

## UBS Investment Research

## European Structured Credit

## CPDO insights on rating actions and manager impact using a new formula

**■ We present a new simple closed-form CPDO formulae**

In our third piece regarding the new CPDO asset class, we introduce a simplified model of CPDO which has the virtue of reducing itself to a closed form formula. First, we will introduce our model and its derived formulae. Then, after a brief description of the shortcomings of our model, we define 3 specific CPDO measures in the context of our model: Maximum drawdown, cash in time and cash out probability.

**■ This model sheds some light on CPDO rating action logic**

According to our simple model, CPDOs could be exposed early to rating downgrades if the market were to tank and implied volatility to rise, but similarly the CPDOs should be quickly upgraded (assuming his initial rating isn't AAA) if the market rallies early. In almost all scenarios a CPDO ends up upgraded up to AAA as soon as the early volatility / risk period has passed. We believe that this phenomenon is well understood and taken into account by rating agencies, it is our understanding that they base their rating not on the initial CPDO Nav, but from a stressed one, reducing therefore drastically the probability of rapid rating action. We tried to quantify this adjustment.

**■ It also show why a manager could minimize the CPDO drawdown**

In our framework, we can show how a good manager can actively reduce the likelihood of downgrades by reducing the maximum drawdown of the CPDO. Because we assume that a manager can only influence the information ratio of its active credit strategy and not the CPDO leverage, we also find the counter intuitive result that a good manager will extend the cash-in time rather than reduce it

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## CPDO insights on rating actions and manager impact using a new closed form formula

In our third piece regarding the new CPDO<sup>1</sup> asset class, we introduce a simplified model of CPDO which has the virtue of reducing itself to a closed form formula. This new flexibility is allowing us to develop some intuitions to some of the most frequently asked questions about CPDO:

1. What is the likelihood of downgrades?
2. What impact can an active manager have on a CPDO strategy?

It also allows us to revisit our analysis of one the main (and least understood) driver of risk of CPDO, i.e. the implied volatility of the market.

In a first part, we will introduce our model and its derived formulae (the very detailed math can be found in annex). Then, after a brief description of the short comings of our model, we define the 3 specific CPDO measures in the context of our model: Maximum drawdown, cash in time and cash out probability.

Then using our model we address the rating actions and manager impact. Our main conclusions are:

1. According to our simple model, CPDOs could be exposed early to rating downgrades if the market were to tank and implied volatility to rise, but similarly the CPDOs should be quickly upgraded (assuming his initial rating isn't AAA) if the market rallies early. In almost all scenarios a CPDO ends up upgraded up to AAA as soon as the early volatility / risk period has passed. We believe that this phenomenon is well understood and taken into account by rating agencies, it is our understanding that they base their rating not on the initial CPDO Nav, but from a stressed one, reducing therefore drastically the probability of rapid rating action.
2. In our framework, we can show how a good manager can actively reduce the likelihood of downgrades by reducing the maximum drawdown of the CPDO. Because we assume that a manager can only influence the information ratio of its active credit strategy and not the CPDO leverage, we also find the counter intuitive result that a good manager will extend the cash-in time rather than reduce it.

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<sup>1</sup> "CPDO an asset class on its own or a glorified bearish Rated Equity?", 12-Dec-06, Varloot & Charalampidou (Rolling CPDO)

"A Rating-floored 10Y Buy & Hold strategy makes better CPDOs", 2-Feb-06, Varloot (Buy and Hold CPDO)

## Two different ways of approaching this article

There are multiple ways of reading this article: one may want to skip through the mathematical formulae and focus on the charts and texts to get an intuition. Those readers could easily only read the four following pages:

- Impact of volatility,
- Fair Value of a CPDO,
- Rating Dynamic of a CPDO
- Impact of a CPDO manager

Or one may want to follow our formula derivation in details; in this case the model description and the mathematical annexes should be of interest.

## Reasons for using a CPDO model

CPDO is a path dependent product like CPPI and is sensitive to several market micro phenomena. Thus the first reaction when we saw the product was to build a realistic, yet complex, Monte Carlo engine to help us gain insight into its nature. This was extensively analysed in our previous publications. Monte Carlo has served its purpose but it also has limitations. It is prohibitively difficult to establish sensitivity / Greeks using MC and also some of the insights are blurred due to noise. This is the approach currently used by the rating agencies to rate those products.

We were intrigued by the idea that there could be a closed formula for the CPDO (or at least an approximation) in which case we would gain considerable flexibility and further important insights into the product. Our intuition was to see if the “CPDO” formula (Chart 1) could be reduced using simple Brownian diffusion models.

In this publication we present our modelling setup that gave us this analytical tractability and the power to address questions we couldn’t tackle before with the MC approach.

The more quantitative readers will probably frown when they will realize some underlying short cuts we did to compute on cash in time or our cash out probability. We are comfortable with those approximations as they allow leveraging efficiently the model to illustrate our points regarding rating actions and managed CPDOs rather than digging into more precise numerical resolutions. We are convinced that our formula can be extended and improved, CPDO research is in its infancy and to our knowledge this is the first attempt to formalize them through a mathematical approach.

### Chart 1 The CPDO Formula:

$$Lev = \frac{CashInAmount - Nav}{PV(returns)}$$

Source: UBS

## Introducing the CPDO formula

The change in NAV is the leveraged notional multiplied by the return of the long credit investment minus the coupon promised by the CPDO product

The change in the net asset value (NAV,  $N_t$ ) of the CPDO,  $dN_t$  is the return from the credit investment  $dS_t$  of a leveraged notional  $Lev_t$  minus the coupon paid to the investor  $c$ . We assume for simplicity that the coupon is paid continuously. Notice that we are abstracting ourselves from discounting back to time zero.

$$dN_t = Lev_t dS_t - c dt$$

The return from the credit investment is modelled as normal

The return from the long credit position  $dS_t$  is given by  $dS_t$  an arithmetic Brownian motion which models the uncertainty of the investment return of the long credit position. These returns are therefore modelled to be normal:

$$dS_t = m dt + s dW_t$$

Obviously there are tons of better models to consider (log normal, mean reverting, jumpy), but for each level of “realism” they bring, they introduce mathematical difficulties leading to numerical solution rather than closed form. The normal model is in fact not so badly suited to model a managed portfolio strategy with an information ratio of  $ir = \mu/s$

Leverage is the shortfall between the Cash in NPV (TBP) and the current NAV ( $N_t$ ) divided by the expected investment return and adjusted by a fudge factor.

The leverage of the CPDO at  $t$  ( $Lev_t$ ) is determined by the shortfall between the Cash in NPV (TBP) and the current NAV ( $N_t$ ) divided by the expected income from the long credit position over the remaining period.

$$Lev_t = \frac{TBP_t - N_t}{f m (T - t)}$$

The factor  $f$  is usually called “the CPDO fudge factor”. It is used to make sure that the average cash-in time comes sooner than the maturity of the product. Its direct consequence is that those structures are over collateralized in coupons and therefore can achieve very high ratings.

This formula should stress the major drawback of our simple model: the leverage isn't capped as it is in all the existing products. And it can become very big indeed close to maturity. A similar issue can be found in the close form formula of the CPPI. We will suggest in appendix some adjustments that can be done to limit this effect and still leading to close form formulae. They lead to more realistic results but the formula becomes nastier and somehow unintuitive, so we will stick to the simple model until then.

The shortfall to be paid (TBP) at time  $t$  is given by :

$$TBP_t = 100 + c (T - t)$$

In a CPDO, at time  $t$ , 100 is the value of the future libor payments and of the notional at maturity, this is a riskless par floater and therefore always worth par (We will use  $K=100$  in our equations). However during the remaining trade life  $(T-t)$  we also need to pay the coupon  $c$  whose value<sup>2</sup> is  $c (T-t)$ .

Bringing everything together we find the equation that specifies the evolution of the CPDO NAV ( $N_t$ ) (also in Chart 2).

$$dN_t = \frac{100 + c (T - t) - N_t}{m f (T - t)} (m dt + s dW_t) - c dt$$

From this equation we derive the NAV  $N_t$  at time  $t$ , its expected value  $E[N_t]$ , its variance  $Var[N_t]$  and most importantly its cumulative probability distribution  $N_{t,Q}$  at any time  $t$  for any percentile  $Q$ .

Chart 3 illustrates a simulated path of the model.

It is worth noting that the formula can also be expressed in terms of information ratio, it is therefore useful to test some assumptions regarding the quality of a manager.

$$dN_t = \frac{100 + c (T - t) - N_t}{f (T - t)} (dt + \frac{dW_t}{ir}) - c dt$$

**Our main result: the cumulative distribution at time  $t$ .**

We derive, in annex, the  $Q$  percentile distribution of  $N$  at time  $t$  as (Where  $F$  is the cumulative normal distribution):

$$N_{Q,t} = N_0 + c (T - t) - c T \left( \frac{T - t}{T} \right)^{\frac{1}{f}} \text{Exp} \left( - \frac{s^2}{2 m^2 f^2} \frac{t}{T (T - t)} \right) \text{Exp} \left( \frac{s}{f m} \sqrt{\frac{t}{T (T - t)}} F^{-1} (Q) \right)$$

**Calibrating the formula to market values**

We detail in Chart 4 the value of the various inputs and factors of our model we will use for our analysis going forward ( $K, N_0, c, f, m, s$  and  $T$ ). We consider the initial rolling version of the CPDO with a spread around 30bps. Using the results from our rock bottom analysis, we know that 12bps are due to default losses so our expected income from the long credit strategy is  $m=30\text{bps}-12\text{bps}=18\text{bps}=0.18$ ,  $c = 1$  for a CPDO that pays 100bps p.a.  $s$  the volatility of the returns of the credit strategy is given by the volatility of the spread (we take it to be 40%) multiplied by the dv01 of 5y Globxxx, 4.5 multiplied by the spread 30bps leading to a volatility of 54% ( $0.4*0.3*4.5=54\%$ ). The fudge factor  $f$  is used to adjust the leverage so that sometimes we leverage more than what we need to be able to match future payments. For security reasons, sometimes we may want to leverage less to limit initial volatility. There are variations of CPDOs of both types and the fudge factor allows us to gain some insight into how these variations of the leverage rules affects the product.

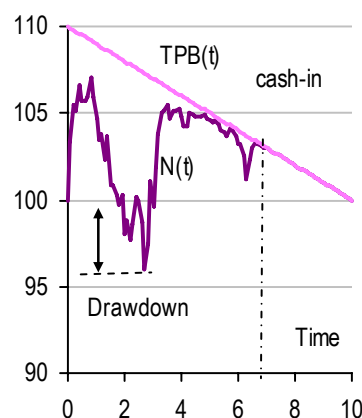
<sup>2</sup> We assume interest rates are zero as from our Monte Carlo analysis and general understanding about CPDO we knew before hand that interest rates do not affect the CPDO significantly. Thus for simplicity and tractability and so we decided to assume no discounting.

**Chart 2: The model for the change in the CPDO NAV,  $N_t$  at time  $t$**

$$dN_t = \frac{K + c (T - t) - N_t}{m f (T - t)} (m dt + s dW_t) - c dt$$

Source: UBS

**Chart 3: Illustrative path of the model**



Source: UBS

**Chart 4: Main model parameters**

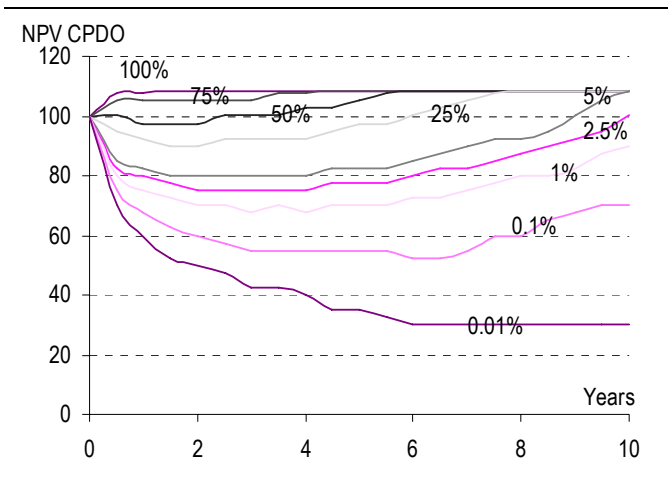
Variable	Description	Value
<b>K</b>	Redemption amount	100
<b>N<sub>0</sub></b>	NAV at start, time $t=0$	99
<b>c</b>	CPDO coupon above Libor	100bps
<b>m</b>	Growth rate per annum	0.18 (18bps)
<b>s</b>	Volatility	0.54
<b>T</b>	Maturity - 10 years	10
<b>f</b>	Leverage fudge factor	0.4

Source: UBS

# Faults and Virtues of our model

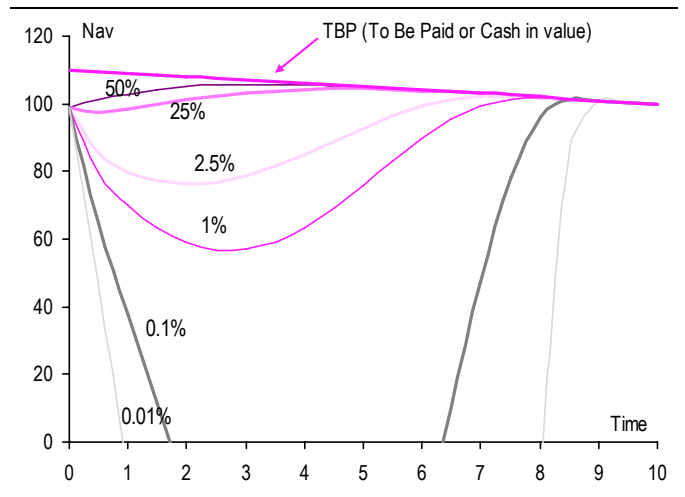
To establish if the model is good enough to approximate the NAV of the CPDO, we extracted the chart presenting the probability distribution of NAV at any time  $t$  and compared it to previous results we add from rolling strategies or static strategies. Chart 5 and Chart 6 compare the monte carlo results to the model results.

**Chart 5:NAV distribution as a function of time  $t$  (publication "CPDO an asset class on its own..." Dec '06 chart 11**



Source: UBS

**Chart 6:NAV distribution derived from the analytical model**



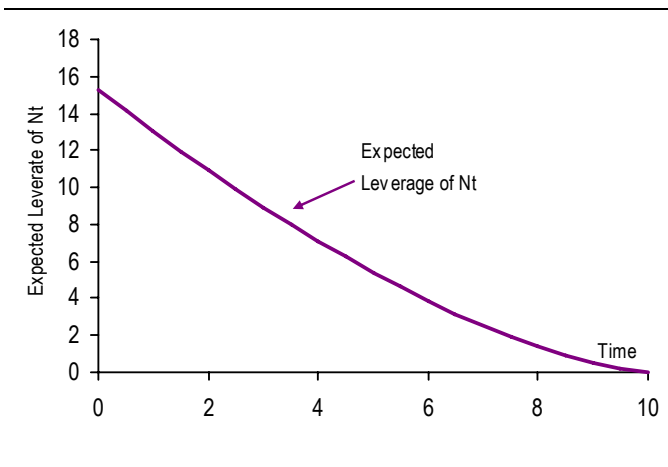
Source: UBS

First, the model does look like a CPDO, especially up to year 7 and for Nav greater than 60, i.e. for most of the realistic paths. (In fact it looks more like a second generation CPDO). It becomes weaker for very low Nav close to Cash out or in the last years very to close to maturity. This is linked to its boundless leverage as in both scenarios the leverage becomes much higher than the usual cap at 15. Nevertheless the expected leverage of the CPDO across the life of the product is given in Chart 7 and looks realistic, whereas the expected NAV as been calibrated (using the value of  $f$ ) to converge toward "cash-in" in year 5.

**Model limitations:**

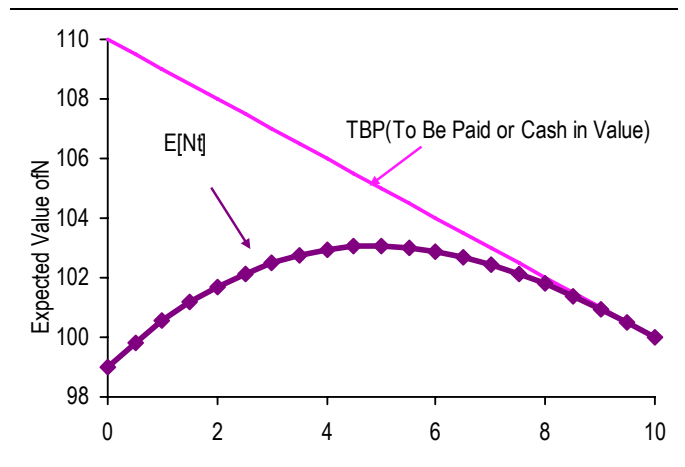
- Leverage is not capped and can become very high for low NAV or close to maturity
- Excess returns are simplistic (no jump to default costs, no fallen angel costs and no mean reversion or positive convexity.)

**Chart 7: Expected Leverage of the CPDO across time**



Source: UBS

**Chart 8:Expected NAV  $N_t$  at any time  $t$  and  $TBP_t$ .**



Source: UBS

# Model's Risk Measures

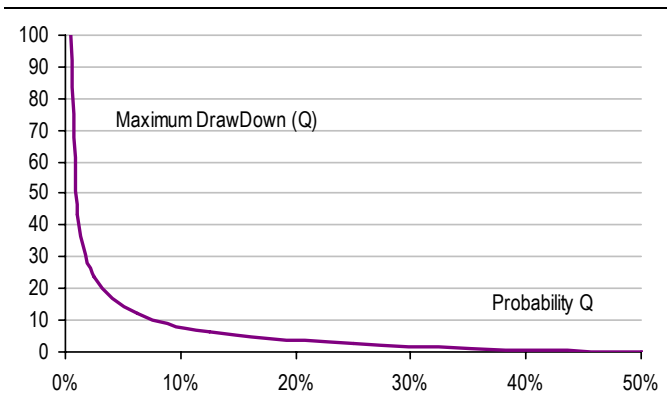
## Calculating Maximum drawdown

As discussed in our other publications, the maximum drawdown is the most meaningful measure of risk for a CPDO. This is the second question asked when one looks at a new CPDO, “how deep can it go?”, the first question being “what are the chances of it blowing up?”. In order to answer this question in a continuous modelling framework, we need to pick a percentile Q as the NAV N<sub>t</sub> can reach any value.

We therefore define the maximum drawdown for a specific percentile Q (e.g. 75%) as 100 minus the minimum NAV N<sub>t</sub> across time t for that percentile. For example, in Chart 6 the maximum drawdown for a certain Q is the minimum of the curve that corresponds to Q. Chart 9 presents the distribution of the max drawn down value, whereas Chart 10: illustrates the “path” of that maximum drawdown point over time. There isn't any closed form formula for this local minimum, but finding the minimum from our closed form percentile equation is trivial.

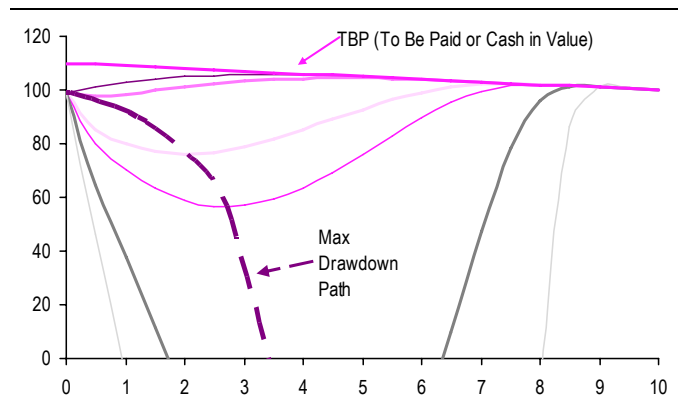
$$MaxDrawDown(Q = q) = N_0 - \text{Min}_t (N_{Q=q,t})$$

**Chart 9: Cumulative distribution for maximum drawdown**



Source: UBS

**Chart 10: maximum drawdown “path” over time**



Source: UBS

## Deriving "Cash in time" and "Cash out probability" from our formula

Because of its unbounded leverage, it should be clear that the model implies that the CPDO always cashes-in at time T. Other points are that (1) it never cashes in before T but the difference between  $TBP_t$  and  $N_t$  can become very small. (2) it never cashes out but  $N_t$  can become negative (3) it never finishes between cash in and cash out as the investor can always double up on the down side.

We will still be able to study those 2 important limit conditions, that is when does CPDO cash in and when does it cash out by approximating:

- The **cash-in time** as the time that the shortfall  $TBP_t - N_t$  equals to 1% of  $cT$  with a percentile Q of 50%, ( $cT$  is the initial gap between  $N_0$  and  $TPB_0$  neglecting fees).

$$timeCashIn = \text{Min}_t (TBP_t - N_{Q=50\%,t}) < 1\% \cdot cT$$

- The **cash out probability** as the maximum percentile Q that the curve  $N_{Q,t}$  lies above the Cash out NAV threshold (we picked 10 as the cash out threshold, i.e. if the CPDO NAV becomes 10 or less at any time  $t_c$  this is the cash out time)

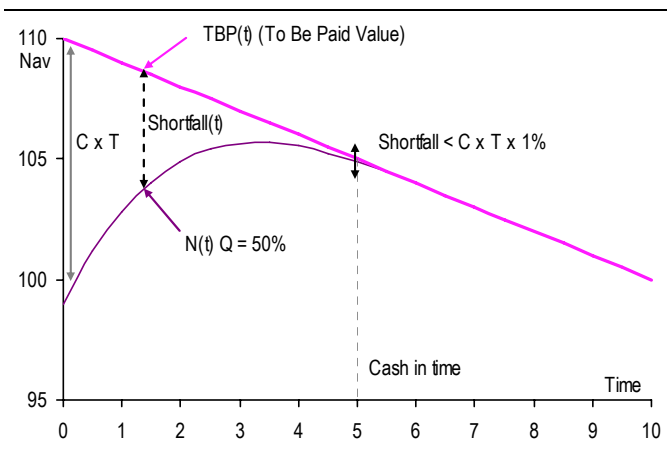
$$probaCashOut = 1 - \text{Max}_Q [N_{Q,t} \neq 10, "t \in (0, T)]]$$

In both cases we are fully aware that those are very rough approximations of the underlying option barrier problem. This approach tends to underestimate the cash out probability and overestimate the cash in time.

The Chart 11 and Chart 12 below illustrate the cash in time and cash out probability approximation computation.

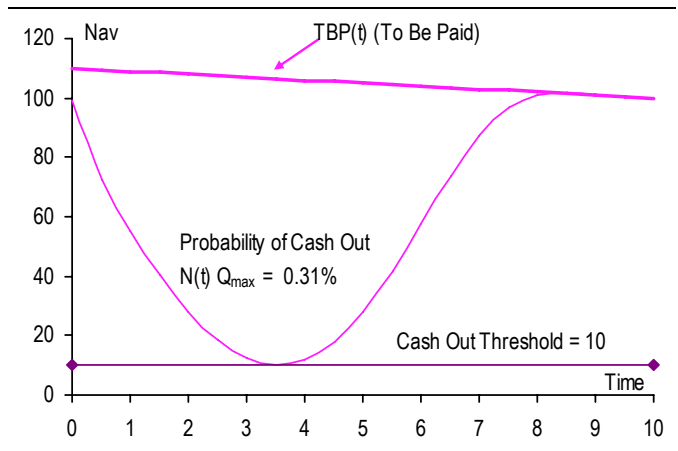
For cash out we find 0.31% with our hypothesis leading to an Aa3 equivalent rating at issuance. For cash in time we find 5.1 years.

**Chart 11: median Cash in time approximation**



Source: UBS

**Chart 12: Cash out Probability approximation**



Source: UBS

## Impact of volatility

First let us illustrate graphically what is the impact of volatility for this product. We are going to double the model implied volatility and see what is the impact on cash in time and cash out probability as defined before<sup>3</sup>. The flexibility of our model, allows us to consider any time  $t$  after issuance of the product ( $t > 0$ ) as well as any Nav for the product. We will assume that the Nav is now 90 at the end of year 2 for illustrative purpose.

The following charts illustrate the results. As expected, increasing volatility did increase our probability of cash out from 1% to 3%. What is more counter intuitive for most of us, is that the cash in time becomes “shorter”, and so would be understood as “better” by most investors. (it is reduced from 6.9 years to 5 years). This is because while increasing volatility generates more “downside scenarios” it also generates more “upside scenarios”. This fundamental result will have repercussions on our discussion about fair value and the impact of an asset manager on rating upgrades & downgrades.

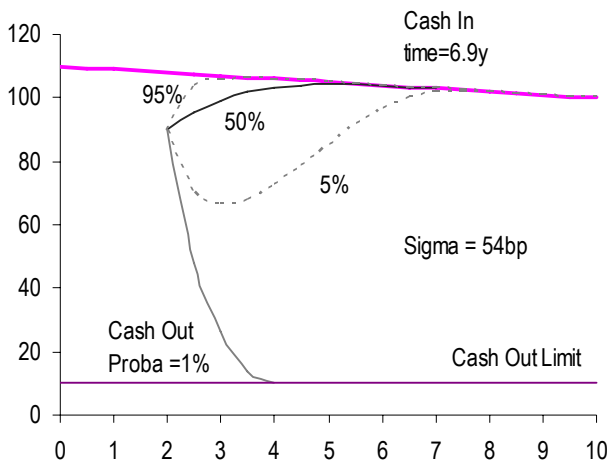
Another way to develop this counter intuitive result is to look at the expected path of  $N_t$  (Chart 8). It is interesting that the formula of this expected value (Chart 13) is not a function of volatility and would therefore be the evolution of the NAV of the CPDO if the markets were totally static. In this riskless case the cash in time is very long (year 8.5), and much longer than the median cash in time (year 5). This is a very well known result for all real CPDOs in the market.

Chart 13:  $E[N_t]$  formula

$$E[N_t] = K + c \cdot (T - t) - (K - N_0 + c \cdot T) \left( \frac{T - t}{T} \right)^{\frac{1}{\sigma}}$$

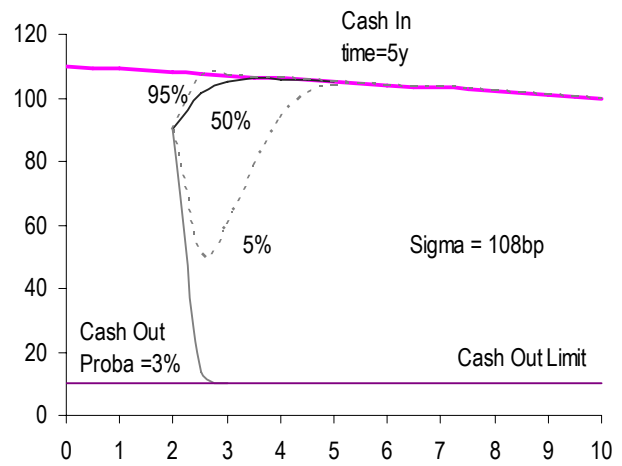
Source: UBS

Chart 14: Model Profile for a 90 Nav in year 2 with low vol



Source: UBS

Chart 15: Model Profile for a 90 Nav in year 2 with high vol



Source: UBS

<sup>3</sup> The nice thing about having a close form model is that doing that type of analysis is instantaneous, whereas current Monte Carlo models take at least 5 mins per simulation.

## Fair Value of a CPDO

We approximate the fair value of this CPDO at time  $t$  as the weighted sum conditional to cash out of:

- (1) TBPt if no cash out occurs. This time we can use the time value of money, with  $TBP_t = 100 + Dv01(T-t) \times \text{coupon}$ .
- (2) 10 (our cash out level) discounted cash out time.

This is rough, but allows us to look at the influence of volatility ( $\sigma$ ) on price.

$$P = (1 - \text{CashOut}(Nt)) (100 + dv01_t \cdot c) + \text{CashOut}(Nt) \text{Exp}(-r \cdot t_{\text{Cashout}})$$

Chart 16 aside illustrate the valuation of our model CPDO at issuance. We find Libor+96 bp as a “fair coupon”. (i.e. the expected average return of that CPDO).

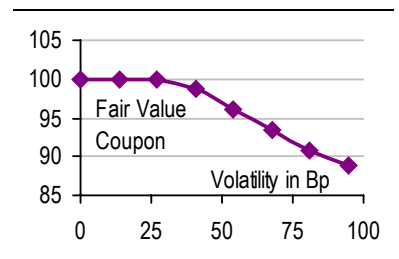
Following the same logic, we can check the “short vega” position of a CPDO using our model. Chart 17 presents the change of fair coupon as a function of sigma. The coupon is going down as volatility is going up, therefore the price of the structure is going down.

Chart 16: CPDO Fair Value.

	Price	Coupon	Proba
TBP	108.08	100	99.69%
Cash Out	8.69	-1130	0.31%
FairValue	107.77	96	100.00%

Source: UBS

Chart 17: Change of Fair Value as a function of volatility.



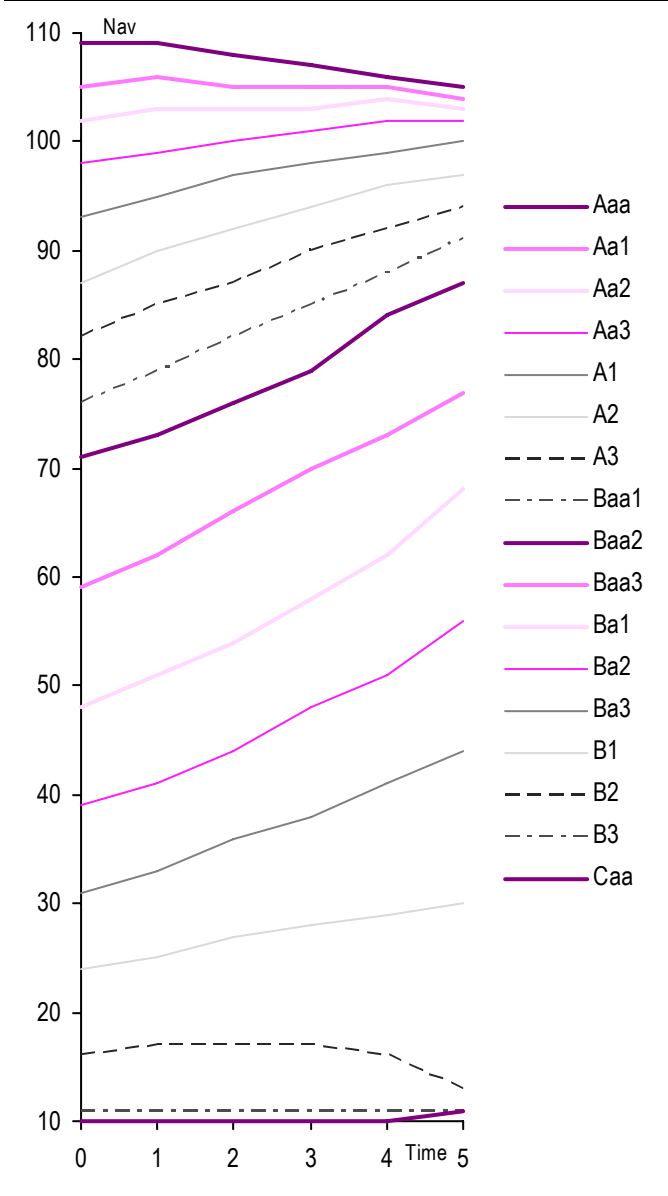
Source: UBS

## Rating dynamic of CPDO.

Another investor understandable concern is the rating stability of a CPDO. Our model allows us to illustrate this. Because we have a probability of default for any level of Nav at any time, we can map that point to a rating using the agencies mapping. Chart 18 present the results using our inputs. As we established before, our initial ratio is Aa3, but it is interesting to see that we would probably lose one notch if the NAV falls to 98 according to our model. (We will see in the next chapter that it isn't the case in real life).

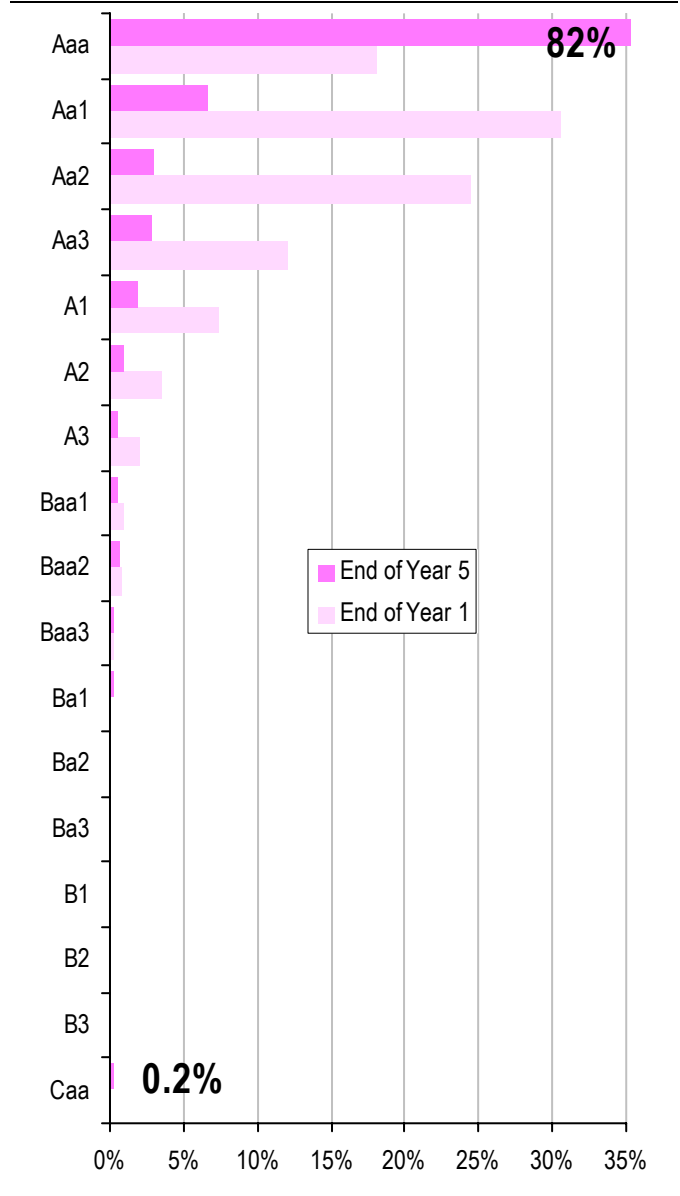
Similarly, using our drawdown approach, we can compute the probability of reaching a given rating at a given date. Chart 19 illustrates this point focussing on the early years where our model is more stable. It appears as time passes by that the CPDO is more and more likely to be upgraded to AAA. Interestingly this model can generate immediate downgrades by just raising implied volatility.

**Chart 18: Nav to Rating Mapping.**



Source: UBS

**Chart 19: Rating distribution as a function of time**



Source: UBS

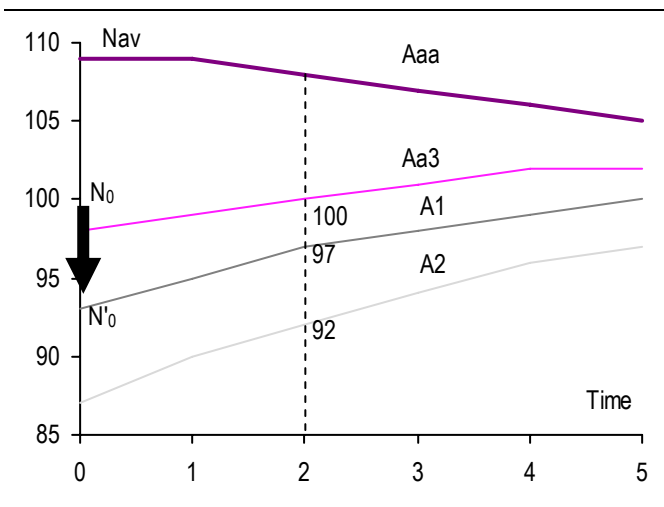
## Agencies solutions to mitigate rating actions

We have just shown that a modest change in the level of the Nav  $N_0$  soon after the issuance of the CPDO would generate an early upgrade / downgrade according to our model. Obviously this is neither realistic nor welcome. It is our understanding that the agencies are fully aware of this theoretical rating volatility and we believe that have protected themselves by using very conservative hypothesis:

1. We believe that they are replacing the initial Nav  $N_0$  by a distressed  $N'_0$ , with  $N'_0$  sensibly lower than  $N_0$ . In order to illustrate the impact of this adjustment in our model, we will assume a reduction of 5% from  $N_0$ . Now, referring to our mapping  $N_t$  to rating, this CPDO would not be initially rated Aa3 but A1 if not A2. This creates a substantial cushion and reduces the probability of downgrade by year 2 from 21.3% to 8.3% (again according to our model and assuming an initial A2 rating).

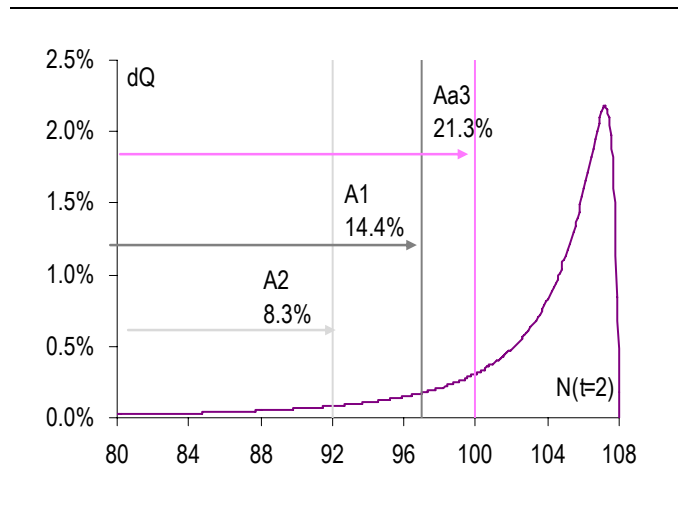
Let us illustrate this calculation. First using our “rating map” moving  $N_0$  from 99 to 94 pushes the rating from Aa3 to A1 or A2 (Chart 20). The same chart shows that  $N_t$  needs to be above 100 to get a Aa3 rating by year 2, 97 for A1 and 92 for A2. Using those value in the  $N(Q,2)$  function we can infer that if we started with a Aa3 rating our chances of downgrades are 21.3%, 14.4% for A1 and 8.3% for A2 (Chart 21).

**Chart 20: Impact of using 95% of  $N_0$  to compute initial rating**



Source:UBS

**Chart 21: Rating distribution by year 2 as a function of N**



Source:UBS

2. We also believe that the agencies use historical realised volatility in their model rather than market implied ones<sup>4</sup>. This means that they should only readjust their volatility assumption if something major was to happen in the overall structure of the market. Also their model already assumes a changing volatility if spreads are widening as it is a log normal model, therefore reducing the frequent need of changing the volatility assumptions.

<sup>4</sup> This would be difficult anyway as only short dated implied volatility is observable in the CDS swaption market.

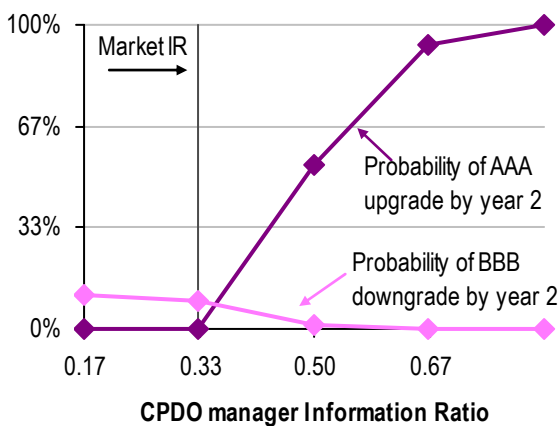
### Impact of a CPDO manager.

Every bank seems to be keen to sell managed CPDOs at the moment. But a fair question is “what for?”. At the end of the day, a CPDO will most likely cash in and pay its agreed coupon. A CPDO presents no upside (contrary to a CPPI or a rated equity, or any first lost product) and very little down side in the long run, therefore the only added value a manager can bring is a reduction of the mark-to-market risk. (In our model, risk is quantified by the maximum drawdown (Chart 10) and the related cash out level (Chart 12)). It is also worth noting that this general statement can be extended to the senior tranches of managed CDO.

So assuming our manager does a good job and does improve the information ratio of the source of credit excess return which is leveraged by the CPDO, what really does happen according to our model? Well, for a given level of returns, the volatility on the credit process goes down. We are therefore confronted to the same discussion on volatility we covered before (Chart 14 & Chart 15). The maximum drawdown will go down – which is good news - but the CPDO cash in time will be extended! Therefore if one picks a very good manager, their relationship may last much longer than expected! The solution of this catch 22 should be to let the manager actively manage its leverage as well as the underlying credit strategy. In that case the manager should try to under-leverage relatively to the CPDO default formula if he were to anticipate a spread widening and over-leverage on a rally Also an active long-short manager may achieve a better than expected credit excess return  $m^* > m$ . In this case the cash in time could also occur sooner than implied by our formula. Simulating those two effects is outside the scope of our simple model.

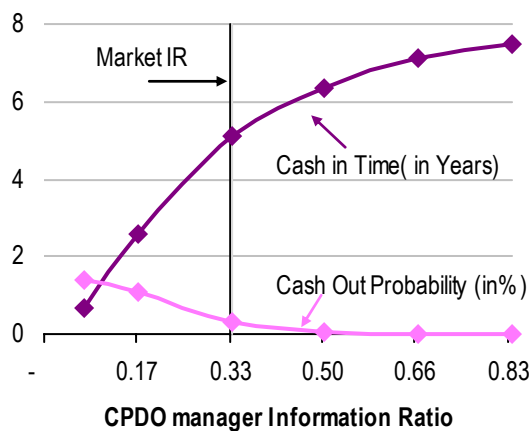
The good news is that a good manager will definitely accelerate upgrades and reduce downgrades as ratings are driven by maximum drawdown probabilities. Chart 22 and Chart 23 illustrate the impact in terms of up / downgrades as a function of the manager’s information ratio (assuming he has no influence on its leverage), as well as the impact on cash in time and maximum drawdown.

**Chart 22: chance of Aa1 and above upgrade & BBB or lower downgrade by year 5 as a function of the manager info ratio.**



Source: UBS

**Chart 23: Maximum drawdown and Cash in time as a function of the manager information ratio.**



Source: UBS

## Technical Appendix

Presenting our model: The shortfall between the Cash-in amount and the NAV at  $t$  is modelled as lognormal.

The change in the CPDO's net asset value (NAV)  $dN_t$  is modelled as the leverage on the underlying strategy at  $t$  multiplied by the return of this strategy minus the CPDO coupon. The leverage is set by dividing the shortfall between the cash in value and the current NAV by the average expected income from the strategy earned until maturity. Therefore the SDE is given by:

$$dN_t = \frac{K + c (T - t) - N_t}{f m (T - t)} [m dt + s dW_t] - c dt$$

Where  $K, c, T, f, m, s$  are constants and  $dW_t$  is a Brownian motion, we chose an arithmetic Brownian motion. We use the following substitution to allow us to get the formula for the NAV at time  $t$   $N_t$ :

$$L_t = K + c (T - t) - N_t \Rightarrow dL_t = -c dt - dN_t$$

Note that  $L_t$  in reality is the shortfall between the cash in value ( $TBP=K+c(T-t)$ ) and the current NAV  $N_t$  at any time  $t \in (0, T)$ . We now have the following SDE to solve:

$$dL_t = -\frac{L_t}{f m (T - t)} [m dt + s dW_t]$$

Solving the SDE to derive  $N_t$ , the NAV distribution at any time  $t \in (0, T)$

The SDE above when solved gives us a lognormal distribution for the shortfall  $L_t$ . The solution is easily derived by using the substitution  $G_t = \ln(L_t)$ .  $G$  is a  $C^2$  (twice differentiable with continuous derivatives) function and a non stochastic function of  $L_t$  and therefore we can apply Ito's lemma:

$$G_t = \ln(L_t)$$

$$dG_t = \frac{\partial G}{\partial L_t} dL_t + \frac{1}{2} \frac{\partial^2 G}{\partial L_t^2} \langle L_t \rangle^2$$

$\langle L_t \rangle^2$  is the quadratic variation for the stochastic process  $L_t$  and is given below:

$$\langle L_t \rangle^2 = \frac{L_t^2}{f^2} \left( \frac{s}{m} \right)^2 \frac{1}{(T - t)^2} dt$$

$$\frac{\partial G}{\partial L_t} = \frac{1}{L_t}, \quad \frac{\partial^2 G}{\partial L_t^2} = -\frac{1}{L_t^2}$$

Below we present in a detailed manner all the steps to derive the value of  $N_t$ .

$$dG_t = -\left( \frac{1}{f (T - t)} + \frac{1}{2} \frac{1}{f^2} \left( \frac{s}{m} \right)^2 \frac{1}{(T - t)^2} \right) dt - \frac{1}{f} \left( \frac{s}{m} \right) \frac{1}{(T - t)} dW_t$$

$$G_t - G_0 = \int_0^t dG_s = -\frac{1}{f} \int_0^t \frac{1}{T - s} ds - \frac{1}{2} \frac{1}{f^2} \left( \frac{s}{m} \right)^2 \int_0^t \frac{1}{(T - s)^2} ds - \frac{1}{f} \left( \frac{s}{m} \right) \int_0^t \frac{1}{T - s} dW_s$$

The authors want to thank **Antonella Buccigliaglia** and **George Tsouderos** for their assistance

$$G_t = G_0 + \frac{1}{f} \left[ \text{Ln}(T-s) \right]_0^t - \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \left[ \frac{1}{(T-s)} \right]_0^t - \frac{1}{f} \left( \frac{s}{m} \right) \int_0^t \frac{1}{T-s} dW_s$$

$$\text{Ln}(L_t) = \text{Ln}(L_0) + \frac{1}{f} \text{Ln}\left(\frac{T-t}{T}\right) - \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} - \frac{1}{f} \left( \frac{s}{m} \right) \int_0^t \frac{1}{T-s} dW_s$$

Now we can have an expression for  $L_t$  and thus  $N_t$ :

$$N_t = K + c (T-t) - (K - N_0 + c T) \left( \frac{T-t}{T} \right)^{\frac{1}{f}} \text{Exp} \left( - \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} - \frac{1}{f} \left( \frac{s}{m} \right) \int_0^t \frac{1}{T-s} dW_s \right)$$

Deriving the first two moments of the NAV distribution at any time t.

The formula that gives the CPDO NAV  $N_t$  at any time t is a very powerful tool. To gain some insights and to demonstrate the derivation of the first two moments of  $N_t$  we rewrite the equation above as:

$$N_t = K + c (T-t) - (K - N_0 + c T) \left( \frac{T-t}{T} \right)^{\frac{1}{f}} \text{Exp} \left( - \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} \right) A_t$$

In the equation for  $N_t$  the only stochastic part is  $A_t$  which is the exponential of an Ito integral  $X_t$ .

$$A_t = \text{Exp} \left( - \frac{1}{f} \left( \frac{s}{m} \right) \int_0^t \frac{1}{T-s} dW_s \right) = \text{Exp}(X_t) \quad X_t = - \frac{1}{f} \left( \frac{s}{m} \right) \int_0^t \frac{1}{T-s} dW_s$$

It is easy to see that the Ito integral  $X_t$  in the exponent is a Gaussian process. Specifically:

$$E[X_t] = 0$$

$$\text{Var}[X_t] = E[X_t^2] - (E[X_t])^2 = E \left[ \frac{1}{f^2} \left( \frac{s}{m} \right)^2 \left( \int_0^t \frac{1}{T-s} dW_s \right)^2 \right] = \frac{1}{f^2} \left( \frac{s}{m} \right)^2 E \left[ \left( \int_0^t \frac{1}{T-s} dW_s \right)^2 \right]$$

To proceed from this point we need to deploy the Ito's isometry that states that for a stochastic process  $z_t$  adapted to the filtration of the Brownian  $W_t$ , the following isometry holds.

$$E \left[ \left( \int_0^t z_s dW_s \right)^2 \right] = E \left[ \int_0^t z_s^2 ds \right]$$

In our case  $z_t$  is non stochastic and  $z_t = \frac{1}{T-t}$ , therefore using the Ito isometry we have:

$$E \left[ \left( \int_0^t \frac{1}{T-s} dW_s \right)^2 \right] = E \left[ \int_0^t \left( \frac{1}{T-s} \right)^2 ds \right] = \int_0^t \left( \frac{1}{T-s} \right)^2 ds = \left[ \frac{1}{T-s} \right]_0^t = \frac{1}{T-t} - \frac{1}{T} = \frac{t}{T(T-t)}$$

$$E[X_t] = 0 \quad \text{Var}[X_t] = \frac{1}{f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)}$$

$$A_t = \text{Exp}(X_t) \quad X_t \text{ is } N \left( 0, \frac{1}{f} \left( \frac{s}{m} \right) \sqrt{\frac{t}{T(T-t)}} \right)$$

We therefore have derived the first two moments of  $X_t$  which is a normal variable. Since  $A_t$  is a lognormal variable and we have the first two moments of its exponent  $X_t$  we also have the first two moments of  $A_t$  if we use the following general mathematical lemma :

$$V = N(m', s') \Rightarrow E[Y] = \text{Exp}\left(m' + \frac{s'^2}{2}\right)$$

$$Y = \text{Exp}(V) \Rightarrow E[Y^2] = \text{Exp}\left(2\left(m' + \frac{s'^2}{2}\right)\right) \Rightarrow \text{Var}[Y] = \text{Exp}\left(2\left(m' + \frac{s'^2}{2}\right)\right) - \text{Exp}\left(2m' + s'^2\right)$$

In our case  $V$  is  $X_t$  and  $Y$  is  $A_t$  and therefore we have

$$E[A_t] = \text{Exp}\left(\frac{1}{2} \frac{\left(\frac{s}{m}\right)^2}{f^2} \frac{t}{T(T-t)}\right)$$

$$\text{Var}[A_t] = \text{Exp}\left(\frac{1}{f^2} \left(\frac{s}{m}\right)^2 \frac{t}{T(T-t)}\right) \left[ \text{Exp}\left(\frac{1}{f^2} \left(\frac{s}{m}\right)^2 \frac{t}{T(T-t)}\right) - 1 \right]$$

Using the formula for  $N_t$  and the above moments for  $A_t$  which is the stochastic part of  $N_t$  we naturally arrive to the following formulas for the two first moments of  $N_t$  which is what we were after all along.

$$E[N_t] = K + c(T-t) - (K - N_0 + cT) \left(\frac{T-t}{T}\right)^{\frac{1}{f}}$$

$$\text{Var}[N_t] = (K - N_0 + cT)^2 \left(\frac{T-t}{T}\right)^{\frac{2}{f}} \left[ \text{Exp}\left(\frac{1}{f^2} \left(\frac{s}{m}\right)^2 \left(\frac{t}{T(T-t)}\right)\right) - 1 \right]$$

Deriving  $N_{Q,t}$  which is the NAV level such that there is  $Q$  probability the NAV at  $t$  will be higher than that level  $N_{Q,t}$ .

Since we know the distribution of the NAV at time  $t$ ,  $N_t$  we can easily derive the probability of  $N_t$  being greater than any level. This ability is central to our analysis. We define  $N_{Q,t}$  as the level of NAV  $N_t$  that is such that there is  $Q$  probability that the NAV will be higher than that level  $N_{Q,t}$  at time  $t$ . Formally :

$$\Pr[N_t \geq N_{Q,t}] = Q$$

We give below in detail the steps leading to the formula for  $N_{Q,t}$ :

$$\Pr\left[ K + c(T-t) - (K - N_0 + cT) \left(\frac{T-t}{T}\right)^{\frac{1}{f}} \text{Exp}\left(-\frac{1}{2} \frac{\left(\frac{s}{m}\right)^2}{f^2} \frac{t}{T(T-t)}\right) A_t \geq N_{Q,t} \right] = Q$$

$$\Pr\left[ A_t \geq \frac{K + c(T-t) - N_{Q,t}}{K - N_0 + cT} \left(\frac{T-t}{T}\right)^{\frac{1}{f}} \text{Exp}\left(\frac{1}{2} \frac{\left(\frac{s}{m}\right)^2}{f^2} \frac{t}{T(T-t)}\right) \right] = Q$$

$$\Pr\left[ \text{Exp}(X_t) \geq \frac{K + c(T-t) - N_{Q,t}}{K - N_0 + cT} \left(\frac{T-t}{T}\right)^{\frac{1}{f}} \text{Exp}\left(\frac{1}{2} \frac{\left(\frac{s}{m}\right)^2}{f^2} \frac{t}{T(T-t)}\right) \right] = Q$$

We know that  $X_t$  is normal with zero mean and a standard deviation  $\text{stdev}(X_t) = s/m/f \sqrt{t/(T-t)}$ . Considering the standard normal random variable  $S_t$  we thus have:

$$\Pr \left[ \frac{1}{f} \left( \frac{s}{m} \right) \sqrt{\frac{t}{T(T-t)}} S_t \leq \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} + \text{Ln} \left( \frac{K+c(T-t)-N_{Q,t}}{K-N_0+cT} \left( \frac{T-t}{T} \right)^{\frac{1}{f}} \right) \right] = Q$$

$$\Pr \left[ S_t \leq f \left( \frac{m}{s} \right) \sqrt{\frac{T(T-t)}{t}} \left[ \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} - \frac{1}{f} \text{Ln} \left( \frac{T-t}{T} \right) + \text{Ln} \left( \frac{K+c(T-t)-N_{Q,t}}{K-N_0+cT} \right) \right] \right] = Q$$

Considering the inverse function of the standard normal probability distribution  $F^{-1}(Q)=Y$   $\Pr[X<Y]=Q$  where X standard normal random variable we have:

$$F^{-1}(Q) = \frac{m}{s} f \sqrt{\frac{T(T-t)}{t}} \left[ \frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} - \frac{1}{f} \text{Ln} \left( \frac{T-t}{T} \right) + \text{Ln} \left( \frac{K+c(T-t)-N_{Q,t}}{K-N_0+cT} \right) \right]$$

$$N_{Q,t} = K+c(T-t) - (K-N_0+cT) \left( \frac{T-t}{T} \right)^{\frac{1}{f}} \text{Exp} \left( -\frac{1}{2 f^2} \left( \frac{s}{m} \right)^2 \frac{t}{T(T-t)} + \frac{1}{f} \frac{s}{m} \sqrt{\frac{t}{T(T-t)}} F^{-1}(Q) \right)$$

The formula above is central for the results of this publication.

For a certain probability Q,  $N_{Q,t}$  is a function of time and it gives us a curve for a certain percentile. These curves for different value of Q can be seen in Chart 6.

We define as the maximum drawdown MD(Q) with respect to a probability Q :

$$MD(Q) = N_0 - \text{Min}_t (N_{Q,t})$$

While the max drawdown time for Q MDT(Q) is such that:

$$N_{Q,MDT(Q)} = \text{Min}_t (N_{Q,t})$$

We are aware that the ways we define the concepts of max drawdown and cash in time or cash out probability are not necessarily equivalent to the corresponding concepts we had when we did our Monte Carlo analysis where everything was based for every Monte Carlo path. We seek understanding of the product and what exactly statistical measures we use to understand the distribution of the stochastic process behind is of secondary value. It seems that in this analysis we have replaced the concept of a Monte Carlo path with the concept of the curve  $N_{Q,t}$ .

### Better Models?

As we have seen throughout the piece the main weakness of our formulae is its implied massive leverage close to maturity or for very low NAV. We currently are toying with alternative model specifications to reduce that effect. One idea we are currently testing is to replace the leverage formulae by a less aggressive function while calibrating the initial leverage to a realist value. For example one can replace in the leverage formula (1) the linear distance between  $TBP_t$  and  $N_t$  by its square root in the nominator to reduce growth (2) T by a longer  $T^*$  in the denominator to avoid having a zero value at time T, this still leads a closed form formula but the lognormal is replaced by a chi2 of first order.

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