Depressing Recoveries

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The current U.S. expansion is the longest and strongest in economic history. The expansion has been a boon for U.S. banking institutions, which have enjoyed a long period of relatively low default rates. And perhaps the economic good times will roll on forever, in a permanent economic high. But an eventual reversal—an economic downturn—seems more likely. If the downturn is severe, the misfortune for banks may be twofold: a higher rate of default among their borrowers, and a lower rate of recovery on defaulted loans.

Defending against economic downturns is the principal reason that banks hold capital. Increasingly, banks manage their capital with guidance from the new portfolio credit models such as CreditManager or CreditRisk+. But when these models look to the next economic downturn, they allow for only one misfortune, the increase in the default rate. The other misfortune, a possible simultaneous decrease in average recovery, evades analysis. If banks depend upon such models, they might enter a severe downturn holding too little capital.

The degree of potential shortfall is easy to grasp. Suppose one of the first-generation credit models accurately projects that in a severe downturn a bank will experience a 10% default rate. Overall credit loss is approximately equal to the default rate times the flip-side of loan recovery, loss-given-default (LGD). But in the first-generation models, LGD does not depend on default. If long-term average LGD is 25%, these models will project that LGD will equal 25% on average in any year, and that capital of 2.5% will withstand the downturn.

However, the same economic conditions that cause default to rise to 10% might also cause LGD to rise above its long-term average. If in the downturn LGD rises to 50%, the need for capital to withstand the downturn would equal 5.0% rather than 2.5%. If credit models overlook the possible doubling of LGD in a severe downturn, they understatement capital by half.

This study examines data on U.S. corporate bonds and finds significant synchrony between default and recovery. It then fits the model presented in Collateral Damage (Risk, April 2000) to this data. Extrapolating the model to the conditions that produce a 10% default rate, recovery falls by 25% in absolute terms from its normal-year average. If that decline pertains to recoveries on bank loans as well on bonds, the LGD at banks could, in fact, double from the normal-year average.

**The Data**

Because the focus of this study is on banks and the credit risk at banks, the ideal data set would be a long history of bank loans, loan defaults, and eventual loan recovery amounts. Though efforts are underway to collect such data, this data set does not exist at present. Fortunately, rating agencies collect equivalent data for rated bonds.
This study uses the Moody’s Default Risk Service database, which contains extensive information about debt issues rated by Moody’s since 1970. It includes information about bank loans, corporate bonds, and sovereign bonds; it includes information about entities domiciled in many different countries; and it includes information about bonds that are guaranteed by second entities as well as about unbacked bonds.

Some selection criteria help make sense of the data. The first selection criterion limits the sample to entities domiciled in the United States. This restriction is desirable because economic conditions in different countries are not perfectly synchronized and it would require a multifactor credit model to capture the difference in timing between countries. The credit model employed here, by contrast, is driven by a single factor.

Moody’s supplies a “Broad Industry Group” and a “Specific Industry Group” for each rated issuer. The second selection criterion eliminates certain broad groups (Banking, Finance, Insurance, Other non-Bank, Real Estate Finance, Securities, Sovereign, Structured Finance, and Thrifts), and, within what remains, it eliminates certain narrow groups such as Finance Conduit. What are left are the non-financial issuers within the broad groups of industrial, public utility, and transportation. Like the first selection criterion, the second criterion is meant to produce issuers that respond to a similar risk factor, while keeping the number of issuers as large as possible. The result of this selection is the default universe of this study.

Figure 1 shows the annual default rate in the default universe. The default rate calculation is the same as that used by Moody’s and equals the ratio of two numbers. The numerator is the number of defaults that occur among issuers that had a Moody’s rating on the first of the year. The denominator is the number of issuers that had a Moody’s rating at the
beginning of the year, less half the number that had their rating withdrawn in the course of the year.¹

The recovery universe is further limited by three additional selection criteria. First, the database contains rated loans as well as rated bonds. To keep this study narrowly focused, the first recovery criterion eliminates loans. Second, some bonds are guaranteed by a second entity. If those bonds default, recovery is markedly greater than on unbacked bonds. Therefore the second recovery criterion eliminates bonds backed by a second entity. Third, there must be a recovery amount available; the third recovery criterion eliminates defaulted bonds for which Moody’s does not have a “default price.”

Moody’s observes the default price one month after the first occurrence of a default event. (Usually this is a missed payment.) The default price therefore represents the market valuation of the probability distribution of possible future recovery. This market valuation differs from the eventual recovery itself. Nonetheless, most bond recovery studies rely on market valuations, and the present study assumes that the market valuations are accurate on average in a given year.

Figure 2 displays for each year the number of defaulting issues and the number of issues that have a default price. The years before 1982 have very few default prices with which to construct an annual average. In 1970, there are 15 default prices but these cover less than one-third of recoveries that year. As a consequence, the years 1970-1981 are eliminated from the data sample. This removes from the recovery universe only 6% of default prices.²
Having defined the default universe and the recovery universe, and having delimited the sample period, Figure 3 gets to the heart of things. It contrasts the distribution of recovery in the high-default years of 1990 and 1991 to the distribution of recovery in all other years. The low-recovery bars at the left of the diagram are dominated by defaults that occur in the high-default years, and the high-recovery bars at the right of the diagram tend to reflect defaults in other years. This suggests that the distribution of recovery is different in a high-default year from a low-default year. The suggestion is confirmed by a simple chi-square test: the difference between the histograms is highly significant. The credit model described next puts a shape on this difference, so that we can develop an idea of expected recovery in a severe economic downturn.

The Model

This study uses a model that is nearly identical to the model presented in Collateral Damage, Risk, April 2000. But while Collateral Damage implies recovery from an equation that determines collateral, this study models recovery directly. The reason is simply that the Moody’s database observes recovery, not collateral.

The model presented here, like the model of Collateral Damage, draws heavily from models suggested by Michael Gordy and Chris Finger. These models are driven by a single systematic risk factor rather than by a multitude of correlation parameters. This simplification of course lacks a great deal of detail, but it is appropriate for studying general influences such as an economic downturn. The present model departs from Gordy and Finger by allowing recovery, as well as default, to depend on the state of the systematic risk factor.
The systematic risk factor, $X$, is the central player. It affects the fortunes of every firm and the amount of every recovery. If $X$ takes a low value, there is a greater than average expected rate of default and a lower than average expected rate of recovery. These effects move the expected rates of default and recovery, but they affect individual events only as tendencies. Thus with high levels of $X$ there can be some defaults and some unfavorable recoveries.

From the standpoint of a given firm $j$ in the model, two factors affect it. First is the level of $X$, which has a simultaneous effect on every other firm. Second is the level of an independent factor $X_j$ that affects only firm $j$. These two—the systematic risk factor $X$ and the idiosyncratic risk factor $X_j$—combine to determine the level of firm $j$’s asset value index, $A_j$:

$$A_j = pX + \sqrt{1-p^2}X_j$$

(1)

The two risk factors, $X$ and $X_j$, are assumed to have independent standard normal distributions, which implies that $A_j$ has a standard normal distribution. One can imagine a mapping from the asset value index to the dollar value of the assets of firm $j$, but the “work” in the model is done by the index, $A_j$.

The parameter $p$ plays an important role in the asset equation (1). It controls how much the systematic factor affects the set of issuers. If an economy has a low value of $p$ (near zero), issuers have little connection to the state of the economy. In such an economy, each issuer finds its independent, idiosyncratic factor to be far more important than the common factor. Therefore the prosperity (or default) of one firm has little connection to the prosperity (or default) of other firms. On the other hand, if an economy has a high value of $p$, each issuer is strongly tied to the general economy. An economy with a high level of $p$ is a highly cyclical economy, and any year that $X$ takes a low value will be a bad year for many issuers. Thus, an economy having a large value of $p$ will have a severe credit cycle.

In the model, a firm defaults when its asset value index falls below a threshold. The level of the threshold is chosen to produce the long-term probability of default of the firm in question. Letting $D_j$ symbolize the default event of firm $j$,

$$D_j = 1 \text{ if } A_j < \Phi^{-1}(PD_j); D_j = 0 \text{ otherwise}$$

(2)

where $PD_j$ is the probability of default of firm $j$.

Like the models of Gordy and Finger, this model assumes that the portfolio is large and fully diversified. The law of large numbers then implies that, conditional on a level of $X$, the observed default frequency approximates its conditionally expected rate:
Turning to the recovery side, the recovery equation is similar to asset equation (1). Recovery in default $j$ depends on the systematic factor $X$ and also on an idiosyncratic factor, $Z_j$, which affects only the recovery in default $j$:

$$R_j = \mu_j + \sigma q X + \sigma \sqrt{1 - q^2} Z_j$$

(4)

$Z_j$ is assumed to have a standard normal distribution independent of $X$. Therefore $R_j$ has a normal distribution with mean $\mu$ and variance $\sigma^2$. The parameters $\mu$, $\sigma$, and $q$ may be interpreted as the quantity, quality, and sensitivity of recovery. The sensitivity parameter, $q$, controls the strength of the effect of the systematic factor on recovery. This role is parallel to the role of $p$ in the asset equation; note that $\text{Corr}(A_j, X) = p$ and $\text{Corr}(R_j, X) = q$.

**Fitting the model to data**

The need to fit equation (3) to data leads to the final criterion defining the data sample. Equation (3) requires a probability of default for every issuer. That probability is estimated by the long-term average one-year default rate of issuers holding the same Moody’s rating. However, there are several ratings for which long-term history is simply not available. In fact, Moody’s changed its ratings system twice over the period 1982-2000. In 1982, Moody’s abandoned the grades of A and B. It reclassified the substantial number of firms having those ratings into the new “alphanumeric” grades of A1, A2, A3, and so forth. Similarly, in 1997, Moody’s began to abandon the Caa grade and began to reclassify Caa-rated obligors into new classes Caa1, Caa2, and Caa3. Therefore the 1982-2000 sample period has very short histories of certain rating grades. The model is fit to the fifteen-year sample period 1983-1997, because that is the longest historical period for which the Moody’s scale is uniform.

The estimation is by conditional maximum likelihood. An overview of the conditional approach is as follows. First, the default data alone is used to estimate $p$ by maximum likelihood. Given $p$, the portfolio generalization of equation (3) implies the level of $X$ each year. These implied levels of $X$ are combined with the recovery data to estimate the other parameters by a second maximization. This approach departs from the ideal because it uses only default data to estimate the levels of $X$, rather than making use of the
information about $X$ that might be contained in the recovery data. On the other hand, it is intuitively appealing that default data is used to estimate $p$ (which governs the severity of the default cycle), and recovery data is used to estimate the other parameters (which govern the quantity, quality, and sensitivity of recovery).

A more detailed discussion of the estimation approach requires restating equations (3) and (4) with detailed subscripts. The subscript $t$ counts the $T = 15$ years of the data sample. The subscript $r$ counts the $R = 19$ rating grades (Aaa, Aa1, Aa2, Aa3, A1, A2, A3, Baa1, Baa2, Baa3, Ba1, Ba2, Ba3, B1, B2, B3, Caa, Ca, C). Then in year $t$, the conditional default rate of a firm rated $r$ equals

$$DF_{t,r} = \Phi \left[ \Phi^{-1}(PD_r) - pX_t \right] \sqrt{1 - p^2}$$

(5)

where $PD_r$ equals the long-term average default rate of firms rated $r$.

Let $h_{t,r}$ represent the fraction of the default universe rated $r$ in year $t$. Then the conditional default rate of the default universe equals

$$DF_t = \sum_{r=1}^{R} h_{t,r}DF_{t,r} = g_p(X_t)$$

(6)

The function $g$ is monotonic, therefore it can be inverted numerically with respect to $X_t$. Given that $X_t$ has a standard normal distribution, the change-of-variable technique produces the density of $DF_t$:

$$\left| \frac{dg^{-1}(DF_t)}{dDF_t} \right| \exp \left[ -\frac{(g^{-1}(DF_t))^2}{2} \right] \frac{1}{\sqrt{2\pi}}$$

(7)

Taking the derivative and assuming independence from year to year, the joint density of the fifteen years of default rate data is then

$$\prod_{t=1}^{T} \sqrt{1 - p^2} \exp \left[ -\frac{(g^{-1}(DF_t))^2}{2} \right]$$

$$\left( p\sqrt{2\pi} \sum_{r=1}^{R} h_{t,r} \phi \left[ \frac{\Phi^{-1}(PD_r) - pg^{-1}(DF_t)}{\sqrt{1 - p^2}} \right] \right)$$
Expression (8) is a function of the default rate data \(\{DF_t\}\), the portfolio proportions \(\{h_{t,r}\}\), the long-term default rates \(\{PD_r\}\), and the unknown parameter \(p\). Maximizing (8) with respect to \(p\) provides the estimate \(\hat{p} = 0.23\). Given \(\hat{p}\), equation (5) says that the default rate in any year depends only on the level of \(X_t\). Therefore we can imply \(X_t\) for each year.

The estimated levels of \(X_t\) are put to use in the recovery equation. The subscript \(j\) now counts the \(J = 4\) seniority classes (senior secured, senior unsecured, senior subordinated, and subordinated). Moody’s assigns these classifications at the time of issuance. Though subjective and unchanging, these classifications provide some information about the amount of recovery in the event of default. Therefore, a distinct level of \(\mu\) is estimated for each seniority class.

Sometimes a defaulting firm will have several outstanding bonds classified, for example, as senior subordinated. The model has nothing to say about the difference in recovery among these bonds. Therefore, the symbol \(R_{t,j,i}\) denotes the dollar-weighted average recovery in year \(t\) of bonds of the \(j\)th seniority class issued by the \(i\)th defaulting issuer:

\[
R_{t,j,i} = \mu_j + \sigma q X_t + \sigma \sqrt{1 - q^2} Z_{t,j,i}
\]

(9)

The symbol \(N_{t,j}\) is the number of recoveries in year \(t\) and seniority class \(j\), and the symbol \(N_t\) is the total number of recoveries in year \(t\). Then, using the recovery equation (9), average recovery in year \(t\) equals

\[
R_t = \frac{\sum_{j=1}^{J} \sum_{i=1}^{I} R_{t,j,i}}{\sum_{j=1}^{J} N_{t,j}}
\]

(10)

\(R_t\) has a normal distribution and

\[
R_t = \frac{\sum_{j=1}^{J} N_{t,j} \mu_j}{N_t} + \frac{X_t \sum_{j=1}^{J} N_{t,j} \sigma q}{N_t} + Y_t
\]

(11)

where \(Y_t\) has a normal distribution with mean zero and
\[
\text{Var}[Y_t] = \frac{\sum_{j=1}^{J} N_{t,j} \sigma^2(1-q^2)}{(\sum_{j=1}^{J} N_{t,j})^2}
\]

(12)

This leads to the likelihood function of the recovery data

\[
f(R_t) = \exp\left[-\left(\frac{\sum_{j=1}^{J} N_{t,j} \mu_j - \sum_{j=1}^{J} N_{t,j} \sigma q^2}{\sqrt{2\pi \text{Var}[Y_t]}} X_t\right)^2\right]
\]

(13)

Maximizing \( \prod_t f(R_t) \) with respect to \( \{\mu_j\} \), \( \sigma \), and \( q \) results in the parameter estimates shown in table A.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>0.23</td>
</tr>
<tr>
<td>( q )</td>
<td>0.17</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.32</td>
</tr>
<tr>
<td>( \mu ) (Senior Secured)</td>
<td>0.47</td>
</tr>
<tr>
<td>( \mu ) (Senior Unsecured)</td>
<td>0.70</td>
</tr>
<tr>
<td>( \mu ) (Senior Subordinated)</td>
<td>0.12</td>
</tr>
<tr>
<td>( \mu ) (Subordinated)</td>
<td>0.41</td>
</tr>
</tbody>
</table>
Sizing up the fit

This section discusses the estimates of the parameter values, the agreement between the model and the data, and the implication of the model for bond recovery in an economic downturn.

The estimate of $\sigma$ implies great uncertainty in any particular recovery event. The difference between zero recovery and 96% recovery is only three standard deviations. Despite this uncertainty, recovery depends on systematic risk: the estimate of $q$ is nearly as great as the estimate of $p$.

The near-equality of $p$ and $q$ was foreseen in Collateral Damage as follows. The two parameters measure the sensitivity to $X$ of two kinds of asset values. These asset values are, of course, the asset values of firms and the amounts recovered on debt instruments. Taken at a very high level of abstraction, assets are assets and there is little reason to suspect a major difference in their sensitivities. Further, since recovery value derives from the asset value of a firm, there is reason to suspect that the two parameters are similar.

The near-equality of $p$ and $q$ supports the “expected loss equivalent” substitution rule discussed in Collateral Damage. This rule says that two debts having the same $EL$ (but having possibly a different division of $EL$ into $PD$ and expected $LGD$) will require the nearly the same amount of capital. Capital estimation within CreditMetrics does not observe the $EL$-substitution rule, and within an $EL$ grade it allocates less capital—too little capital, according to the present model—to debts having a low expected $LGD$. Therefore the present model suggests that CreditMetrics may estimate too little capital overall and that it may misallocate capital between debts.

The pattern of estimates of $\{m_j\}$ would be expected to decline from the strongest seniority class (senior secured) to the weakest (subordinated). Instead there are breaks in the pattern, for example, the relatively low estimate of 0.12 for the senior-subordinated class. The breaks arise because of the distribution of recoveries across years. Only 20% of senior-subordinated recoveries occur in 1990 or 1991, compared to the overall average of 35%. This means that the senior-subordinated class evades somewhat the depressing effect of the systematic risk factor, and the optimization routine instead attributes a low value to $\mu_{(Senior-Subordinated)}$. In fact, none of the seniority classes are distributed as expected across the years. A contributing reason to this is the low number of observations—only 405 cases in the recovery universe and 30 cases in the senior-subordinated class.
Figure 4 compares the data—the annual default rate in the default universe and the average recovery rate in the recovery universe—to the projections of the model. Some data points are specifically labeled. As discussed previously, years 1992, 1998 and 1999 were excluded from the estimation process to avoid making assumptions about the long-term default frequencies where only short-term histories are available. Nonetheless, these years appear to fit the overall pattern of the data sample. Years 1990 and 1991 contain 149 of the 405 recoveries in the recovery universe.

The line for the fitted model is constructed by substituting a range of values of $X$ into equation (6) and equation (10). These calculations assume the average distribution of the portfolio across rating grades and the average distribution of defaults across seniority classes. Because figure 4 compares historical data (which depends on the actual distribution across rating grades and across seniority classes) to a projection that uses the long-term average portfolio, the true fit of the model is better than the impression given by figure 4.

The most interesting part of figure 4 is on the right, which extrapolates the estimated model beyond historical experience. According to the model, if the systematic risk factor pushes the default rate to a level of 10%, average recovery might fall to about 20%. This contrasts to the normal-year average of 45% or so. Thus, in a severe economic downturn, recovery might fall about 25 percentage points from the levels of the mid-1990s.

**Loan recovery in a downturn**

Ideally one could fit the same model to data on loans. However, the first loan recovery in the Moody’s database occurs in late 1996, and there are recoveries from only fifteen defaults in total. Clearly, this data is not sufficient to fit the present model.
Instead, we attempt to adjust the model parameters to better reflect loans. The critical assumption is that the $p$ and $q$ relevant to loans are equal to the values estimated for bonds. It appears that the level of $\sigma$ may be lower for loans than for bonds. The standard deviation of the fourteen senior-secured loan recoveries is 26%, compared to a value of 31% for senior-secured bonds in the same historical period. Therefore we adopt two representative values for $\sigma$ : 0.32 as estimated above, and 0.25. It also appears that loan recovery exceeds bond recovery. The average of the fourteen senior-secured loan recoveries is 69.3%, compared to a value of 47.8% for senior-secured bonds in the same historical period. We adopt the representative value of 30.7% for the expected $LGD$ of loans.

### Table B: Estimated $LGD$ of Bonds and Loans in an Economic Downturn

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>(1) $p$</th>
<th>(2) $p$</th>
<th>(3) $p$</th>
<th>(4) $p$</th>
<th>(5) $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$PD$</td>
<td>1.99%</td>
<td>2.00%</td>
<td>0.20%</td>
<td>2.00%</td>
<td>0.20%</td>
</tr>
<tr>
<td>$ELGD$</td>
<td>59.1%</td>
<td>30.7%</td>
<td>30.7%</td>
<td>30.7%</td>
<td>30.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normal state, $X = 0$</th>
<th>Default</th>
<th>1.8%</th>
<th>1.7%</th>
<th>0.2%</th>
<th>1.7%</th>
<th>0.2%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LGD$</td>
<td>55%</td>
<td>28%</td>
<td>27%</td>
<td>28%</td>
<td>28%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Depressed state, $X = -4.5$</th>
<th>Default</th>
<th>10.4%</th>
<th>14.8%</th>
<th>2.9%</th>
<th>14.8%</th>
<th>2.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LGD$</td>
<td>80%</td>
<td>52%</td>
<td>51%</td>
<td>47%</td>
<td>47%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Increase in $LGD$</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45%</td>
</tr>
</tbody>
</table>

(1) Estimated parameters; Moody's average $PD$ and average $ELGD$
(2) Estimated parameters, low-quality loans
(3) Estimated parameters, high-quality loans
(4) Reduced value of $\sigma$, low-quality loans
(5) Reduced value of $\sigma$, high-quality loans

Table B shows the implications. The first column reflects the average Moody’s portfolio. In this column, the “normal” state and the “depressed” state identify two points along the locus of figure 4. $LGD$ is 80% in the depressed state, a 45% increase from the normal state. The other columns reflect parameters more appropriate for loans. Columns (2) and (3) employ $\sigma = 0.32$, which implies that in the depressed state $LGD$ nearly doubles. The difference between column (2) and column (3) shows that the level of $PD$ has very little influence on systematic recovery risk. Columns (4) and (5) use the estimate $\sigma = 25%$. This implies that in the depressed state $LGD$ increases about 70%.

The message of table B repeats the message that began this article. In a period of high default, it is intuitive that debt recovery would run low. This intuition is confirmed by data on U.S. corporate bonds. Using that data to estimate an appropriate credit model, we can extrapolate that in a severe economic downturn bond recoveries might decline 20-25 percentage points from the normal-year average. Loan recoveries may decline by a similar amount, but from a higher level. This could cause loss given default to increase.
by nearly 100% and to have a proportionate effect on economic capital. Such systematic recovery risk is absent from first-generation credit models. Therefore, these models may significantly understate the capital required at banking institutions.

Jon Frye is Senior Economist in the Policy Group at the Federal Reserve Bank of Chicago. He would like to thank Sarah Mangi for superb research assistance. The views expressed are the author’s and do not necessarily represent the views of the management of the Federal Reserve Bank of Chicago or the views of the Federal Reserve System.

References

Finger C, 1999
Conditional Approaches for CreditMetrics® Portfolio Distributions
CreditMetrics® Monitor, April

Frye J, 2000
Collateral Damage
Risk, April, pages 91-94

Gordy M, 2000
A Comparative Anatomy of Credit Risk Models
Journal of Banking & Finance, January, pages 119-149

Moody’s Investors Service, 2000
Default Risk Service Database

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1 Moody’s justifies this procedure as follows. The withdrawal of a rating is not generally associated with a loss for the investor—the bonds mature or they are called or defeased. After the withdrawal, a default of the issuer does not appear in the database. Therefore, from the perspective of the database, a firm with a withdrawn rating is subject to default for only the fraction of the year prior to withdrawal. On average this fraction is one-half, which accounts for the adjustment to the denominator.

2 In several years within the 1970-1981 period, the average default price was low despite favorable default rate experience. This anomaly might be explained by low liquidity in the defaulted-bond market in those years.

3 The Chi-square statistic of the count data equals 40.7 with eight degrees of freedom, significant at a level of 0.000003. This test combines the recoveries greater than 80 in order to keep expected cell frequencies greater than 5. Both this test and figure 3 employ the dollar-weighted averaging and the 1983-1997 sample period that are introduced later.

4 Thus the distribution of the assets that govern recovery ($R_j$) is similar to the distribution of the assets that govern default ($A_j$). An alternative specification of recovery might be bounded on $[0,1]$, e.g., recovery might have a beta distribution. However, Moody’s default prices are not bounded at 100%, as shown in figure 3. Further, annual average recovery tends to normality as the number of recoveries increases.

5 The variance of $Y$ has not been corrected for the fact that some defaulting issuers have outstanding bonds in more than one seniority class. The associated recoveries have idiosyncratic factors that probably fail to be independent. Three defaults affect three seniority classes and 43 defaults affect two seniority classes among the 405 events in the default universe.

6 This is according to the set of loan codes provided by Moody’s.