ND} > y\} - \int_{-\infty}^{\infty} \frac{1}{2} \mathbb{P}\{S_D = y\} d\mathbb{P}\{S_{ND} > y\}.
\]

By transforming we get

\[
\text{AUROC} = \int_{-\infty}^{\infty} \left[ \mathbb{P}\{S_D > y\} + \frac{1}{2} \mathbb{P}\{S_D = y\} \right] dF_{S_{ND}}(y)
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ 1_{\{x>y\}} + \frac{1}{2} 1_{\{x=y\}} \right] dF_{S_D}(x) dF_{S_{ND}}(y)
\]

\[
= \mathbb{P}\{S_D > S_{ND}\} + \frac{1}{2} \mathbb{P}\{S_D = S_{ND}\}
\]

where \(F_{S_{ND}}\) and \(F_{S_D}\) are the non-defaulters’ and defaulters’ distribution functions, respectively. The last line follows from the assumption of independent draws out of the two populations. For continuous random variables, we have \(\mathbb{P}\{S_D = S_{ND}\} = 0\). An AUROC of 0.5 (area of the orthogonal, isosceles triangle) reflects random forecasts, while AUROC = 1 (area of the square) implies perfect forecasts. For any reasonable rating model, the AUROC lies between 0.5 and 1.

We note that the discussion of power must always take place with the understanding that, if the goal is to compare two models, calculation of power needs to be done on the same population. Power statistics are especially sensitive to the sample chosen when the number of defaults is limited, as is typically the case in commercial lending. Therefore, differences in samples may lead to different assessments of power (see also Hamerle, Rauhmeier, and Roesch (2003) and Stein (2002)). In the subsequent analysis, we report the discriminatory power as a measure for the entire loan market and, consequently, on the same population.
II. Credit Decision

In this section, we extend the ROC analysis and link it to a NPV analysis to deduce optimal credit decisions. First, we discuss the derivation of profit-maximizing threshold values (cutoff regime). Second, we show how the pricing curve can be derived from the ROC curve (pricing regime). We note that a credit scoring system is not the only lever for a commercial bank to assess the creditworthiness of potential customers. Competitive predictors of the overall default frequency through macroeconomic indicators, loss rate estimation, an exact estimate of a default term structure, rating migration risk, or an accurate valuation of the relationship benefit have the potential to further increase profitability. However, since our emphasis is on the analysis of credit scoring models, we do not consider such extensions of our model framework here.

A. Cutoff regime

The basic use of ROC analysis is to provide guidance for setting the lending cutoffs. The user can define model scores below which a loan will be granted and above which it will not. However, the determination of the cutoff level is an arbitrary choice, most often based on qualitative arguments such as, e.g., business constraints. From a value-maximizing perspective, such a determination is in general suboptimal.

A more rigorous criterion can be derived from knowledge of the prior probability of default and the associated costs and revenues. To this end, we make the simplifying assumptions that there are exogenously determined costs of default and a single market premium $R$ for bearing credit risk. We note that these assumptions are not restrictive for our analysis and could be generalized. For instance, we do not consider explicitly portfolio effects.

Given this risk premium, a bank can either accept or reject a loan. Table 2 gives an overview of the cash-flows involved in case of default and in case of non-default, respectively.

---

2 For a general discussion of various performance criteria for scoring functions, see Hand (1997), chapter 8.
3 For a credit risk model with portfolio effects, see, e.g., Egloff, Leippold, and Vanini (2004) or Kealhofer and Bohn (2001).
Given the default probability and the expected cash-flows, net present value NPV per unit of credit amount (i.e., one US Dollar) dependent on the credit score \( t \) can be written as

\[
\text{NPV}(t) = -1 + \frac{1}{1 + \delta} \left[ \mathbb{P} \{ Y = 1 | S = t \} (1 - \text{LGD}) + \mathbb{P} \{ Y = 0 | S = t \} (1 + R + C) \right],
\]

where \( \delta \) is the risk-adjusted discount rate. We assume all quantities to be adjusted for a one-period time horizon. LGD denotes loss-given default and includes recovery costs. Recovery costs or workout fees depend heavily on law enforcement and liquidity of the collateral. We treat LGD as an exogenously determined constant value or expected value, respectively.\footnote{Note that if LGD were stochastic and uncorrelated with the credit score \( S \), then the expected values we are reporting in the following are unaffected.}

In equation (1), \( R \) represents the interest to be paid at maturity, \( \mathbb{P} \{ Y = 1 | S = t \} \) is the conditional probability of default given knowledge of the credit score. Relationship managers often use the argument of “strategic value” when taking on loans with seemingly negative NPV. We capture this strategic value by \( C \), which is an (real) option to make follow-on business such as, e.g., private banking activities.

In the subsequent analysis, we will assume that the bank does not invest in negative NPV-projects. Thus, the lender rejects all obligors that do not fulfill either of the following two inequalities (2) and (3) below:

\[
R \geq \frac{\mathbb{P} \{ Y = 1 | S = t \}}{\mathbb{P} \{ Y = 0 | S = t \}} \text{LGD} - C + \frac{\delta}{\mathbb{P} \{ Y = 0 | S = t \}} \quad (2)
\]

\[
\geq \frac{\mathbb{P} \{ Y = 1 | S = t \}}{\mathbb{P} \{ Y = 0 | S = t \}} \text{LGD} - C \quad (3)
\]

\[
= -\frac{\mathbb{P} \{ Y = 1 \}}{\mathbb{P} \{ Y = 0 \}} \frac{d \alpha(t)}{dt} \text{LGD} - C \quad (4)
\]

\[
= -\frac{\mathbb{P} \{ Y = 1 \}}{\mathbb{P} \{ Y = 0 \}} \frac{d \beta(t)}{dt} \frac{d \beta(t)}{dt} \text{LGD} - C. \quad (5)
\]
A bank loan that fulfills inequality (3) is profitable. If it meets inequality (2), then the loan deal adds value to the bank’s credit portfolio. We deduce equation (4) from the fact that

\[
\alpha(t) = \mathbb{P}\{S \leq t|Y = 1\} = \frac{1}{\mathbb{P}\{Y = 1\}} \int_{-\infty}^{t} \mathbb{P}\{Y = 1|S = s\} dF(s),
\]

\[
1 - \beta(t) = \mathbb{P}\{S \leq t|Y = 0\} = \frac{1}{\mathbb{P}\{Y = 0\}} \int_{-\infty}^{t} \mathbb{P}\{Y = 0|S = s\} dF(s),
\]

where \(F\) is the distribution function of random variable \(S\). We use the differential quotient somewhat informally in equation (4). The curve defined by a ROC analysis may not be differentiable. Nevertheless, we use it to convey the conventional meaning when interpreting ROC curves as if they were continuous and differentiable at least once.

Therefore, the required risk premium \(R(t)\) for a specific credit risk score \(t\) increases a) with the steepness of the ROC curve, i.e., with \(-\frac{d\alpha(\beta)}{d\beta}\), b) with the discount factor \(\delta\), c) with the loss-given default LGD, and d) with the expected default frequency \(\mathbb{P}\{Y = 1\}\). In contrast, the required risk premium \(R(t)\) decreases with the value of the real option \(C\).

In equation (3), we can set the discount rate equal to zero, i.e., \(\delta = 0\), the net present value equals profit. Rearranging equation (2), we arrive at

\[
-\frac{d\alpha(\beta(t))}{d\beta(t)} \leq \frac{\mathbb{P}\{Y = 0\}}{\mathbb{P}\{Y = 1\}} \frac{R + C}{LGD} =: s. \tag{6}
\]

The left-hand side of the inequality represents the slope of the ROC curve at point \(t\). A bank certainly refuses all obligors with negative expected profit. Hence, the bank rejects all applicants with a score \(t\) and a corresponding slope of the ROC curve higher than \(s\). The numerator in (6) represents the probability-weighted opportunity cost of withholding lending to non-defaulters. The denominator represents the probability-weighted recovery cost of accepting defaulters.

We call the straight line in the ROC graph with slope \(s\) an iso-profit line. All points on the straight line achieve the same profit. The point at which the line with slope \(s\) forms a tangent
to the ROC curve defines the optimal cutoff \( t^* \). At this point, expected marginal profit is zero, i.e.,

\[
-\mathbb{P} \{ Y = 0 \} (R + C) \frac{d\beta(t)}{dt} \bigg|_{t=t^*} = \mathbb{P} \{ Y = 1 \} LGD \frac{d\alpha(t)}{dt} \bigg|_{t=t^*},
\]

or, using (4),

\[
\mathbb{P} \{ Y = 0 | S = t^* \} (R + C) \quad \mathbb{P} \{ Y = 1 | S = t^* \} LGD
\]

The above findings are consistent with our general intuition. At the optimal cutoff, the probability-weighted marginal cost of a mistake has to equal the marginal benefit for a correct decision. In other words, all obligors with a conditional expected revenue higher than or equal to the conditional expected loss are accepted.

To give some intuition of how the optimal truncation values behave, consider the examples given in Figure 3 that show how the cutoffs and the ranking of the various models change as we vary cash-flow assumptions. We plot four different cash-flow scenarios for each of the models, i.e., the perfect and the random model, and Model I and II. On the dashed line we have constant profit and the closer the line to the point \((0,1)\), the higher is the profit. For tangency between the ROC curve and iso-profit line, the slopes must be equal.

In practice, it is often the case that a particular model will outperform another model under some specific set of cash-flow assumptions, but can be disadvantageous under a different set of assumptions. If the ROC curve for two models cross, then neither model is unambiguously better than the other with respect to a general cutoff. When one ROC curve completely dominates the other, such as the perfect model, the dominant model will be preferred for any possible threshold value. For example, in the upper left panel of Figure 3, we are indifferent between Model I and Model II, but we prefer both of them over the random model. By slight changes of cash-flow assumptions, we prefer either Model I or Model II.

[Figure 3 about here]
If we have to accept or reject applicants based on the random model, it is either optimal to accept all applicants
\[ \mathbb{P} \{ Y = 1 \} \cdot LGD < \mathbb{P} \{ Y = 0 \} \cdot R, \]
or to refuse all
\[ \mathbb{P} \{ Y = 1 \} \cdot LGD > \mathbb{P} \{ Y = 0 \} \cdot R. \]
Under some extreme cash-flow assumptions, it might be best to work with no model at all (see lower right panel of Figure 3).

Today, risk spreads for bank loans are not stale and depend on the creditworthiness of each obligor. Banks set prices based on the predictions of their models, and there is no longer a single market-price for bank loans. Therefore, it is important to have a powerful model in order to derive competitive prices.

B. Pricing regime

In a pricing regime, the bank sets the price of the loan according to the credit score. The bank will accept all applicants paying this price. Therefore, the main challenge for the bank lies in the determination of the appropriate and optimal price.

As an economically meaningful criterion, we assume that the bank does not invest in negative NPV projects. Then, the bank sets minimum prices for a borrower based on the predictions of its model, i.e., according to equation (5),
\[
R(t) \geq -\frac{\mathbb{P} \{ Y = 1 \}}{\mathbb{P} \{ Y = 0 \}} \frac{d\alpha(\beta(t))}{d\beta(t)} \cdot LGD - C, \tag{8}
\]
where \( R(t) \) is the credit risk spread, now as a function of score value \( t \). If equality applies the expected profit would be zero.

\footnote{In this paper, we interpret risk-adjusted pricing as a pricing that is based on the obligor’s creditworthiness only, but not on the profitability measured by economic capital.}
We rewrite (8) by introducing \(k \geq 0\),

\[
R(t) + C = \frac{\mathbb{P}\{Y = 1|S = t\}}{\mathbb{P}\{Y = 0|S = t\}} \text{LGD} + k.
\]  

(9)

In the long run, a bank cannot offer loans for a revenue \(R(t) + C\) lower than the right-hand side of (8). Therefore, this term marks a lower bound, i.e., a minimum price. Since the slope of the ROC curve determines the minimum price, we can link pricing rule and ROC curve, both graphically and functionally.

Figure 4 shows two credit scoring models with the same discriminatory power but different minimum pricing schemes. We also plot the random rating model. For this model, the slope of the ROC curve is one. Therefore, every borrower has to pay the same interest rate, ceteris paribus. The bank cannot apply a price discrimination strategy. On the other side, if a bank disposes of a perfect scoring model, all defaulting obligors are refused and the others have to pay the risk-free rate or even less if they are “relationship” customers. Note that with a steeper ROC curve, the bank can put in place a more effective price discrimination strategy to avoid defaulting obligors.

[ Figure 4 about here ]

C. Mixture of cutoff and pricing regime

Credit specialists question both cutoff and pricing regime, since both approaches do not reflect a real credit environment. A cutoff regime oversimplifies today’s lending practice in which risk-adjusted pricing is common practice. A pure pricing regime has also its shortcomings. First, it is questionable whether the risk premium is strictly exogenous. One could imagine that a high risk premium may backfire, in the sense that it triggers a failure. A high risk premium may therefore increase the default probability. Second, one should challenge the assumption that obligors pick or change their current bank only based on slight pricing differences. A good relationship between an obligor and a bank is of value to the obligor and might keep obligors from switching banks, even though they would have been charged a lower risk premium by another bank.
With the above arguments in mind, we suggest a different approach that consists of a mixture of cutoff and pricing regimes. We construct such a mixture model as follows. First, we start with the pricing rule (9). Unlike the pure pricing regime, the risk premium \( R(t) \) is rounded, i.e., toward the next quarter of a percentage point. By doing so, we capture the impact of transactional costs. Slight changes in pricing do not necessarily lead to a loan deal for the bank offering at the lowest exact rate as in a pricing regime. If two or more banks offer at the same rounded rate, the bank is selected by random. Second, obligors whose risk premia exceed an upper threshold value are rejected. This assumption mitigates possible feedback effects of high risk premium on default events.

III. Loan Market Model

Given the profit-maximizing cutoffs or optimal pricing rules, a bank can run into severe adverse selection problems. A rational credit applicant closes the deal with the lender that provides the most favorable terms. When the bank derives the price from the credit score, a low-power model results in a non-competitive pricing system. Hence, a marginal power improvement may lead to a sizeable profitability increase.

Moreover, the quality of the credit scoring model may play a decisive role on how the market is shared among the competing banks. For example, imagine two financial institutions, one with a very high-power model (or even the perfect model), whereas the other bank does the credit business without any model. The first bank will experience almost no defaults, as most of them are absorbed by the second bank. Even worse, the second lender will place almost no loan contracts with non-defaulting obligors, due to overpricing.

To better understand the economic impact of scoring methods with given discriminatory power, we present a stylized loan market model. This stylized market model could be generalized to incorporate other aspects of the banks’ lending decision. However, since our goal is to understand the impact of using credit scoring models with different discriminatory power, we restrict our analysis to the three market regimes as described in the previous section, i.e., the cutoff regime, the pricing regime, and the mixture of cutoff and pricing regime.
The cutoff regime is rather conservative, since the bank uses the credit score to either accept or reject customers at an exogenously given risk premium. The probability of acceptance depends on the credit score. In the pricing regime, the lender is able to either attract or alienate obligors by applying a “smart” pricing scheme. All applicants are offered loans with corresponding risk adjusted credit spreads. Hence, the accepting probability is fixed at one, but the credit risk premium varies with credit scores. By mixing cutoff and pricing, both probability of acceptance and credit spread are attached to the score. Therefore, the mixture regime is closer to common market practice.

We assume a market that is composed of three lenders with credit rating scores $S_1$, $S_2$, and $S_3$. By $Y^*$, we denote the unobservable creditworthiness. We make the following distributional assumption

$$
\begin{pmatrix}
S_1 \\
S_2 \\
S_3 \\
Y^*
\end{pmatrix} \sim N
\begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix},
\begin{pmatrix}
1 & \rho & \rho & \rho_1 \\
\rho & 1 & \rho & \rho_2 \\
\rho & \rho & 1 & \rho_3 \\
\rho_1 & \rho_2 & \rho_3 & 1
\end{pmatrix}.
$$

(10)

We note that, since $S_1$, $S_2$, $S_3$ and $Y^*$ represent ordinal measures, we can assume normalized variables without loss of generality. By the principle of parsimony, we adopt equal correlation among credit scores $\rho$. If the correlation $\rho$ is high and if there are differences in discriminatory power, then the result will be a high profit difference, ceteris paribus. The default indicator $Y$ is defined as follows

$$
Y = \begin{cases}
0 & : \ Y^* \leq c, \\
1 & : \ Y^* > c,
\end{cases}
$$

where the threshold $c$ is calibrated to match the unconditional probability of default $P\{Y = 1\} = \Phi(-c)$. The conditional probabilities of default are computed as

$$
P\{Y = 1|S_i\} = \Phi\left(\frac{c - \rho_i S_i}{\sqrt{1 - \rho_i^2}}\right),
$$

(11)

$$
P\{Y = 1|S_i, S_j\} = \Phi\left(\frac{c - \rho_i S_i - \frac{\rho_j - \rho \rho_i}{1 - \rho^2} (S_j - \rho S_i)}{\sqrt{1 - \rho_i^2 - \frac{(\rho_j - \rho \rho_i)^2}{1 - \rho^2}}}\right).
$$

(12)
The conditional default probability (11) corresponds to the unconditional probability $P\{Y = 1\}$, if credit score $S_i$ and creditworthiness $Y^*$ are independent, or equivalently if $\rho_i = 0$. Later, we will also investigate the economic impact when a bank finds out about the scoring method of a competitor and employs this knowledge to increase forecasting accuracy. Then, the conditional probability of default can be conditioned on two different scores. This conditional probability is computed in equation (12).\(^6\)

A. Calibration and simulation

To calibrate the models, we resort to the experience of credit officers as well as statistical findings. Since the information for making default predictions is typically based on financial key figures, computed by balance sheet and income statement, the credit scores between two banks are highly correlated in absolute terms. Therefore, we fix $\rho$ at 0.8. The remaining correlation coefficients $\rho_1$, $\rho_2$, and $\rho_3$ are set at 0.48, 0.5, and 0.52, that corresponds to AUROC levels in practice. An LGD of 0.4 results as a weighted average of collateralized and unsecured credits. We further fix the unconditional probability of default at $P\{Y = 1\} = 0.02$ and the corresponding threshold at $c = 2.0537$. These values are in line with practical experience.

In the mixture regime, the prices are rounded toward the next quarter of a percentage point. Credit officers argue that they still keep obligors from moving, even if the bank’s credit spread is, on average, around one eighth of a percentage point higher than risk premia of competitors, by virtue of connecting with customers. In case of a mixture regime, all credit applicants with rounded risk premia higher than 2.5% are rejected. In both the pricing and mixture regime, we set $k - C$ in (9) equal to 30 basis points for all obligors. In practice, customer relationship manager of typical retail banks rarely impose credit risk spreads higher than 2.5%. For the cutoff regime, we assume a constant risk premium $R$ of 75 basis points. Finally, we fix the market potential for bank loans in the economy at USD 100 billion.

Starting from this basic setting, we are in a position to calculate different statistics for all three market regimes, e.g., AUROC, price per creditworthiness, market share per creditwor-

---

\(^6\)The derivation of the mathematical formulas can be found in the appendix. Furthermore, in the appendix, we generalize the loan market model to a market consisting of $n$ competitors.
thinness, market share defaulters and non-defaulters, loss, revenue, profit. To calculate these statistics, we use Monte-Carlo simulation. To briefly illustrate the simulation approach, we look at the first two draws from the Monte-Carlo simulation (see equation (10)),

\[
\begin{pmatrix}
0.4991 \\
-0.1177 \\
0.0208 \\
0
\end{pmatrix}
, \quad
\begin{pmatrix}
1.5059 \\
0.8904 \\
1.6744 \\
2.3263
\end{pmatrix}
, \quad \ldots
\]

(13)

The first draw represents a median client. One half of the population have better, the other half lower creditworthiness \( Y^* \). The random credit scores of 0.4991 (Bank 1), -0.1177 (Bank 2), and 0.0208 (Bank 3), lead to default probabilities of 0.0193, 0.0074 and 0.0084 according to equation (11). The second draw represents a defaulting client, because the creditworthiness exceeds threshold \( c = 2.0537 \). The corresponding conditional probabilities are 0.0646, 0.0316, and 0.0830.

**B. Cutoff regime**

In case of evaluating the economic impact in a cutoff regime, the assumption is that all three banks lend at the same risk premium. Therefore, price is not a discriminator. In a first step, the obligor chooses, with equal probability, one bank at random. If the score of the borrower was below the cutoff of the credit scoring model the bank is using, the loan will be assigned to that bank. If the score was above the cutoff, the borrower picks, again at random, one of the remaining two banks. If the score was below that cutoff, the loan is assigned to the bank of second choice. If not, the borrower’s score on the last model is compared to that model’s cutoff. If not accepted at the last bank, it is assumed that the loan would be denied by all three banks.

For our calibration defined above, the optimal cutoffs \( t^* \), according to (7), are 0.4628, 0.4912 and 0.5199. This means that Bank 3 is willing to accept more applicants than Bank 1 and Bank 2. Figure 5 depicts the derivation of profit-maximizing threshold values. By referring to sample vector one in (13) we see that this actual non-defaulter is rejected by Bank 1 (credit
score of 0.4991 is higher than cutoff of 0.4628) and accepted by Bank 2 and Bank 3. Therefore, the revenue of $R = 0.0075$ is expected to being split between Bank 2 and Bank 3. In the second simulation (defaulting loan) in (13), all three credit scores exceed the corresponding cutoffs. Thus, the potential borrower is rejected by all three lenders.

[ Figure 5 about here]

Figure 6 plots probability of acceptance versus creditworthiness (Panel A) and the expected offering premium versus creditworthiness (Panel B). The risk premium is fixed at $R = 0.0075$ and not variable (solid horizontal line). What we observe is that a median borrower has a probability of 0.7011 (0.7146, 0.7285) of being accepted by Bank 1 (Bank 2, Bank 3). Comparing the high-power model (Bank 3) to both the medium-power (Bank 2) and low-power model (Bank 1), the probability of acceptance is higher for “good” and lower for “bad” borrowers. The three curves intersect at around 91%. Therefore, bad borrowers including the defaulting obligors that are located on the right-hand side of the solid vertical line at 98% (since we set the unconditional probability of default equal to 2%) have the greatest chance of getting a loan from Bank 1 (low-power). Therefore, with the high-power model, Bank 3 closes more loan deals and accepts less defaulting loans compared with its competitors Bank 1 and Bank 2.

[ Figure 6 about here]

The differences in the discriminatory power of the scoring models used by the banks will eventually influence the market share and revenues. Note that in the cutoff regime, not all demand for credit will be supplied, since we have a fixed risk premium $R$. Figure 7 plots market share versus creditworthiness (Panel A) and the expected revenue versus creditworthiness (Panel B). In the cutoff regime, the expected market share is a decreasing function of creditworthiness. The same holds for the expected revenue. A median loan exhibits a probability of 0.2760 (0.2857, 0.2960) of going to Bank 1’s (Bank 2’s, Bank 3’s) loan portfolio. The expected revenues on a median loan are 20.7, 21.4 and 22.2 basis points, respectively, as can be observed from the green lines. The market share and the expected revenue curves both cross at around 91%.
Table 3, Panel A, shows expected market shares, profit, revenue, and loss in the cutoff regime. We split up the expected market shares for defaulters and non-defaulters. The market share for the high-power Bank 3 amounts to 28.03% compared to their competitors (26.58% for Bank 1 and 27.29% for Bank 2). Since in the cutoff regime, not all demand for loans is matched by supply, the total market share that is supplied is only 81.89%. By splitting up the market shares, we observe that the low-power Bank 1 is biased towards attracting defaulters (12.08% market share). In contrast, equipped with the best-performing model, Bank 3 closes more loan deals and accepts less defaulting loans (10.64% market share). These differences in market shares imply differences in profit, revenues, and losses. In particular, Bank 3’s profit will be more than 20% higher than Bank 1’s profit (123.5 versus 100.9). Thus, a high-power model leads to a more profitable credit-portfolio and a smaller recovery portfolio. This result underscores the adverse selection problem for banks with weaker models in a competitive market.

C. Pricing regime

Unlike cutoff, the pricing regime allows variable credit spreads. All banks derive risk premia for loans according to equation (9), with $k = 0.0030$, and with no relationship benefit, $C = 0$. This means, all three banks price each loan with conditional expected profit equal to 30 basis points, corresponding to their credit scoring models. “Conditional” refers to the information set which consists of the credit score. Therefore, using only the banks’ credit scores, information about the baseline default rate and knowledge of the cash-flows of lending, all banks set prices for loans.

To illustrate the pricing and selection mechanism, we refer to the two sample vectors in (13). According to the pricing rule in equation (9), the two simulations result in exact premia of 109, 60, 64 basis points (first draw, non-defaulter) as well as 306, 161, 392 basis points (second draw, defaulter). In both simulation draws the loan is granted by Bank 2. In the first
draw, Bank 2 makes a revenue of 60 basis points. However, in the second draw, Bank 2 has to write off a loss of 40%.

Figure 8 shows both probability of acceptance (Panel A) and expected premium offered by the corresponding bank (Panel B). Around two thirds of all the borrowers can expect lower risk premia at Bank 3 with the high-power model. The remaining third of the population consists of “bad obligors.” They are attracted by the lower premium of Bank 1 and Bank 2 that are operating with the medium-power and low-power model, respectively.

[Figure 8 about here]

Figure 9 plots the expected market share (Panel A) and expected revenue versus creditworthiness (Panel B). The better the credit quality of the obligor, the more likely the obligor is closing the deal with Bank 3 using the high-power model. In contrast to the cutoff regime, the pricing regime implies an expected market share that is no longer a decreasing function of creditworthiness. These differences arise, since in the pricing regime supply and demand for loans are cleared through the risk premium and the market clears.

Given a defaulting borrower, it is more probable that Bank 1, with a low-power model, is granting the credit. The probability that a defaulter (non-defaulter) closes the deal with the high-power Bank is 28.7% (39.6%). The corresponding probabilities for the low-power Bank are 38.2% and 27.5%, respectively.

[Figure 9]

The above analysis shows that the bank pricing with the most powerful model, due to a better estimate of the probability of default, can price the loans more attractively for those borrowers that are less likely to default, and more expensively for those that are more likely to default. On average, the poorer credits end up going to banks with weaker models. Borrowers with low default probability will end up doing business with banks that apply more powerful scoring models. The bank using the weaker model is in effect creating adverse selection for itself. These differences in pricing and market shares will necessarily lead to different profits.
Table 3, Panel B, shows expected market shares, profit, revenue, and loss in the pricing regime. In contrast to the cutoff regime, all loans demanded will be supplied. Therefore, the total market share amounts to 100%. Furthermore, compared to the cutoff regime, the differences between market shares are much larger. Looking at the figures for Bank 1 and Bank 3, we see that the market shares differ almost by 12 percentage points (27.71% versus 39.37%) for the whole population, and by more than 10 percentage points for the defaulters (38.21% versus 28.70%). These large differences also have a significant effect on profit. In particular, the profit for Bank 1 will be negative (-74.9), since Bank 1 accepts too many of the bad obligors. Even Bank 2 will incur negative profits (-19.0), whereas Bank 3 manages to stay profitable (36.0) due to its high-power scoring model and accurate pricing.

D. Mixture of pricing and cutoff regime

In the previous two subsections, we discussed the cutoff and pricing regimes. Credit officers and customer relationship managers alike would probably discard both regimes. Consequently, we propose a mixture model that is closer to an actual credit banking environment.

Consider again the sample vectors in (13). These draws result in exact premia of 109, 60, 64 basis points (first draw) as well as 306, 161, 392 basis points (second draw). Rounding these figures results in risk premia of 1%, 0.5%, 0.75% and 3%, 1.50%, 4%, respectively. Bank 2 gets both loans – the former (non-defaulter) for a revenue of 0.5%, the latter (defaulter) for a loss of 40%. Bank 1 and Bank 3 reject the defaulting obligor, since the risk premium would exceed the upper bound of 2.5%.

Figure 10 shows both the probability of acceptance (Panel A) and the expected risk premium (Panel B). Unlike pricing and cutoff, both measures depend on the nonobservable creditworthiness. Bank 3 (high-power) is inclined to accept more creditworthy borrowers at lower risk premia than Bank 2 (medium-power) and Bank 1 (low-power). Right from the vertical line, where we find defaulting borrowers, the story goes the other way round. These findings on a microstructure can be confirmed on a market level, as illustrated in Figure 11.
Table 3, Panel C, summarizes the findings in Figure 10 and 11. Since the mixture model is a compromise between the cutoff regime and the pricing regime, we expect the market figures to lie in between the numbers displayed in the previous panels. Indeed, the profit difference among banks amounts to up to USD 69.1 millions. The mixture regime is more competitive than cutoff. In the cutoff regime, the profit difference reaches only USD 22.6 millions. However, the mixture regime is less aggressive than the pricing regime, in which we observe a maximum profit gap of USD 110.6 millions. Loosely speaking, profit differences in a cutoff and pricing approach determine lower and upper bounds for profit gaps in an actual banking environment.

E. Impact of Model Improvement and Additional Information

The previous section restricts the analysis to a cross-sectional perspective. Here, we are concerned with the situation in which one of the competing banks changes its rating system from one period to the next. Within the mixture regime, we study two different settings. In the first setting, one competitor improves discriminatory power. In the second setting, the rating methodology of one lender becomes public, either on purpose or because information has leaked. Then, competitors will use the additional information in order to increase their forecasting accuracy.

E.1. Improving the Credit Scoring Model

In the first setting, Bank 1’s summary statistic AUROC increases, from 0.8134 to 0.8300, e.g., by means of switching from an expert-based to a statistical scoring procedure. This improvement will induce a steeper pricing curve. Compared to the previous setting, true non-defaulters have to pay less and actual defaulters more. For instance, for a median borrower with a creditworthiness of 0.5, the expected offered risk spread shifts from 83 basis points to 79 basis points. Similarly, Bank 1 would have accepted less investment grade loans and more loans close to default than its competitors. After the switch to the more accurate model, Bank 1 performs better but still lags behind Bank 3.
Bank 1’s rating switch to an improved scoring model affects competitors as well, since market shares change. In particular, the market shares will change in favor of Bank 1 and at the expense of both Bank 2 and Bank 3. The median borrower now features a probability of 0.372 of heading towards Bank 1, up from 0.286. Given the median borrower, Bank 1 (Bank 2, Bank 3) will now make higher (lower) expected revenue of 20.2 (19.3, 21.2) basis points, compared to 18.8 (20.6, 22.6) basis points before. Table 4 summarizes the impact of the improved credit scoring model of Bank 1 on the different profitability and market figures.

[ Table 4 about here ]

### E.2. Additional Information

We now assume that Bank 1 is applying its improved scoring model, but somehow the rating knowledge has leaked. Bank 1 can only resort to its own scoring model, but Bank 2 and Bank 3 combine their own model with Bank 1’s model for their default estimates. Clearly, the combined models achieve higher accuracies than their stand-alone counterparts.

Table 5 collects the market statistics when Bank 2 and Bank 3 are able to use Bank 1’s rating model in order to improve their default forecasts. Not surprisingly, Bank 1’s profit drops. Its profit difference amounts to USD 11.3 million, compared to the situation when the rating methodology is not public. Hence, the information leak offsets a large part of a profit increase caused by an improved scoring methodology. On the other side, both Bank 2 and Bank 3 can improve their profits significantly by around USD 60 million, just by knowing the rating methodology of one competitor.

[ Table 5 about here ]

### IV. Conclusion

In this paper, we provide rules how the credit score can be used for loan pricing by linking the ordinal ROC curve with the pricing curve. We show that the slope of the ROC curve enters
the pricing rule. It turns out that the credit spread increases with the steepness of the ROC curve. When the loan market does not allow risk adjusted pricing, we derive the profit-optimal cutoff level values from the ROC statistics.

The profitability of a rating model depends also on other competitors’ discriminatory power as well. In a stylized loan market, we study the economic impact of the discriminatory power of the scoring models. Weak models will attract more “bad” borrowers and therefore, the competitor with a low discriminatory power will incur lower revenues and bigger losses. In case of a cutoff regime, banks judging obligors based on a poor unsound credit rating system grant more loans to subsequent defaulters and refuse more non-defaulting borrowers. We show, in quantitative terms, that these banks become unwittingly market leader in the segment of distressed loans, resulting in a sizable recovery portfolio. In case of pricing and mixture regime, a bank with a low-power model attracts bad customers through (too) high prices for credit-worthy and (too) low risk premia for not-credit-worthy borrowers. An increase in the discriminatory power of one bank affects the profits as well as the market shares of all other lenders.

Common to all regimes is the fact that the better the scoring model, the lower is the risk of adverse selection and the higher the added value to the bank. A lender can significantly increase its loan portfolio by improving its rating system, with the positive side effect that the recovery portfolio decreases. The increase takes place even in a saturated loan market. Therefore, in an emerging and growing loan market, the rise in profit and market share will be even greater.

Credit scoring is regarded as a core competence of commercial banking. We end our discussion of the loan market model by giving a quantitative example that highlights the economic disadvantage when this core competence is lost. If competitors can exploit another bank’s rating knowledge, they can improve their own profits at the expense of the bank whose knowledge has leaked. Therefore, banks should pay attention to whom they communicate their scoring methodologies.
Appendix

We construct a credit market with \( n \) banks as supplier of loans and obligors whose unobservable creditworthiness is Gaussian distributed. If an obligor cannot pay interest or repay the loan at maturity, the loan will be in default. Each bank makes default predictions based on credit scores and deduces probabilities of default. The market has the structure

\[
\begin{pmatrix}
S \\
Y^*
\end{pmatrix} 
\sim \mathcal{N}
\left(
\begin{pmatrix}
0 \\
0
\end{pmatrix}
, 
\begin{pmatrix}
\Omega & \rho \\
\rho^\top & 1
\end{pmatrix}
\right)
\]

where \( S \) represents the \( n \times 1 \) column vector of credit scores \( S_i, i = 1, \ldots, n \), of competitor \( i \), \( 0 \) is the \( n \times 1 \) column vector of zeros, and \( Y^* \) is the non-observable creditworthiness. The \( n \times n \) matrix \( \Omega = [\rho_{ij}], i, j = 1, \ldots, n \), captures the correlation structure of the credit scores \( S \) and \( \rho = [\rho_i], i = 1, \ldots, n \), represents the \( n \times 1 \) column vector of correlations between \( Y \) and \( S \). We define the default indicator \( Y \) as

\[
Y = \begin{cases}
0 & : Y^* \leq c, \\
1 & : Y^* > c,
\end{cases}
\]

where \( c \) is a threshold value, calibrated form the unconditional probability of default \( p := \mathbb{P}\{Y = 1\} \) and \( q := \mathbb{P}\{Y = 0\} \), respectively.

A. Conditional distributions

We can write the conditional distribution \( S|Y^* \) as \( S|Y^* \sim \mathcal{N}(\rho Y^*, \Omega - \rho \rho^\top) \). Furthermore, we have

\[
Y^*|S \sim \mathcal{N}(\rho^\top \Omega^{-1}S, 1 - \rho^\top \Omega^{-1}\rho).
\]

Then,

\[
\mathbb{P}\{Y = 1|S_i\} = \int_c^\infty f_{Y^*|S_i}(y^*)dy^* = \Phi \left(-\frac{c - \rho_i S_i}{\sqrt{1 - \rho_i^2}}\right), \quad \text{(A.1)}
\]

\[
\mathbb{P}\{Y = 1|S_i, S_j\} = \int_c^\infty f_{(Y^*|S_i, S_j)}(y^*)dy^* = \Phi \left(-\frac{c - \rho_i S_i - \frac{\rho_i^2 - \rho_i \rho_j}{1 - \rho_i^2}(S_j - \rho_{ij} S_i)}{\sqrt{1 - \rho_i^2 - \frac{(\rho_i^2 - \rho_i \rho_j)^2}{1 - \rho_{ij}^2}}}\right), \quad \text{(A.2)}
\]

\(^7\text{See, e.g., Hamilton (1994), pages 100 – 102.}\)
where \( f_{Y \mid S_i} \) and \( f_{Y \mid S_i, S_j} \) denote the density functions of the corresponding random variables, and \( \Phi \) is the standard normal distribution function. By taking the first derivative

\[
\mathbb{P} \{ S \leq s \mid Y \} = (1 - Y) \frac{\mathbb{P} \{ S, Y^* \leq c \}}{\mathbb{P} \{ Y^* \leq c \}} + Y \frac{\mathbb{P} \{ S, Y^* > c \}}{\mathbb{P} \{ Y^* > c \}},
\]

we obtain

\[
f_{S \mid Y}(s) = \frac{1 - Y}{q} f_S(s) \int_{-\infty}^{c} f_{Y \mid S}(y) \, dy^* + \frac{Y}{p} f_S(s) \int_{c}^{\infty} f_{Y \mid S}(y) \, dy^*.
\]

By the same reasoning, we can derive the density functions for the conditional univariate random variables \( S_i \mid Y^* \) and \( S_i \mid Y \) as

\[
f_{S_i \mid Y^*}(s) = \frac{1 - Y}{q} \phi(s) \Phi \left( \frac{s - \rho_i Y^*}{\sqrt{1 - \rho_i^2}} \right),
\]

and

\[
f_{S_i \mid Y}(s) = \frac{1 - Y}{q} \phi(s) \Phi \left( \frac{c - \rho_i s}{\sqrt{1 - \rho_i^2}} \right) + \frac{Y}{p} \phi(s) \Phi \left( \frac{c - \rho_i s}{\sqrt{1 - \rho_i^2}} \right),
\]

where \( \phi \) denotes the standard Gaussian density function.

**B. ROC curve**

On the basis of the density function \( f_{S_i \mid Y} \), we calculate \( \alpha \)- and \( \beta \)-error, dependent on the threshold value \( t \), namely

\[
\alpha_i(t) = \mathbb{P} \{ S_i \leq t \mid Y = 1 \}
= \frac{1}{p} \int_{-\infty}^{t} \mathbb{P} \{ Y = 1 \mid S_i = s \} \phi(s) \, ds \tag{A.3}
= \int_{-\infty}^{t} f_{S_i \mid Y = 1}(s) \, ds
= \frac{1}{p} \int_{-\infty}^{t} \phi(s) \Phi \left( \frac{c - \rho_i s}{\sqrt{1 - \rho_i^2}} \right) \, ds \tag{A.4}
\]
The first line reflects the definition of the $\alpha$-error, under the null hypothesis that obligors with a credit score higher than $t$ are defaulters. With the second type of error, we can proceed the same way

$$\beta_i(t) = \mathbb{P}\{S_i > t|Y = 0\}$$
$$= \frac{1}{q} \int_{-\infty}^{\infty} \mathbb{P}\{Y = 0|S_i = s\} \phi(s) ds$$
$$= \int_{-\infty}^{\infty} \bar{f}(S_i|Y = 0)(s) ds$$
$$= \frac{1}{q} \int_{-\infty}^{\infty} \phi(s) \Phi \left( \frac{c - \rho t s}{\sqrt{1 - \rho^2}} \right) ds$$

(A.5)

(A.6)

For a given threshold level $t$, $(\beta(t), 1 - \alpha(t))$ constitutes one coordinate of the ROC curve. The curve is drawn by running $t$ from $-\infty$ to $+\infty$. AUROC denotes the area below the ROC curve and is computed by

$$\text{AUROC}_i = - \int_{-\infty}^{\infty} (1 - \alpha_i(t)) d\beta_i(t)$$
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1 \{x > t\} \bar{f}(S_i|Y = 0)(x) \bar{f}(S_i|Y = 1)(t) dx dt$$
$$= \frac{1}{pq} \int_{-\infty}^{\infty} \phi(t) \Phi \left( \frac{c - \rho t}{\sqrt{1 - \rho^2}} \right) \int_{-\infty}^{\infty} \phi(x) \Phi \left( -\frac{c - \rho x}{\sqrt{1 - \rho^2}} \right) dx dt$$

(A.7)

The first two lines are true for all continuous credit scores, and the last line follows by the normal assumption. From (A.7) AUROC can be nicely interpreted as the probability that the continuous credit score of a non-defaulting obligor is lower than the one of a defaulting obligor, given the two scores were drawn independently. The slope of the ROC curve, $s(t) := - \frac{d\alpha(t)}{d\beta(t)}$, can be represented as

$$s_i(t) = - \frac{d\alpha_i(t)}{d\beta_i(t)} = \frac{q}{p} \Phi \left( -\frac{c - \rho t}{\sqrt{1 - \rho^2}} \right)$$

(A.8)

where the second equality follows from (A.4) and (A.6).
Figure 1. Construction of a ROC curve. The $\alpha$- and $\beta$-errors refer to the null hypothesis that the higher the credit score, the higher the default risk. In the panels labeled Cutoff 1, 2, and 3, the dark area under the population of defaulters represents the $\alpha$-error and the shaded area represents the $\beta$-error. The ROC curve summarizes the relation between $\alpha$- and $\beta$-errors for every possible cutoff level.
Figure 2. Comparison of ROC curves. The figure shows three different credit scoring models, Model I and II and a random model, and plots their corresponding α- and β-errors. The dark area under the population of defaulters represents the α-error and the shaded area represents the β-error. The models result in different ROC curves as displayed in the lower panel on the right. For the random model, the ROC curve is just the diagonal. The point (0,1) represents the perfect model in the ROC graph.
Figure 3. Trade-off between $\alpha$- and $\beta$-error under different cost (LGD) and revenue ($R$) assumptions. The straight lines are iso-profit lines defining the optimal cutoff at the tangential point with the ROC curve. At the optimal cutoff, the marginal profit is zero. The figure analyzes four different cash-flow scenarios by varying the relative magnitude of conditional expected losses and revenues.
Figure 4. The left panel plots the ROC curves of two models with the same discriminatory power. We also plot the random model. The left panel shows the pricing regimes for the different models. The random model results in a flat line and, therefore, does not discriminate. Although, the two other models have the same area below ROC curve, they impose different pricing schemes due to the differences in the slope of their ROC curve.
Figure 5. ROC curves of the three banks with model-specific optimal cutoffs and corresponding error rates, for given cash-flow assumptions. The solid curve represents the ROC curve of Bank 1, the dashed curve of Bank 2, and the dash-dotted curve of Bank 3. We also plot the iso-profit lines. The tangential points define the optimal cutoff levels. Bank 1 (Bank 2, Bank 3) accepts 67.8% (68.8%, 69.8%) of all obligors according to profit-optimal cutoffs of 0.4628, 0.4912 and 0.5199.
Figure 6. Cutoff regime. Panel A shows the probability of acceptance versus creditworthiness. Compared to Bank 1 (solid line) and Bank 2 (dotted line), the probability of acceptance of the high-power Bank 3 (dashed line) is higher for good and lower for bad borrowers. The three curves intersect at around 91%. Defaulting obligors, located on the right-hand side of the dashed vertical line, have the greatest chance of getting a loan from the low-power Bank 1. Panel B displays the risk premium represented by the horizontal line at $R = 0.0075$, which is equal among all banks.
Figure 7. Cutoff regime. Panel A plots expected market share versus creditworthiness. For Bank 3 (dashed line) the expected market share is larger for good borrowers and lower for bad borrowers, compared to the competitors Bank 1 (solid line) and Bank 2 (dotted line). Panel B plots expected revenue versus creditworthiness. In comparison to Bank 1 (solid line) and Bank 2 (dotted line), Bank 3 (dashed line) will have more revenues generated from lending to good than to bad borrowers.
Figure 8. Pricing regime. Panel A shows the probability of acceptance versus creditworthiness. In the pricing regime, this probability equals 1 for all three banks. Panel B displays the expected offered premium versus creditworthiness. Bank 3 (dashed line) will, compared to its competitors, offer a lower premium to the borrowers at all levels of creditworthiness. Bank 1 (solid line) will offer the highest premium. Bank 2 (dotted line) will offer a risk premium between those of Bank 1 and Bank 3.
Figure 9. Pricing regime. Panel A plots expected market share versus creditworthiness. For Bank 3 (dashed line) the expected market share is larger for good borrowers (with creditworthiness lower than 91%) and lower for bad borrowers, compared to the competitors Bank 1 (solid line) and Bank 2 (dotted line). Panel B plots expected revenue versus creditworthiness. In comparison, Bank 3 (dashed line) will have more revenues generated from lending to good borrowers than to bad borrowers. Bank 1 (solid line) will have more revenues generated from lending to bad borrowers beyond the 79% creditworthiness.
Figure 10. Mixture regime. Panel A shows the probability of acceptance versus creditworthiness. Compared to Bank 1 (solid line) and Bank 2 (dotted line), the probability of acceptance of the high-power Bank 3 (dashed line) is higher for good and lower for bad borrowers. Defaulting obligors, located on the right-hand side of the dashed vertical line, have the greatest chance of getting a loan from the low-power Bank 1. Panel B shows the expected risk premium. The risk premium offered by Bank 3 (dashed line) is lowest for all borrowers.
Figure 11. Mixture regime. Panel A plots expected market share versus creditworthiness. For Bank 3 (dashed line) the expected market share is larger for good borrowers and lower for bad borrowers, compared to the competitors Bank 1 (solid line) and Bank 2 (dotted line). Panel B plots expected revenue versus creditworthiness. In comparison, Bank 3 (dashed line) will have more revenues generated from lending to good borrowers than to bad borrowers. Bank 1 (solid line) will have more revenues generated from lending to bad borrowers beyond the 83% creditworthiness.
Tables

<table>
<thead>
<tr>
<th>Model</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Credit Quality</td>
</tr>
<tr>
<td>Low Credit Quality</td>
<td>Correct assessment</td>
</tr>
<tr>
<td>High Credit Quality</td>
<td>α-error</td>
</tr>
</tbody>
</table>

Table 1

α- and β-errors.

The table summarizes the different errors of the scoring model. An α-error occurs in the situation in which the model assigns a low risk, when the true risk is high. A β-error occurs in the situation in which the model indicates a high risk, when the true risk is low.

<table>
<thead>
<tr>
<th>Cash flow</th>
<th>Default</th>
<th>Non-Default</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loss given default, recovery cost</td>
<td>Interest and fees + relationship benefit</td>
</tr>
<tr>
<td></td>
<td>LGD</td>
<td>R(t) + C</td>
</tr>
<tr>
<td>Probability</td>
<td>P{Y = 1</td>
<td>S = t}</td>
</tr>
</tbody>
</table>

Table 2

Conditional cash-flows and probabilities. The table shows the different cash-flows involved depending on whether an obligor defaults or does not default, given the score S = t.
### Panel A: cutoff regime

<table>
<thead>
<tr>
<th>Bank</th>
<th>AUROC</th>
<th>Market share</th>
<th>Share non-defaulters</th>
<th>Share defaulters</th>
<th>Profit</th>
<th>Revenue</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>0.8134</td>
<td>0.2658</td>
<td>0.2687</td>
<td>0.1208</td>
<td>100.9</td>
<td>197.5</td>
<td>96.6</td>
</tr>
<tr>
<td>Bank 2</td>
<td>0.8245</td>
<td>0.2729</td>
<td>0.2762</td>
<td>0.1135</td>
<td>112.1</td>
<td>203.0</td>
<td>90.8</td>
</tr>
<tr>
<td>Bank 3</td>
<td>0.8354</td>
<td>0.2803</td>
<td>0.2838</td>
<td>0.1064</td>
<td>123.5</td>
<td>208.6</td>
<td>85.1</td>
</tr>
<tr>
<td>Total</td>
<td>0.8189</td>
<td>0.2729</td>
<td>0.2803</td>
<td>0.1064</td>
<td>336.5</td>
<td>609.1</td>
<td>272.6</td>
</tr>
</tbody>
</table>

### Panel B: pricing regime

<table>
<thead>
<tr>
<th>Bank</th>
<th>AUROC</th>
<th>Market share</th>
<th>Share non-defaulters</th>
<th>Share defaulters</th>
<th>Profit</th>
<th>Revenue</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>0.8134</td>
<td>0.2771</td>
<td>0.2749</td>
<td>0.3821</td>
<td>-74.6</td>
<td>231.0</td>
<td>305.6</td>
</tr>
<tr>
<td>Bank 2</td>
<td>0.8245</td>
<td>0.3292</td>
<td>0.3291</td>
<td>0.3308</td>
<td>-19.0</td>
<td>245.5</td>
<td>264.6</td>
</tr>
<tr>
<td>Bank 3</td>
<td>0.8354</td>
<td>0.3937</td>
<td>0.3959</td>
<td>0.2870</td>
<td>36.0</td>
<td>265.5</td>
<td>229.5</td>
</tr>
<tr>
<td>Total</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>-57.7</td>
<td>742.0</td>
<td>799.7</td>
</tr>
</tbody>
</table>

### Panel C: mixture regime

<table>
<thead>
<tr>
<th>Bank</th>
<th>AUROC</th>
<th>Market share</th>
<th>Share non-defaulters</th>
<th>Share defaulters</th>
<th>Profit</th>
<th>Revenue</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>0.8134</td>
<td>0.2846</td>
<td>0.2848</td>
<td>0.2709</td>
<td>-27.3</td>
<td>189.3</td>
<td>216.6</td>
</tr>
<tr>
<td>Bank 2</td>
<td>0.8245</td>
<td>0.3200</td>
<td>0.3216</td>
<td>0.2426</td>
<td>7.6</td>
<td>201.6</td>
<td>194.0</td>
</tr>
<tr>
<td>Bank 3</td>
<td>0.8354</td>
<td>0.3597</td>
<td>0.3626</td>
<td>0.2171</td>
<td>41.8</td>
<td>215.4</td>
<td>173.6</td>
</tr>
<tr>
<td>Total</td>
<td>0.9643</td>
<td>0.9691</td>
<td>0.9691</td>
<td>0.7306</td>
<td>22.1</td>
<td>606.4</td>
<td>584.3</td>
</tr>
</tbody>
</table>

**Table 3**

Different market figures (in million USD) in the cutoff regime (Panel A), in the pricing regime (Panel B), and in the mixture regime (Panel C). As market figures, we list for each regime the AUROC of the banks’ scoring models, their market shares, profit, revenues and losses.
### Table 4

The table lists the changes in market shares, profit, revenue and loss (in million USD) when Bank 1 improves AUROC from 0.8134 to 0.8300. In brackets, we calculate the difference to the benchmark case, i.e., with Bank 1’s AUROC of 0.8134.

<table>
<thead>
<tr>
<th>market figure</th>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUROC of model</td>
<td>0.8300</td>
<td>0.8245</td>
<td>0.8354</td>
<td>—</td>
</tr>
<tr>
<td>market share</td>
<td>0.3208</td>
<td>0.3026</td>
<td>0.3401</td>
<td>0.9635</td>
</tr>
<tr>
<td></td>
<td>(+0.0362)</td>
<td>(-0.0174)</td>
<td>(-0.0196)</td>
<td>(-0.0008)</td>
</tr>
<tr>
<td>share non-defaulters</td>
<td>0.3225</td>
<td>0.3036</td>
<td>0.3424</td>
<td>0.9685</td>
</tr>
<tr>
<td></td>
<td>(+0.0376)</td>
<td>(-0.0180)</td>
<td>(-0.0202)</td>
<td>(-0.0005)</td>
</tr>
<tr>
<td>share defaulters</td>
<td>0.2393</td>
<td>0.2526</td>
<td>0.2258</td>
<td>0.7177</td>
</tr>
<tr>
<td></td>
<td>(-0.0316)</td>
<td>(+0.0106)</td>
<td>(+0.0087)</td>
<td>(-0.0128)</td>
</tr>
<tr>
<td>profit</td>
<td>7.0</td>
<td>-10.0</td>
<td>24.4</td>
<td>21.4</td>
</tr>
<tr>
<td></td>
<td>(+34.3)</td>
<td>(-17.6)</td>
<td>(-17.4)</td>
<td>(-0.7)</td>
</tr>
<tr>
<td>revenue</td>
<td>198.4</td>
<td>192.1</td>
<td>205.1</td>
<td>595.6</td>
</tr>
<tr>
<td></td>
<td>(+9.1)</td>
<td>(-9.5)</td>
<td>(-10.3)</td>
<td>(-10.7)</td>
</tr>
<tr>
<td>loss</td>
<td>191.4</td>
<td>202.1</td>
<td>180.7</td>
<td>574.2</td>
</tr>
<tr>
<td></td>
<td>(-25.2)</td>
<td>(+8.1)</td>
<td>(+7.1)</td>
<td>(-10.0)</td>
</tr>
</tbody>
</table>

### Table 5

The table lists the changes in market shares, profit, revenue and loss (in million USD) when Bank 2 and Bank 3 know rating methodology of Bank 1. In brackets, we calculate the difference to benchmark case of proprietary rating methodologies, i.e., when no bank has information on any other bank’s rating methodology.

<table>
<thead>
<tr>
<th>market figure</th>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUROC of model</td>
<td>0.8300</td>
<td>0.8245</td>
<td>0.8354</td>
<td>—</td>
</tr>
<tr>
<td>market share</td>
<td>0.2663</td>
<td>0.3115</td>
<td>0.3593</td>
<td>0.9372</td>
</tr>
<tr>
<td></td>
<td>(-0.0545)</td>
<td>(+0.0089)</td>
<td>(+0.0192)</td>
<td>(-0.0264)</td>
</tr>
<tr>
<td>share non-defaulters</td>
<td>0.2669</td>
<td>0.3141</td>
<td>0.3628</td>
<td>0.9439</td>
</tr>
<tr>
<td></td>
<td>(-0.0556)</td>
<td>(+0.0105)</td>
<td>(+0.0204)</td>
<td>(-0.0247)</td>
</tr>
<tr>
<td>share defaulters</td>
<td>0.2378</td>
<td>0.1830</td>
<td>0.1889</td>
<td>0.6097</td>
</tr>
<tr>
<td></td>
<td>(-0.0015)</td>
<td>(-0.0696)</td>
<td>(-0.0369)</td>
<td>(-0.1081)</td>
</tr>
<tr>
<td>profit</td>
<td>-4.3</td>
<td>57.1</td>
<td>83.3</td>
<td>136.0</td>
</tr>
<tr>
<td></td>
<td>(-11.3)</td>
<td>(+67.0)</td>
<td>(+58.9)</td>
<td>(+114.6)</td>
</tr>
<tr>
<td>revenue</td>
<td>186.1</td>
<td>203.7</td>
<td>234.7</td>
<td>624.5</td>
</tr>
<tr>
<td></td>
<td>(-12.3)</td>
<td>(+11.5)</td>
<td>(+29.6)</td>
<td>(+28.8)</td>
</tr>
<tr>
<td>loss</td>
<td>190.5</td>
<td>146.6</td>
<td>151.4</td>
<td>488.5</td>
</tr>
<tr>
<td></td>
<td>(-0.9)</td>
<td>(-55.5)</td>
<td>(-29.3)</td>
<td>(-85.8)</td>
</tr>
</tbody>
</table>
References


Kealhofer, S., and J. Bohn, 2001, “Portfolio Management of Credit Risk,” Discussion paper, Moody’s KMV.


